Dynamical aspects of emittance coupling in intense linac beams

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Overview

- "Open" questions around stability charts
- Behaviour for crossing of "main resonance" (k_z~k_x)
- Above main resonance (k_z>>k_x)
- Below main resonance (k_z<<k_x and $\varepsilon_z=\varepsilon_x$)
- Conclusion

Discuss behavior of **non-EP beams** and its complexity!

- design relevance of EP or non-EP is a different issue - to be **discussed separately**!

Some applications and questions

Main observation:

No need for EP from beam physics point of view – large "white areas"!

Questions:

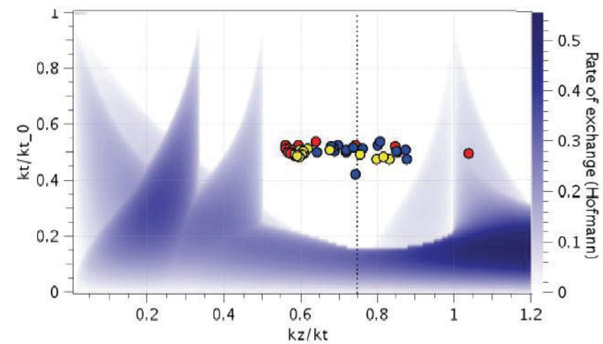
1. Are the colored regions (stop-bands) always of concern?
2. Are the "white" regions (EP or non-EP) safe?
3. Crossing speed of resonances?
4. Is a single chart enough?
5. What is the physics meaning of the EP condition $\varepsilon_z k_z : \varepsilon_x k_x = 1$
6. ....

$$\text{EP condition}: \frac{E_z}{E_x} \equiv \frac{\varepsilon_z k_z}{\varepsilon_x k_x} = 1$$

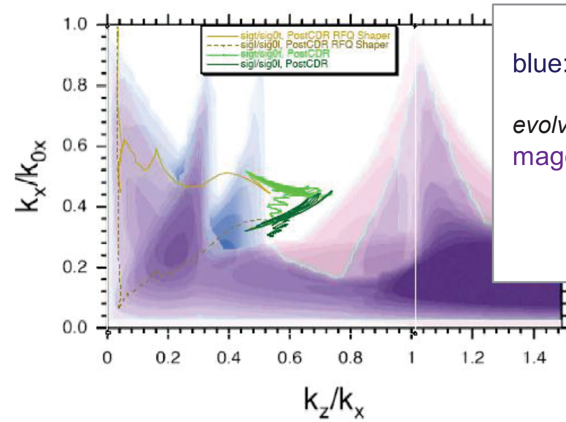
is based on energy of harmonic oscillator $a^2 k^2$

and ensemble average of single particle emittance $\varepsilon \propto a k^2$

Charts indicate (colored) regions, where space charge coupling (by low order space charge modes) may occur



M. *Eshraqi et al.* (IPAC11): ESS sc linac for equipartitioned design



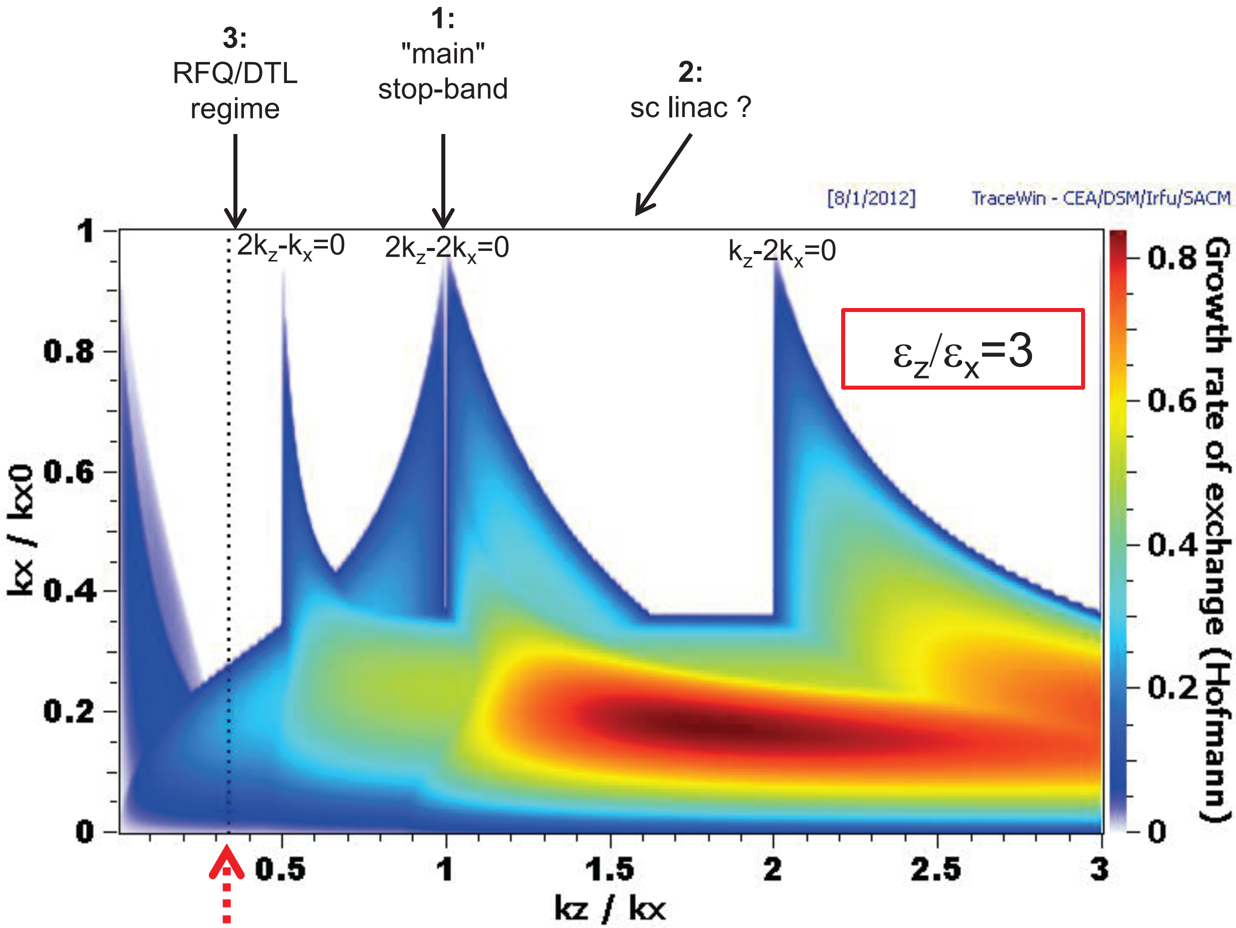
blue: $\varepsilon_l/\varepsilon_{tr}=1.4$

evolving towards:
magenta: $\varepsilon_l/\varepsilon_{tr}=2$

R.A. Jameson (ORNL-Report, 2007): Alternative IFMIF-RFQ design with equipartitioned beams

Examine 3 practically relevant regimes

chose (arbitrarily) $\varepsilon_z/\varepsilon_x=3$ or 1 (here: $\varepsilon=\varepsilon_{normalized}$)



$$\frac{E_z}{E_x} \equiv \frac{\varepsilon_z k_z}{\varepsilon_x k_x} = 1$$

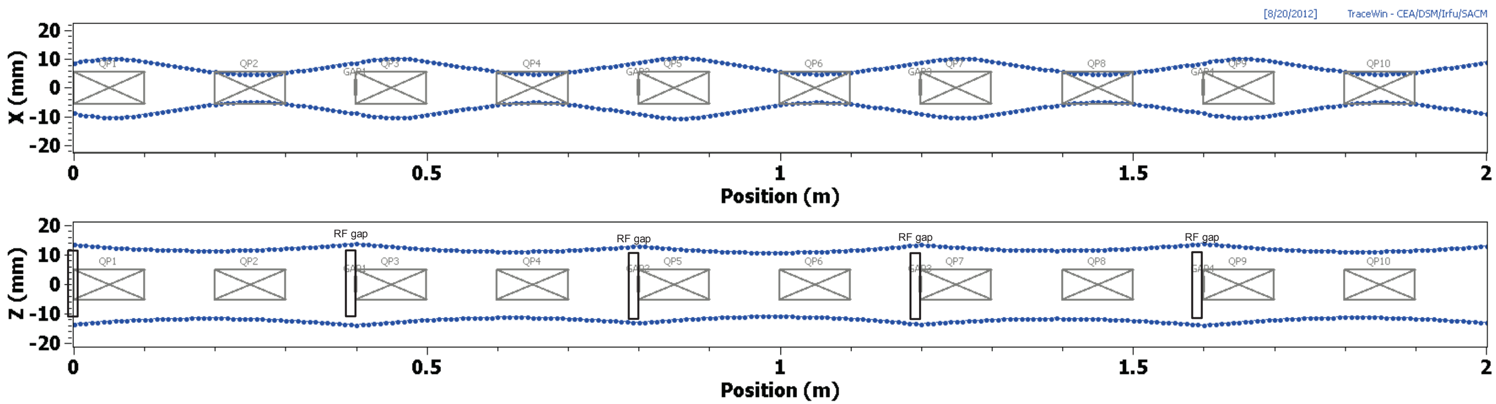
Charts are based on:

- 4D self-consistent Vlasov perturbation theory for modes from 2nd to 4th order
- verified against PIC simulation in 4D and 6D – mostly under transport conditions, partly under "linac conditions"
- validated in 1 experimental case at UNILAC

TRACEWIN simulations in idealized FODO lattice with RF

Idealized problem to symmetric FODO lattice

- acceleration irrelevant – only tunes and emittances
- cell length 40 cm
- 1 RF gap per cell
- transverse WB distribution
- longitudinal uniformly filled ellipse



Acknowledge help from Didier Uriot / CEA with advice on Tracewin

Regime 1: Crossing of main stop-band ($k_{oz}=69^0$)

crossing speed dependent

Crossing over 5 FODO cells

$k_{0x}=85^0...65^0$

- slight exchange
- too fast!

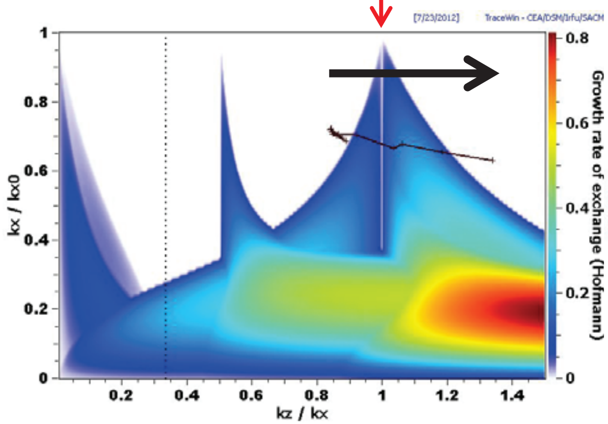
$$\boxed{\frac{E_z}{E_x} \equiv \frac{\varepsilon_z k_z}{\varepsilon_x k_x} \approx 3}$$

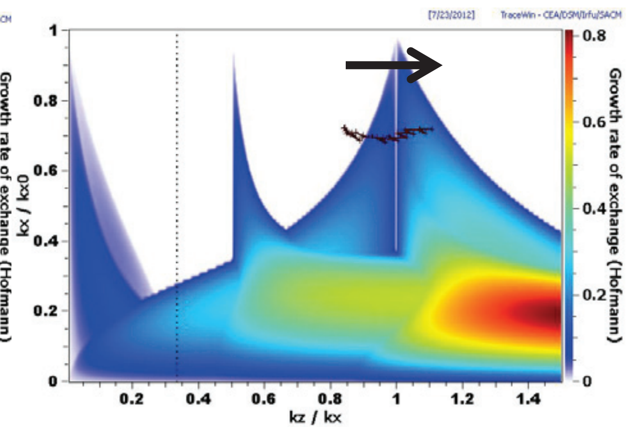
Crossing over 20 FODO cells

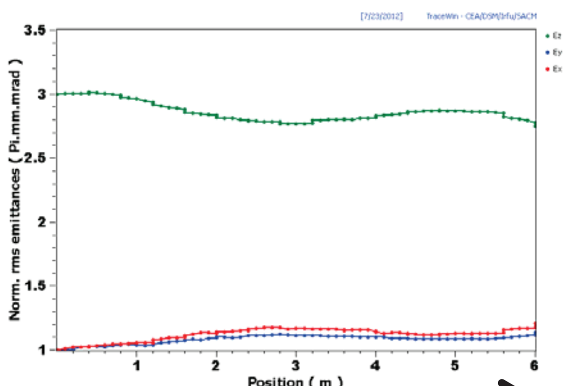
$k_{0x}=85^0...65^0$

•leads to ~EP

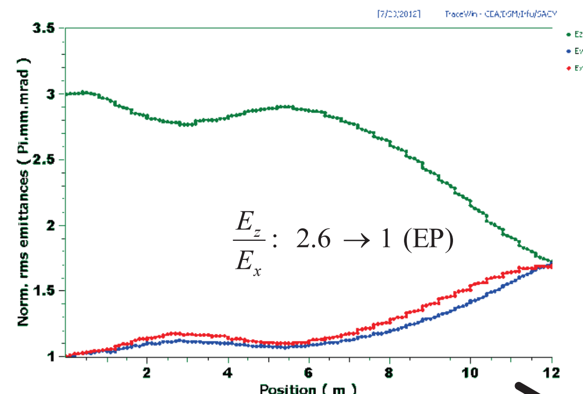
•notice about 2x less swing in k_z/k_x







ramp



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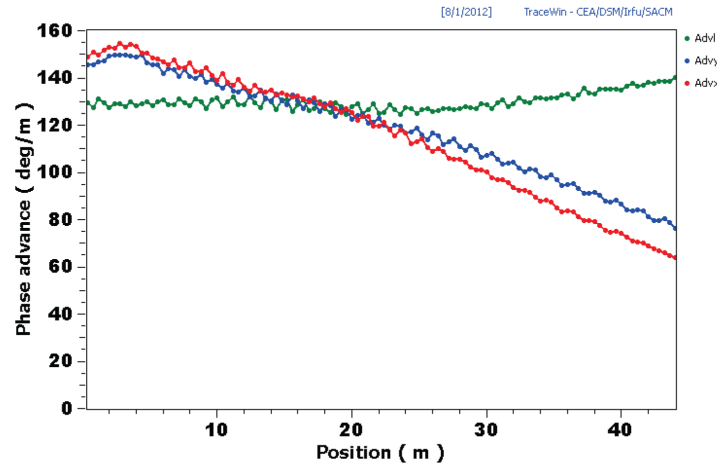
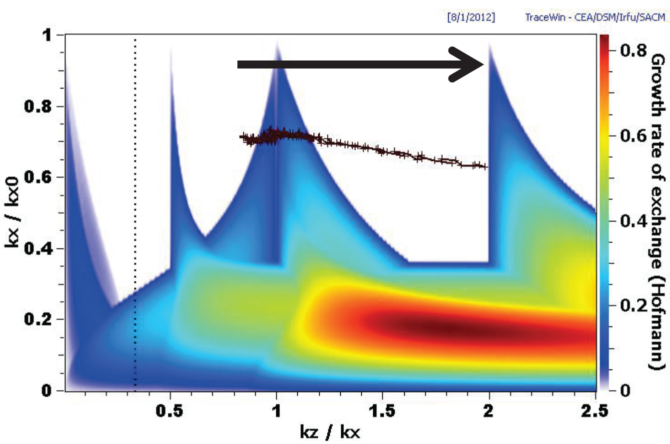
Complex behaviour far beyond ($k_{oz}=69^0$)

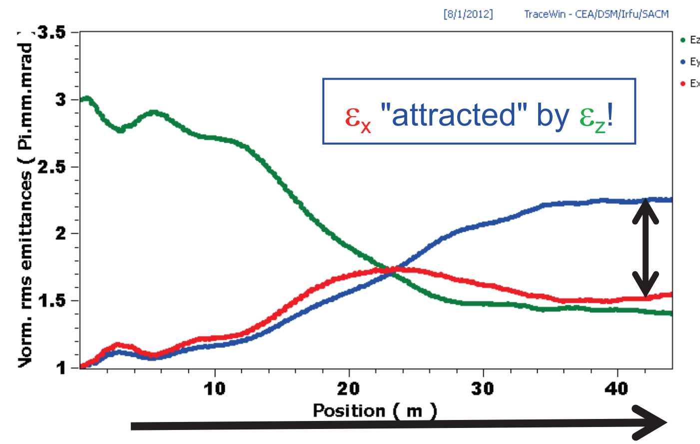
splitting of transverse emittances – x-y away from initial EP!

Crossing over 100 FODO cells
$k_{0x}=85^0 \ldots 45^0$
•full exchange
•splitting of x and y

$$\frac{E_z}{E_x}: \quad 2.6 \rightarrow 1 \rightarrow 2$$
$$\frac{E_z}{E_y}: \quad 2.6 \rightarrow 1 \rightarrow 1.2$$
$$\frac{E_x}{E_y}: \quad 1 \rightarrow 1 \rightarrow 0.6$$







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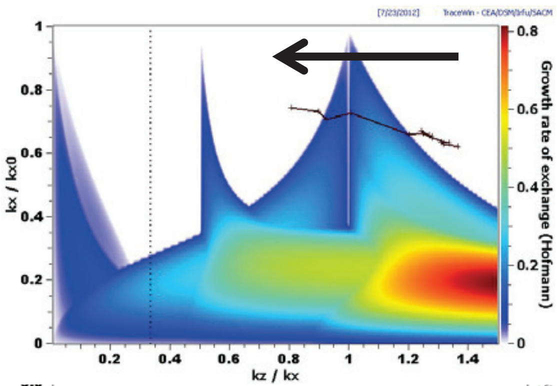
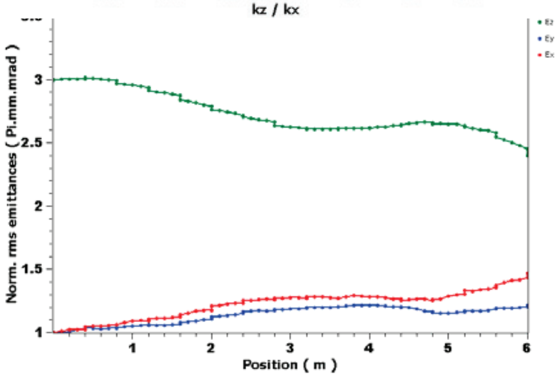
Adverse crossing of main stop-band similar

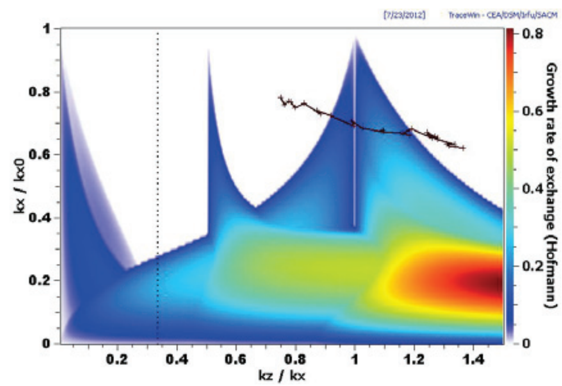
~ similar + splitting of transverse emittances

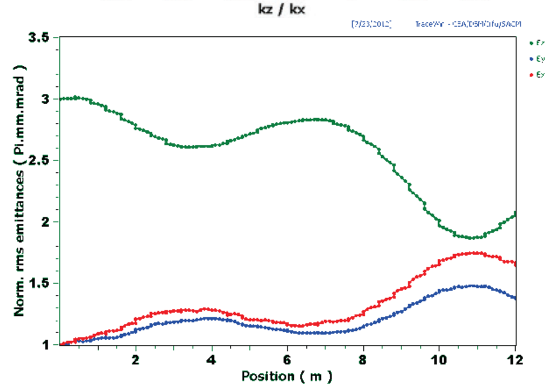
Crossing over 5 FODO cells
$k_{0x}=65^0...85^0$
- slight exchange
- too fast!



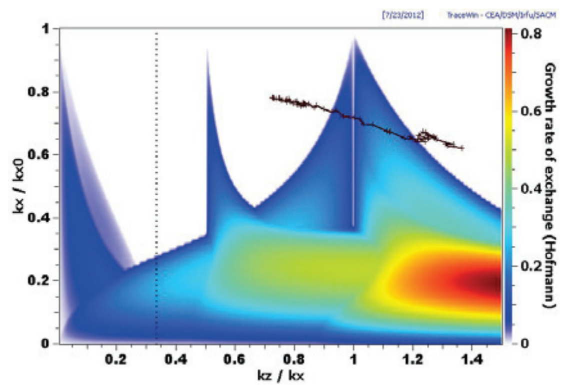


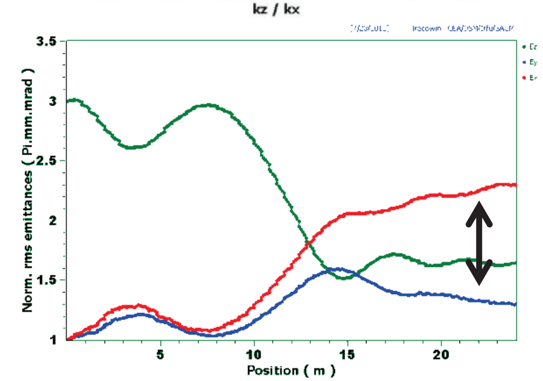
Crossing over 20 FODO cells
$k_{0x}=65^0...85^0$
•splitting of x and y





Crossing over 50 FODO cells
$k_{0x}=65^0...85^0$
•**enhanced** splitting





Semi-empirical scaling laws
(semi-analytical - for rough orientation):

scaling of stop-band width:

$$\Theta \approx \frac{3}{2}\left(\sqrt{\frac{\varepsilon_z}{\varepsilon_x}}-1\right)\frac{\Delta k_z}{k_z}$$

$$\text{with} \quad \Theta = \delta\left(\frac{k_z}{k_x}\right)$$

Example: $\Delta k_z/k_z \sim 0.25$; $\varepsilon_z/\varepsilon_x = 3$ → $\Theta \sim 0.27$
in good agreement with charts

scaling for emittance growth ~ 1/speed:

$$\Delta\varepsilon_x/\varepsilon_x \approx \alpha_{2,2} \times \frac{\left(\sqrt{\frac{\varepsilon_z}{\varepsilon_x}}-1\right)^2 \cdot \left(\frac{\Delta k_z}{360}\right)^2}{\frac{dk_{0z}/360}{dn}}$$

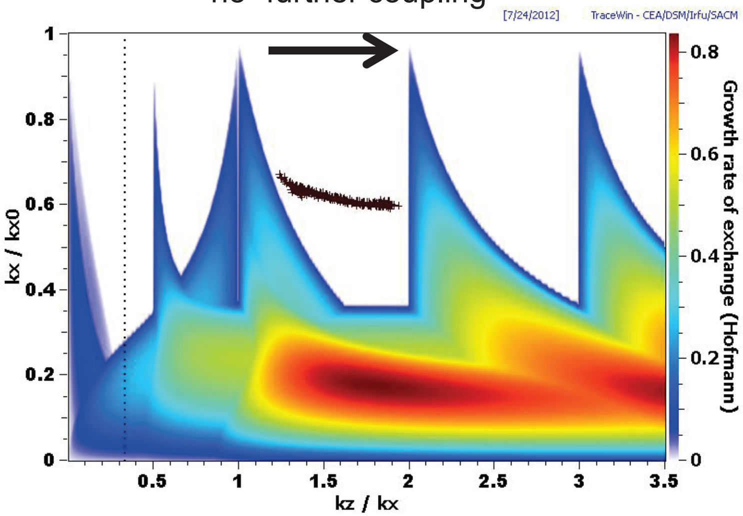
$$\Delta\varepsilon_x/\varepsilon_x \approx 2 \times \frac{\left(\sqrt{3}-1\right)^2 \cdot \left(\frac{17}{360}\right)^2}{\frac{20/360}{10}} \approx 1$$

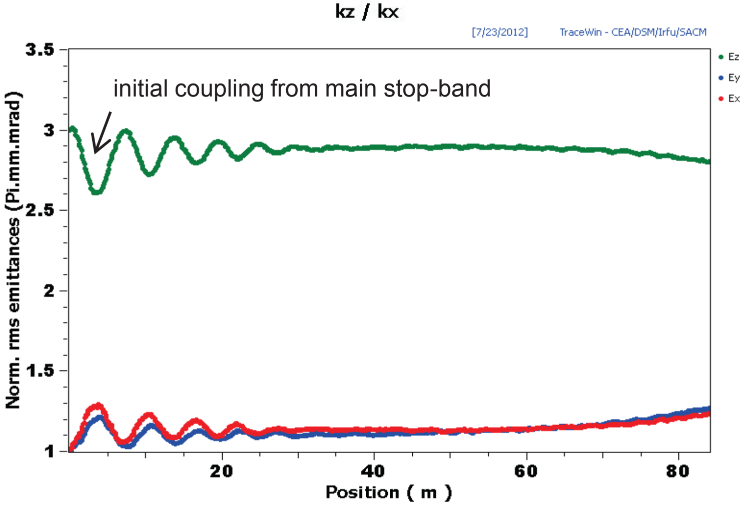
Tracewin emittance exchange for "slow" crossing
also shows good confirmation of the scaling

Regime 2: "Above" main stop-band (k_{oz}=69^0)

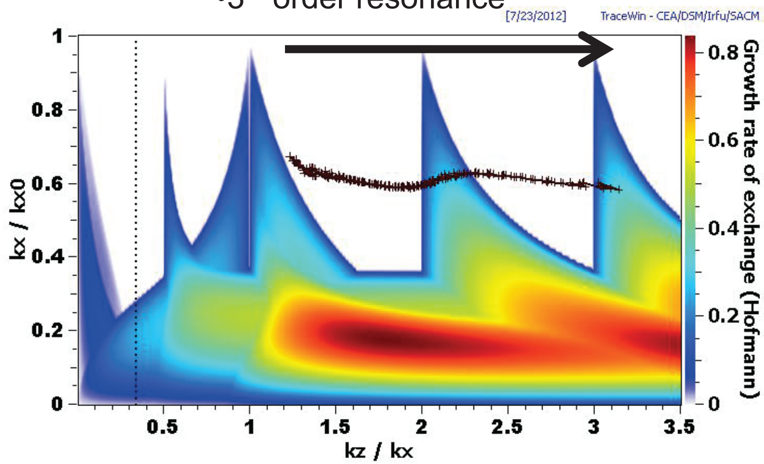
→ confirm absence of emittance exchange in "white" regions
"safe" - even though highly non-equipartitioned!

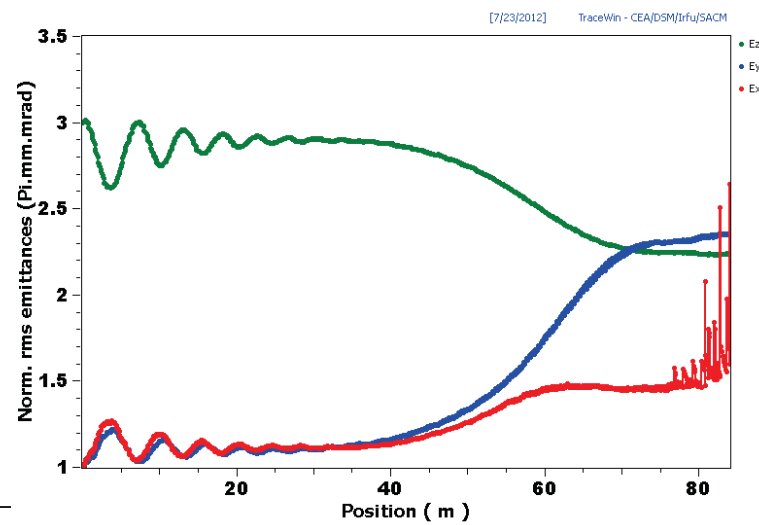
Crossing over 200 cells
k_{0x}=65^0...50^0
•"no" further coupling





Crossing over 200 cells
k_{0x}=65^0...35^0
•again splitting phenomenon
•3rd order resonance



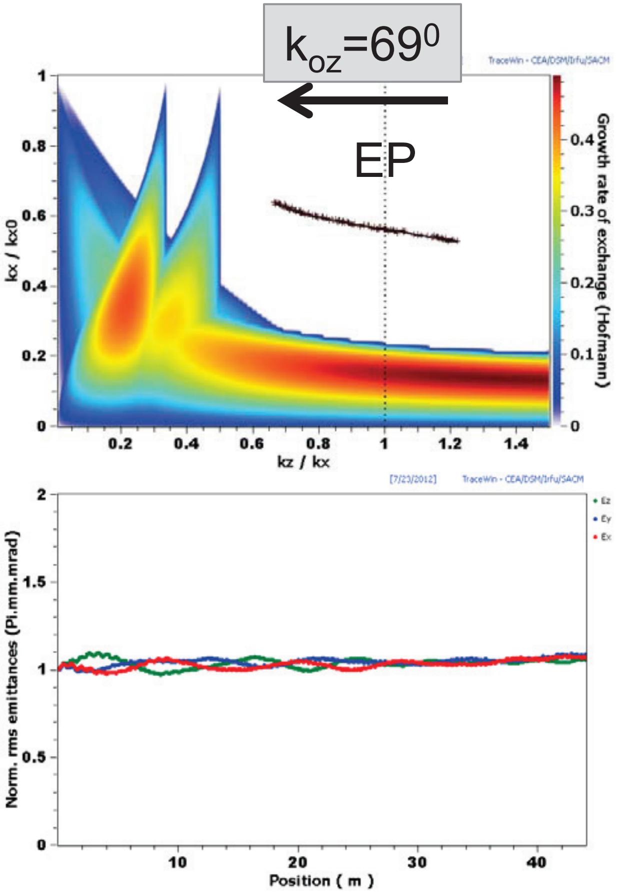


Regime 3: For $\varepsilon_z/\varepsilon_x$=1 condition ($k_{oz}$=69/47^0)

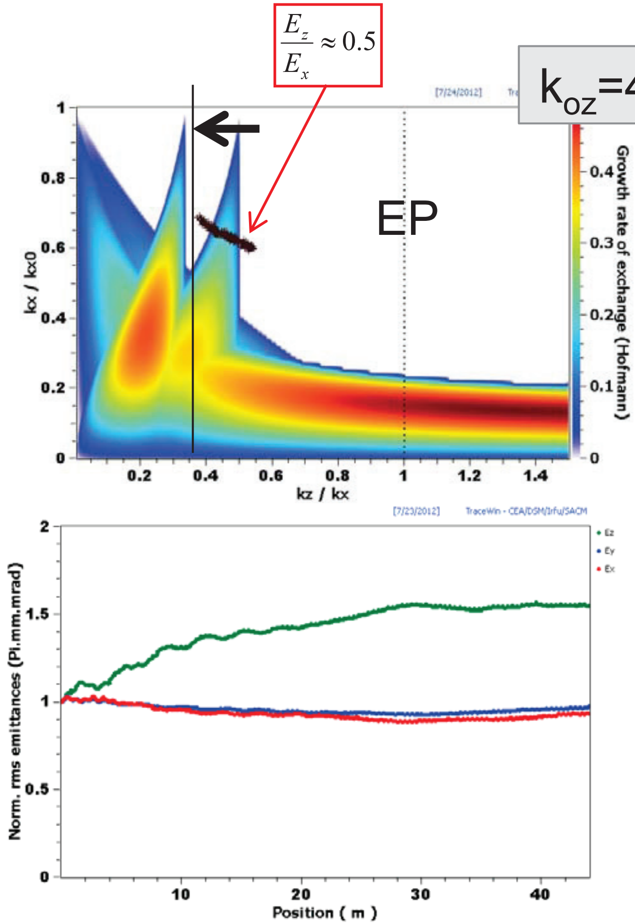
on EP: "safe" (no coupling)

on lower stop-band: reversed emittance exchange towards EP

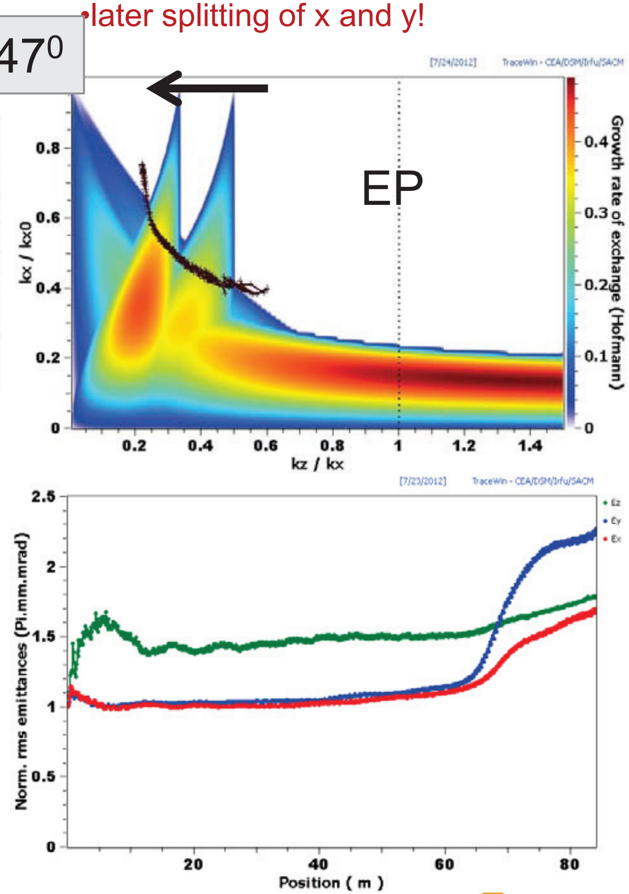
Crossing over 100 FODO cells
k_{0x}=65^0...85^0
- "no" coupling



Crossing over 100 FODO cells
k_{0x}=65^0...85^0
•slow exchange towards EP



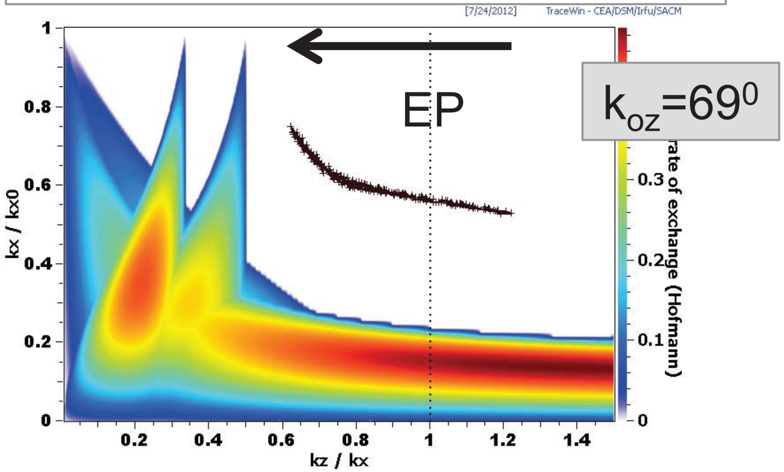
Crossing over 200 FODO cells
k_{0x}=65^0...95^0
•current 2.7 times
•fast initial exchange towards EP
•later splitting of x and y!

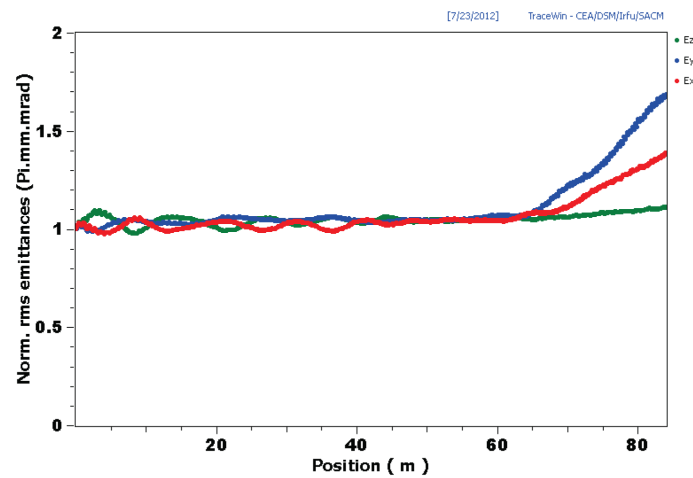


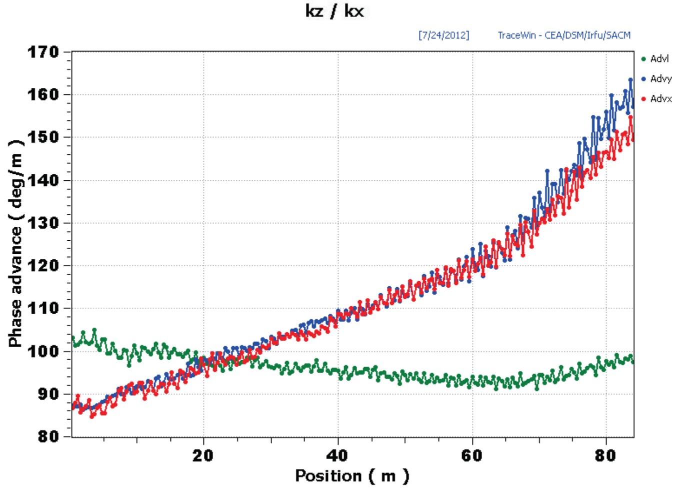
Additional feature: Structure resonances may occur

Crossing over 200 FODO cells: $k_{0x}=65^0\ldots85^0$

- slow enough to excite transverse 3rd order (structure) instability: (stop-band ~ $k_x<60^0$)
- $k_x= 35^0\ldots62^0$







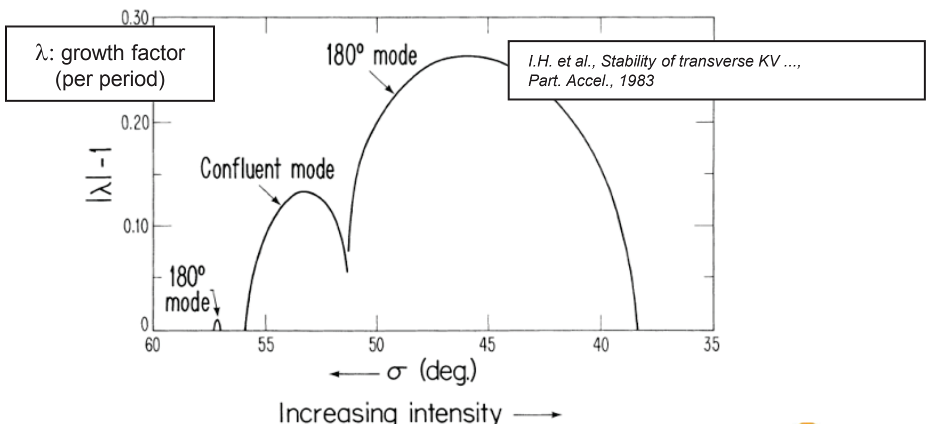


FIGURE 9. Behavior of third-order mode for a quadrupole system with $\eta = 1/10$ and $\sigma_0 = 90°$.

Summary

- Dynamical tune behavior confirms charts - white regions "safe" → EP not necessary!
- Emittance exchange depends on crossing speed (inversely proportional) of resonance stop-bands
- On resonances emittance evolution "towards" EP, but sometimes complex details (splitting of transverse emittances)
- Additional physics **not** addressed by stability charts:
  - Structure resonances or instabilities – how slow tune change?
  - Robustness against mismatch & errors?
  - ...