THRESHOLDS OF THE HEAD-TAIL INSTABILITY IN BUNCHES WITH SPACE CHARGE

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INTRODUCTION

Head-tail instabilities are expected to be one of the main limitations of the high-intensity operation in the future SIS100 synchrotron of the FAIR facility [1], especially for the heavy-ion bunches [2]. This instability is already beginning to limit the operation at the highest intensities in the ISIS spallation neutron source [3] at the Rutherford Appleton Laboratory in the UK. General ISIS bunch parameters, especially the space-charge conditions, are similar to the expected heavy-ion beams in SIS100, thus it might be possible to use the physical insight and the experience from the ISIS studies for anticipating the transverse stability in the SIS100 high-intensity bunches. Of particular interest is the dependence of unstable beam modes on the configuration of the RF system (single or dual harmonic), the influence of high space charge levels, the key role of the betatron tune and the determination of driving beam impedances.

OBSERVATIONS IN ISIS

A dedicated experimental campaign of three shifts has been performed at the ISIS synchrotron in November 2013, with the primary goal to understand more about the fast losses and associated vertical oscillations around 2 ms of the ISIS cycle, see Fig. 1. These losses are a concern for the high-intensity operation and have been usually attributed to head-tail instabilities. In standard ISIS operation, a 2RF system is used. In order to be able to compare with classical theories, and to simplify the first comparisons with simulations, the most of the study was made with the 1RF ($h = 2$) operation. Approximately one-third of the measurements were done with different types of 2RF ($h = 2, 4$).

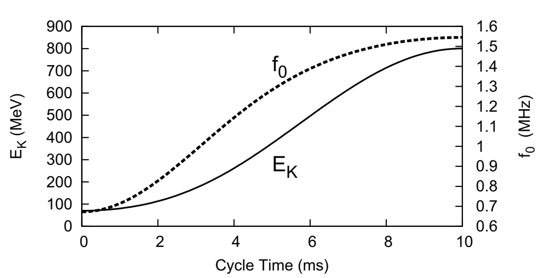

Figure 1: ISIS cycle: the proton kinetic energy (solid line) and the revolution frequency (dashed line), $C = 163.26$ m.

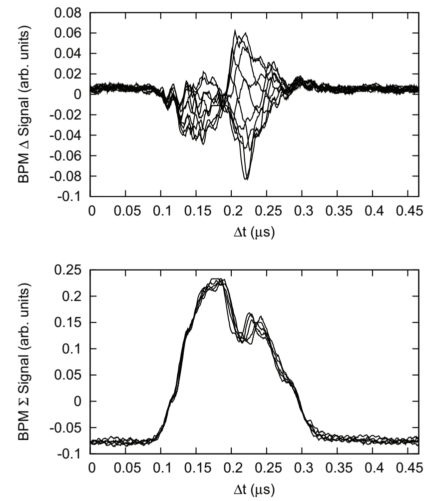

Figure 2: Consecutive bunch traces from the vertical BPM delta (top) and sum (bottom) signal of a typical instability for a 1RF bunch around Cycle Time 2 ms.

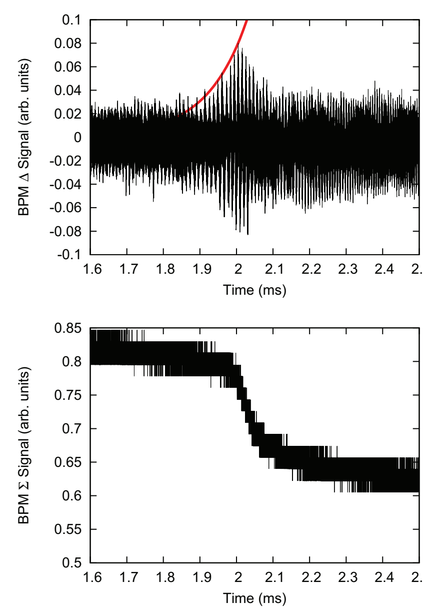

Figure 3: Time evolution of the BPM delta (top) and sum (bottom) signal for the instability from Fig. 2. The red line is an exponential with the growth time $\tau = 0.1$ ms.

According to the experience at ISIS [4–6], the instability appears if the vertical tune is set closer to integer from below. The normal tune ramp at ISIS applies $Q_v = 3.85$ at 0 ms decreasing to $Q_v = 3.68$ at 10 ms, with $Q_v = 3.758$ at 2 ms. In order to focus on the operation-type instability, we have pushed the vertical tune higher around 2 ms cycle time. Once tune reaches $Q_v \approx 3.86$, reproducible strong losses and vertical collective oscillations appear. Figures 2, 3 present typical BPM signals around 2 ms. The over-plotted bunch traces in Fig. 2 show a standing wave pattern

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with one node, with chromaticity wiggles inside. The oscillation grows with the growth time $\tau = 0.1$ ms (Fig. 3), until it is distorted by beam losses. This proves a classical unstable $k = 1$ head-tail mode. In the majority of the recorded signals, it was hard to determine a clear growth rate, probably due to early losses. This might be attributed to large beam sizes which intentionally maximise use of aperture. More examples of the instabilities in ISIS, in 1RF and in 2RF, are given in the slides of this presentation.

For every beam and machine parameter set, an intensity scan has been performed.

The results of the intensity scans are summarized in Fig. 4. The left plot in Fig. 4 is dedicated to the 1RF instability around 2 ms (red) and around 3 ms (blue). The right plot in Fig. 4 compares different RF settings, the flat-bunch 2RF (or lengthening mode) around 2 ms, and the operation-type 2RF (stable asymmetric settings) around 2 ms. There is no instability at low intensities, which is a usual observation for collective instabilities, due to natural non-linearities in the machine optics. Hence, there is the common "bottom" thresholds. Surprisingly, it was observed that instabilities vanish above certain intensities, which we describe as "top" thresholds. This phenomenon was clearly observed and reproducible, in intensity scans upwards as well as downward.

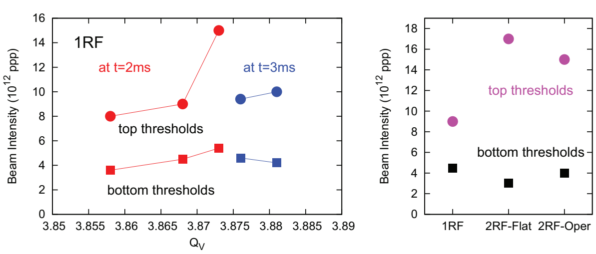

Figure 4: Summary for the intensity thresholds of the head-tail instability observed in ISIS. Beam loss and oscillations appear between the bottom and the top thresholds.

ANALYTICAL CALCULATIONS

We use the theory of Sacherer [7] in order to identify the driving impedance and to estimate the growth rates. The center of the power spectrum is shifted by $\Delta f = f_0 Q\xi/\eta$, $\xi = -1.4, \gamma_t = 5.034$, see Fig. 5. The positive frequencies in the plot do not contribute to the instability drive, according to this theory. In the case of the ISIS bunches, with the full bunch length $t_b \approx 200$ ns around 2 ms, the head-tail modes $k = 2$ should be unstable. In order to explain the observed $k = 1$ modes, an effective bunch length $0.5t_b$ has been assumed. The observation that the instabilities appear as the tune approaches integer from below, is a strong indication that an impedance with a Resistive-Wall frequency dependence $\text{Re}(Z_\perp) \propto 1/f$ (indicated with the black line in Fig. 5) should be the driving force. However, if we assume the Resistive Thick-Wall Impedance with the pipe radius $b_{\text{pipe}} = 50$ mm (i.e., overestimated impedance), much slower instabilities than observed (τ <20 ms) are predicted, see Fig. 5.

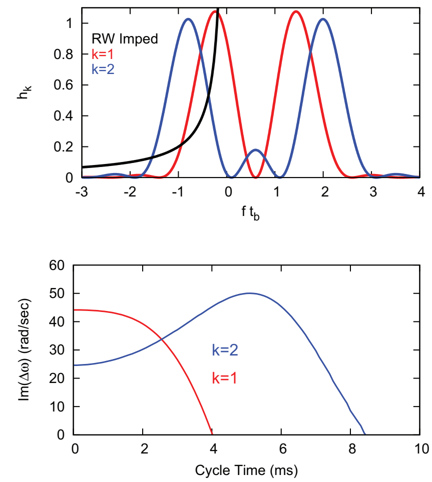

Figure 5: Calculations with the Sacherer theory for the ISIS 1RF bunches. Top: power spectrum of the head-tail modes (see [7]) and the real part of the normalized Resistive-Wall impedance in the unstable range. Bottom: the resulting growth rates.

SPACE CHARGE AND LANDAU DAMPING

The effect of self-field space-charge on head-tail modes in bunches can be analytically solved for an airbag bunch [8]. This theory is also very useful and rather accurate for Gaussian bunches, as it was observed in particle tracking simulation [10] and in experiments in the SIS18 synchrotron at GSI Darmstadt [11]. Landau damping due to space-charge in bunches was predicted analytically in [12] and confirmed with simulation studies in [10].

The additional effect of a coherent tune shift, for example from the imaginary impedance of image charges, has been included into the airbag theory in [9],

$$\Delta Q_k = -\frac{\Delta Q_{\text{sc}} + \Delta Q_{\text{coh}}}{2} \pm \sqrt{\frac{(\Delta Q_{\text{sc}} - \Delta Q_{\text{coh}})^2}{4} + k^2 Q_s^2}\,, \quad (1)$$

where "+" is for modes $k \geq 0$, the notation corresponds to [11].

It is suggested that the combination of space-charge and coherent force should have an effect on Landau damping. Figure 6 shows the head-tail modes with and without a coherent tune shift, and the border of the active Landau damping area, $\Delta Q_{\max} = -0.23 Q_s q + k Q_s$ [11]. The lines are shifted by $(-k)$ for a better comparison. A head-tail mode is affected by Landau damping if the frequency is below damping border, the dashed line in Fig. 6. We see that the $k = 0$ mode is not damped (it is not affected by space-charge, see the left plot). The $k = 1$ mode is damped for $0 \lesssim q \lesssim 3$ without coherent force, and for $0 \lesssim q \lesssim 6$ with ΔQ_{coh}. The $k = 2$ mode is damped for $0 \lesssim q \lesssim 6$ without coherent force, and for $0 \lesssim q \lesssim 12$ with ΔQ_{coh}.

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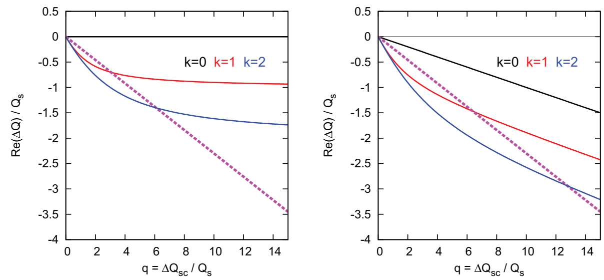

Figure 6: Effect of the image charges on the head-tail modes according to the airbag theory. Left plot: $\Delta Q_{\rm coh} = 0$, right plot: $\Delta Q_{\rm coh} = 0.1\Delta Q_{\rm sc}$. The dashed line indicates the border of Landau damping.

In order to observe the modifications of Landau damping due to image charges, we consider the mode $k = 1$ and compare the areas of strong damping for different strength of the coherent forces. According to the airbag theory and the Landau damping border prediction, with $\Delta Q_{\rm coh} = 0.1\Delta Q_{\rm sc}$ the area of damping should be larger than for the case without image charges, see Fig. 7. For $\Delta Q_{\rm coh} = 0.2\Delta Q_{\rm sc}$ the coherent line remains under damping for all q values considered in Fig. 7. We perform PIC simulations using the code PATRIC [13], similar to the work reported in [10], but with the coherent effect included, which varies along the bunch according to the local slice line density. The simulation results are summarized in Fig. 8. This shows a good qualitative confirmation of the effect of image charges on the space-charge induced Landau damping.

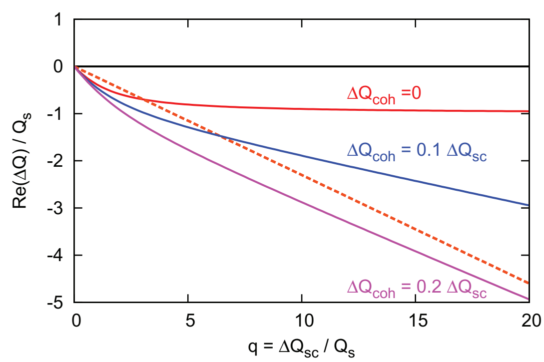

Figure 7: Effect of the image charges on the $k = 1$ head-tail mode according to the airbag theory. The dashed line indicates the border of Landau damping.

LANDAU DAMPING IN THE ISIS BUNCHES

The parameters of the bunches in ISIS correspond to the regime of rather strong space-charge, as demonstrated in Fig. 9 with the estimations for the parameters in our 1RF experiments. The main uncertainty is associated with the transverse emittance, here we have assumed the rms

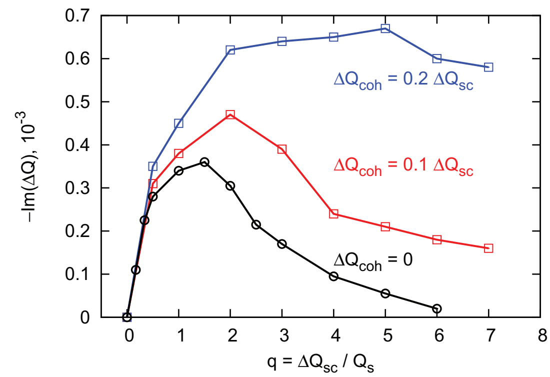

Figure 8: Damping decrement of the $k = 1$ head-tail mode obtained from the PATRIC simulations for a Gaussian (longitudinally and transversally) bunch, $Q_s = 0.01$, for different strengths of the coherent tune shift due to image charges.

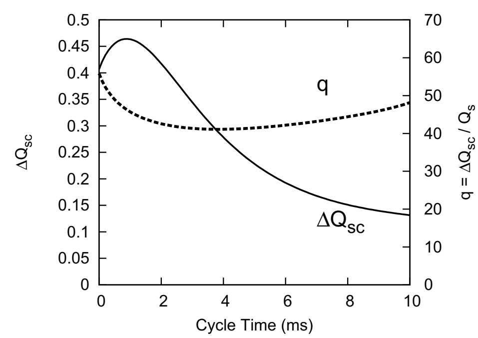

Figure 9: Space-charge tune shift and the space-charge parameter for the 1RF bunches in ISIS at the beam intensity 4×10^{12} ppp.

non-normalized $\varepsilon = 50$ mm mrad at the start of the cycle, in the vertical plane. The space-charge parameter $q = \Delta Q_{\rm sc}/Q_s \gg 1$ implies strong space-charge regime for the bunch head-tail dynamics [10, 12]. We can also conclude that, although the space-charge tune shifts decrease during the ramp, the space-charge parameter stays stable around $q \approx 40$ in this case.

The ISIS vacuum pipe is a rectangular, conformal, stainless steel vessel [14], see Fig. 10. The coherent tune shift due to image charges is proportional to the space-charge tune shift,

$$\Delta Q_{\rm coh} = \Delta Q_{\rm sc} 2\xi_{h,v}\frac{a^2}{h^2}, \qquad (2)$$

where a is the beam radius, h is the characteristic pipe size, and $\xi_{h,v}$ depends on the pipe geometry. For a round pipe, h is equal to the pipe radius, and $\xi_{h,v} = 0.5$. For a rectangular pipe, $\xi_{h,v}$ can be calculated from analytical functions, see [15]. The results are presented in Fig. 10. The overall tune shift can be calculated as an average over the ring, $\langle 2\xi_h(s)/h^2(s)\rangle H^2 = 0.528$, and $\langle 2\xi_v(s)/h^2(s)\rangle H^2 = 1.13$, where $H = \langle h(s)\rangle = 63.42$ mm is the average pipe half-height. This results in a vertical coherent tune shift of

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$\Delta Q_{\rm coh} \approx 0.12 \Delta Q_{\rm sc}$ for the 1RF bunch parameters with 4×10^{12} ppp.

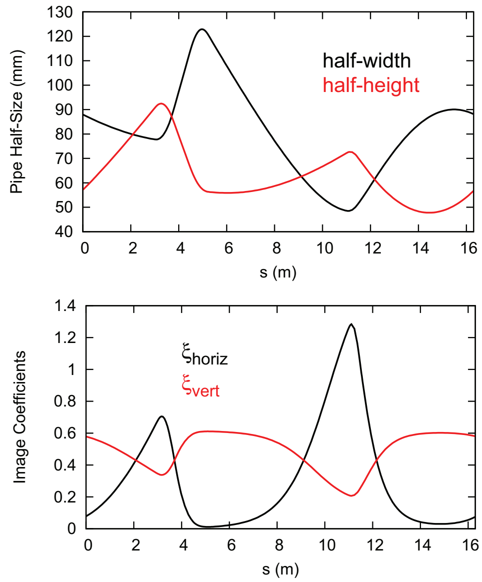

Figure 10: Top plot: half-apertures of the beam pipe in ISIS over a superperiod. Bottom plot: image coefficients for the rectangular ISIS pipe.

In order to analyse the changes in the tune shifts with the increasing beam intensity, we consider three different scenarios of the transverse emittance increase,

$$\Delta \varepsilon_\perp = k_N \frac{\Delta N_p}{N_{p0}} \varepsilon_{\perp 0}. \qquad (3)$$

The case $k_N = 0$ means that the emittance does not change (most unrealistic), $k_N = 1$ implies a constant phase-space density, and $k_N = 0.5$ is an intermediate case. The space-charge tune shift is $\Delta Q_{\rm sc} \propto N_p/\varepsilon_\perp$, thus it stays constant for $k_N = 1$ and increases linearly for $k_N = 0$, see Fig. 11. Differently, the coherent tune shift is $\Delta Q_{\rm coh} \propto N_p$, thus it increases linearly for all the k_N-scenarios. As a result, the relative strength $\Delta Q_{\rm coh}/\Delta Q_{\rm sc}$ stays constant for $k_N = 0$ and it increases for the both $k_N = 1$ and $k_N = 0.5$. The Landau damping for the $k = 1$ mode, as predicted by our model with the airbag theory, is presented in Fig. 12. Similarly to Fig. 7, the mode is under active Landau damping if the coherent line (solid line) is below a dashed line. We see that for the unrealistic case $k_N = 0$ there is no damping, but for an increasing emittance, there is a border intensity where the head-tail mode dips into Landau damping.

CONCLUSIONS

- Collective oscillations and beam losses have been systematically observed in ISIS around 2 ms Cycle Time for various beam and machine parameters. In most cases, it has been identified as the $k = 1$ head-tail mode with the typical growth time $\tau \approx 0.1$ ms. For 2RF operation, the mode structure can be more complicated.

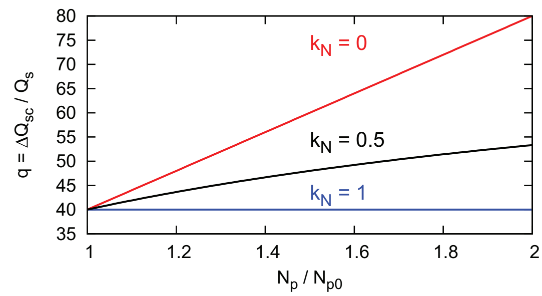

Figure 11: The space-charge parameter in the ISIS bunches for different emittance increase scenario.

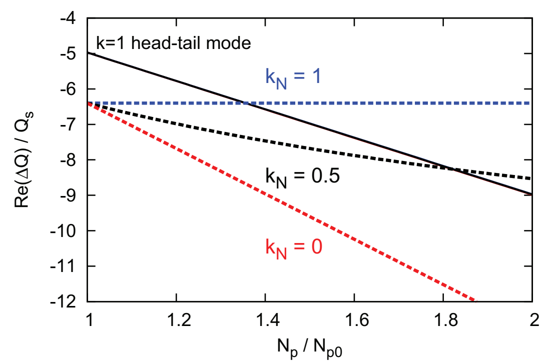

Figure 12: Illustration of the Landau damping for an increasing beam intensity. The black solid line is the coherent tune of the $k = 1$ mode. The dashed lines indicate the borders of Landau damping for different emittance increase scenario.

- With the help of the Sacherer theory, it can be concluded that the driving impedance has a Resistive-Wall type behaviour (resonant at low frequencies), because the instability appears if the vertical tune is set closer to integer from below. However, calculations with the Sacherer theory require a shorter effective bunch length in order to predict the $k = 1$ mode observed. The predicted growth rates are also much smaller than in the observations.
- Intensity thresholds have been identified for 1RF, 2RF settings, and for different driving tunes, Fig. 4. In addition to the common "bottom" thresholds, unexpected "top" intensity thresholds have been observed.
- Coherent tune shifts from the image charges modify Landau damping due to space charge in the bunches. It can be qualitatively analyzed using an airbag theory and a model for the Landau damping. Our PATRIC simulations confirm the enhanced damping due to image charges.
- Detailed calculations show that, due to a unique conformal beam pipe in ISIS, the image charges should produce strong coherent tune shifts. Additionally, the

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bunches in ISIS are in a regime of rather strong space charge, $q = \Delta Q_{sc}/Q_s \approx 40$. The largest uncertainty in the ISIS experiments is due to the measurements of the transverse emittance.

- Calculations for the coherent frequency of the $k = 1$ head-tail mode and for Landau damping with different scenarios of the transverse emittance increase suggest an intensity threshold, above which the $k = 1$ mode can be stabilized. This effect is due to different development of the space-charge tune shift and the image-charge tune shift for an increasing intensity.

- Enhanced Landau damping with image charges may be achieved with a controlled transverse emittance blow-up. This has been observed in the experiments reported here. The instability in 1RF bunches around 2 ms has been cured by increasing the transverse emittance, keeping the rest of the beam parameters fixed. This observation can be interpreted as a "top" threshold in transverse emittance. It should be noted that even with the beam stabilised, the associated increase in emittance can still lead to high levels of beam loss.

- Landau damping due to space charge with the effect of image charges should be taken into account in the stability studies for SIS100. The discrepancies between the Sacherer theory and the ISIS observations imply a possibility for much faster instabilities than expected so far.

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