THO1B

Measurement of Extended Twiss parameters and space charge effects

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Contents

- Measurement of Extended Twiss parameters using turn-by-turn monitor.
- Linear envelope theory using the measured E-Twiss parameters.
- Simulation of space charge effects using the measured E-Twiss parameters.

One turn map containing space charge force

$$\mathcal{M}(s) = \prod_{i=0}^{N_{sc}-1} \mathcal{M}_0(s_{i+1}, s_i) e^{-:\Phi(s_i):}$$

$\mathcal{M}_0(s_{i+1}, s_i)$ Transfer map from s_i to s_{i+1}

$e^{-:\Phi(s_i):}\boldsymbol{p} = \boldsymbol{p} - \dfrac{\partial \Phi(s_i)}{\partial \boldsymbol{x}}$ Space charge force

$\Phi(s_i)$ Calculated by solving Poisson equation with the beam distribution.

Integration step: $\Delta s \ll \beta_{x,y}$

One turn map for nonlinearity lattice

$$\mathcal{M}_0(s) = \prod_{i=0}^{N_{nl}-1} M_0^{-1}(s_{i+1}, s_i) e^{-:H_{nl}(s_i):}$$

Transfer matrix from s_i to s_{i+1} Nonlinear transformation at s_i

$$e^{-:H_{nl}(s_i):}\boldsymbol{p} = \boldsymbol{p} - \frac{\partial H_{nl}(s_i)}{\partial \boldsymbol{x}}$$

$$H_{nl}(s_i) = \frac{K_2(s_i)}{6}(x^3 - 3xy^2) \qquad K_2 = \frac{eB"}{p_0}$$

ex. Sextupole magnet

One turn map including the space charge force is expressed by the nonlinear maps and the transfer matrix

$$\mathcal{M}(s) = \prod_{i=0}^{N_{sc}-1} \mathcal{M}_0(s_{i+1}, s_i) e^{-:\Phi(s_i):} = \prod_{i=0}^{N-1} M_0(s_{i+1}, s_i) e^{-:H_I(s_i):}$$

$$N = N_{nl} + N_{sc} \quad H_I = \Phi \text{ or } H_{nl}$$

Linear dynamics

- Linear Motion is represented by symplectic matrix transformation of the dynamic variables x.

$$\boldsymbol{x}(s) = (x, p_x, y, p_y, z, \delta)^t \qquad z = v(t_0 - t) \quad \delta = \frac{\Delta p}{p_0}$$

- Revolution matrix, M(s).

$$\boldsymbol{x}(s + C) = M_0(s)\boldsymbol{x}(s)$$

- Diagonalize 2x2 blockwisely

$$V_0(s)M_0(s)V_0(s)^{-1} = \begin{pmatrix} U_X & 0 & 0 \\ 0 & U_Y & 0 \\ 0 & 0 & U_Z \end{pmatrix} \equiv U_0 \qquad U_i \equiv \begin{pmatrix} \cos\mu_i & \sin\mu_i \\ -\sin\mu_i & \cos\mu_i \end{pmatrix}$$

$$i = X, Y, Z$$

- Split into three modes (X,Y,Z), with tunes

$$\mu_i = 2\pi\nu_i$$

Twiss parameter and normal mode

- Diagonalizing (eigenvector) matrix, V_0, is parametrized

$$V_0(s) = B_0(s)R_0(s)H_0(s)$$

$$R = \begin{pmatrix} r_0 & 0 & -r_4 & r_2 & 0 & 0 \\ 0 & r_0 & r_3 & -r_1 & 0 & 0 \\ r_1 & r_2 & r_0 & 0 & 0 & 0 \\ r_3 & r_4 & 0 & r_0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \qquad H = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & -\eta_x \\ 0 & 1 & 0 & 0 & 0 & -\eta'_x \\ 0 & 0 & 1 & 0 & 0 & -\eta_y \\ 0 & 0 & 0 & 1 & 0 & -\eta'_y \\ \eta'_x & -\eta_x & \eta'_y & \eta_y & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$B = \begin{pmatrix} B_X & 0 & 0 \\ 0 & B_Y & 0 \\ 0 & 0 & B_Z \end{pmatrix} \qquad B_i = \begin{pmatrix} \frac{1}{\sqrt{\beta_i}} & 0 \\ \frac{\alpha_x}{\sqrt{\beta_i}} & \sqrt{\beta_i} \end{pmatrix} \qquad i = X, Y, Z$$

- V=BRH is represented by Extended Twiss parameters (α,β,r_1-r_4,η).

$$r_0 = \sqrt{1 - r_1 r_4 + r_2 r_3}$$

- Normal coordinates **X** are defined by V,

$$\boldsymbol{X}(s) = B_0(s)R_0(s)H_0(s)\boldsymbol{x}(s) = V_0(s)\boldsymbol{x}(s) \qquad \boldsymbol{X}(s+C) = U_0\boldsymbol{X}(s)$$

$$\boldsymbol{X} = (X, P_X, Y, P_Y, Z, P_Z)^t \qquad J_X = \frac{X^2 + P_X^2}{2}$$

Transfer matrix

$$V_0(s_2)M_0(s_2, s_2)V_0(s_1)^{-1} = \begin{pmatrix} U_{u,21} & 0 & 0 \\ 0 & U_{v,21} & 0 \\ 0 & 0 & U_{w,21} \end{pmatrix} \equiv U_{21}$$

$$U_{i,21} \equiv \begin{pmatrix} \cos(\phi_i(s_2) - \phi_i(s_1)) & \sin(\phi_i(s_2) - \phi_i(s_1)) \\ -\sin(\phi_i(s_2) - \phi_i(s_1)) & \cos(\phi_i(s_2) - \phi_i(s_1)) \end{pmatrix} \qquad i = X, Y, Z$$

- betatron phase difference

Betatron motion and Extended Twiss parameters

- Linear optics parameters, B, R, H and betatron (synchrotron) phases are measurable.
- Betatron oscillation (4x4 formalism, omit 5,6 components)

$$\delta(\boldsymbol{x}^T A_X^R \boldsymbol{x} - W_X)\delta(\boldsymbol{x}^T A_Y^R \boldsymbol{x} - W_Y)$$

- Courant-Snyder invariant

$$W_{X,Y} = 2J_{X,Y} = \boldsymbol{x}^T A_{X,Y}^R \boldsymbol{x}$$

$$A_i^R \equiv R S_4 A_i R^{-1}$$

$$S_4 = \begin{pmatrix} S_2 & 0 \\ 0 & S_2 \end{pmatrix} \quad S_2 = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

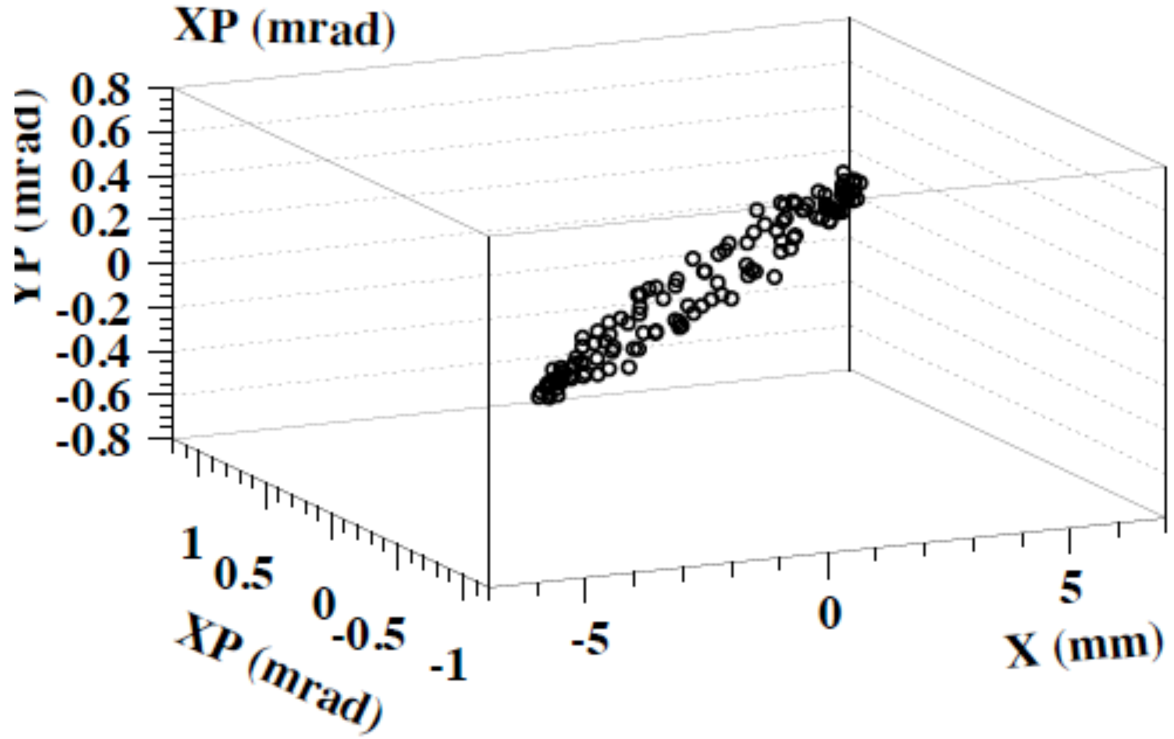
$$A_X = \left(\begin{array}{cc|c} \gamma_X & \alpha_X & \\ \alpha_X & \beta_X & 0 \\ \hline \multicolumn{2}{c|}{0} & 0 \end{array}\right) \qquad A_Y = \left(\begin{array}{c|cc} 0 & \multicolumn{2}{c}{0} \\ \hline & \gamma_Y & \alpha_Y \\ 0 & \alpha_Y & \beta_Y \end{array}\right)$$

Measurement of E-Twiss parameters

- X mode is induced by x injection error. X~x, W_Y~0
- Elliptical trajectory in 4 dimensional phase space (x, p_x, y, p_y).

$$\delta(\boldsymbol{x}^T A_X^R \boldsymbol{x} - W_X)$$

- The phase space trajectory is reconstructed by turn-by-turn monitor



Projection on 3 dim space (x, p_x, p_y)

Determine E-Twiss parameters

- Extended Twiss parameters are determined by the measured 2nd order moment matrix with the formula.

use ☐ parts

- X mode excitation

$$\langle \boldsymbol{x}\boldsymbol{x}^T \rangle = \frac{1}{2\pi} \oint \boldsymbol{x}\boldsymbol{x}^T \delta(\boldsymbol{x}^T A_X^R \boldsymbol{x} - 2J_X) d\boldsymbol{x}$$

$$= J_X \begin{pmatrix} r_0^2\beta_X & -r_0^2\alpha_X & r_0(-\beta_X r_1 + \alpha_X r_2) & r_0(-\beta_X r_3 + \alpha_X r_4) \\ & r_0^2\gamma_X & r_0(\alpha_X r_1 - \gamma_X r_2) & r_0(\alpha_X r_3 - \gamma_X r_4) \\ & & \beta_X r_1^2 - 2\alpha_X r_1 r_2 + \gamma_X r_2^2 & \beta_X r_1 r_3 - \alpha_X(r_1 r_4 + r_2 r_3) + \gamma_X r_2 r_4 \\ & & & \beta_X r_3^2 - 2\alpha_X r_3 r_4 + \gamma_X r_4^2 \end{pmatrix}$$

polluted by small J_y component

- Y mode excitation

$$\langle \boldsymbol{x}\boldsymbol{x}^T \rangle = \frac{1}{2\pi} \oint \boldsymbol{x}\boldsymbol{x}^T \delta(\boldsymbol{x}^T A_Y^R \boldsymbol{x} - 2J_Y) d\boldsymbol{x}$$

polluted by small J_x component

$$J_Y \begin{pmatrix} \beta_Y r_4^2 + 2\alpha_Y r_2 r_4 + \gamma_Y r_2^2 & & & \\ \beta_Y r_3 r_4 + \alpha_Y(r_1 r_4 + r_2 r_3) + \gamma_Y r_1 r_2 & \beta_Y r_3^2 + 2\alpha_Y r_1 r_3 + \gamma_Y r_1^2 & & \\ r_0(\beta_Y r_4 + \alpha_Y r_2) & -r_0(\beta_Y r_3 + \alpha_Y r_1) & r_0^2\beta_Y & \\ -r_0(\alpha_Y r_4 + \gamma_Y r_2) & (\alpha_Y r_3 + \gamma_Y r_1) & -r_0^2\alpha_y & r_0^2\gamma_Y \end{pmatrix}$$

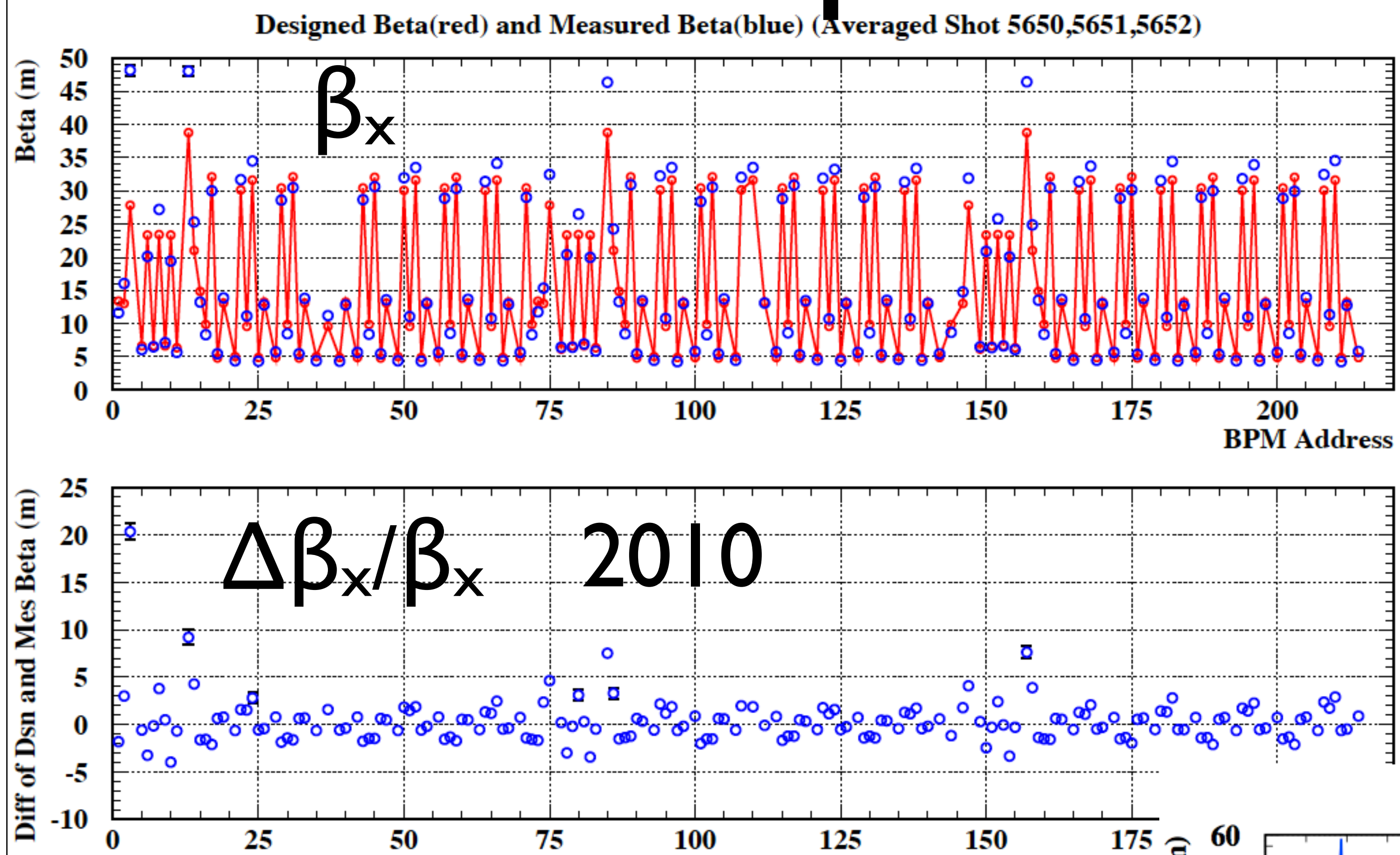
Measurement of Betatron phase

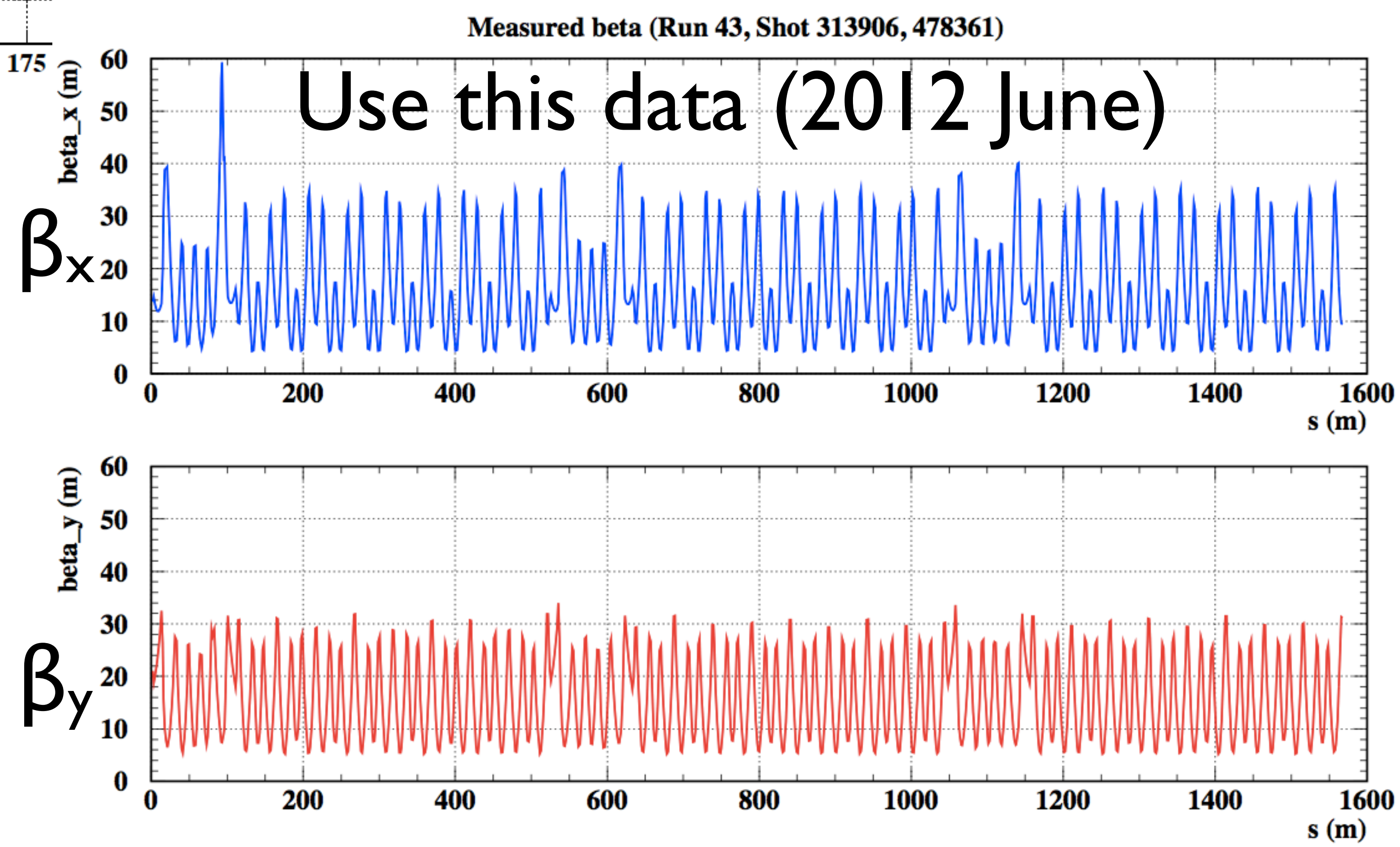
- Correlation between two monitors

$$\langle x(s_{i+1})x(s_i)\rangle = J_x\sqrt{\beta_x(s_{i+1})\beta_x(s_i)}\cos(\phi_{x,i+1}-\phi_{x,i})$$

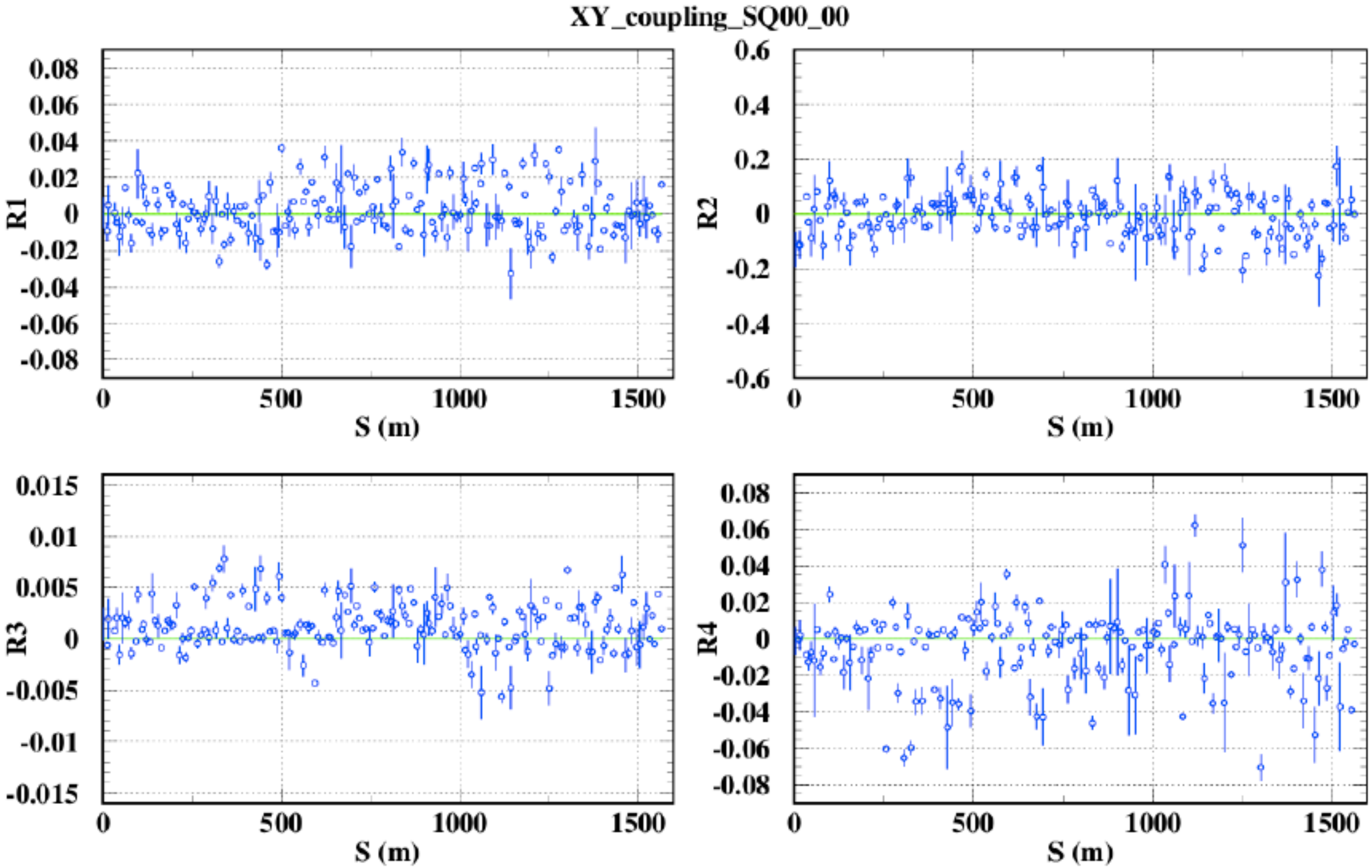
$$\cos(\phi_{X,i+1}-\phi_{X,i}) = \frac{\langle x(s_{i+1})x(s_i)\rangle}{\sqrt{\langle x(s_{i+1})^2)\rangle\langle x(s_i)^2\rangle}}$$

Example of measurement beta function

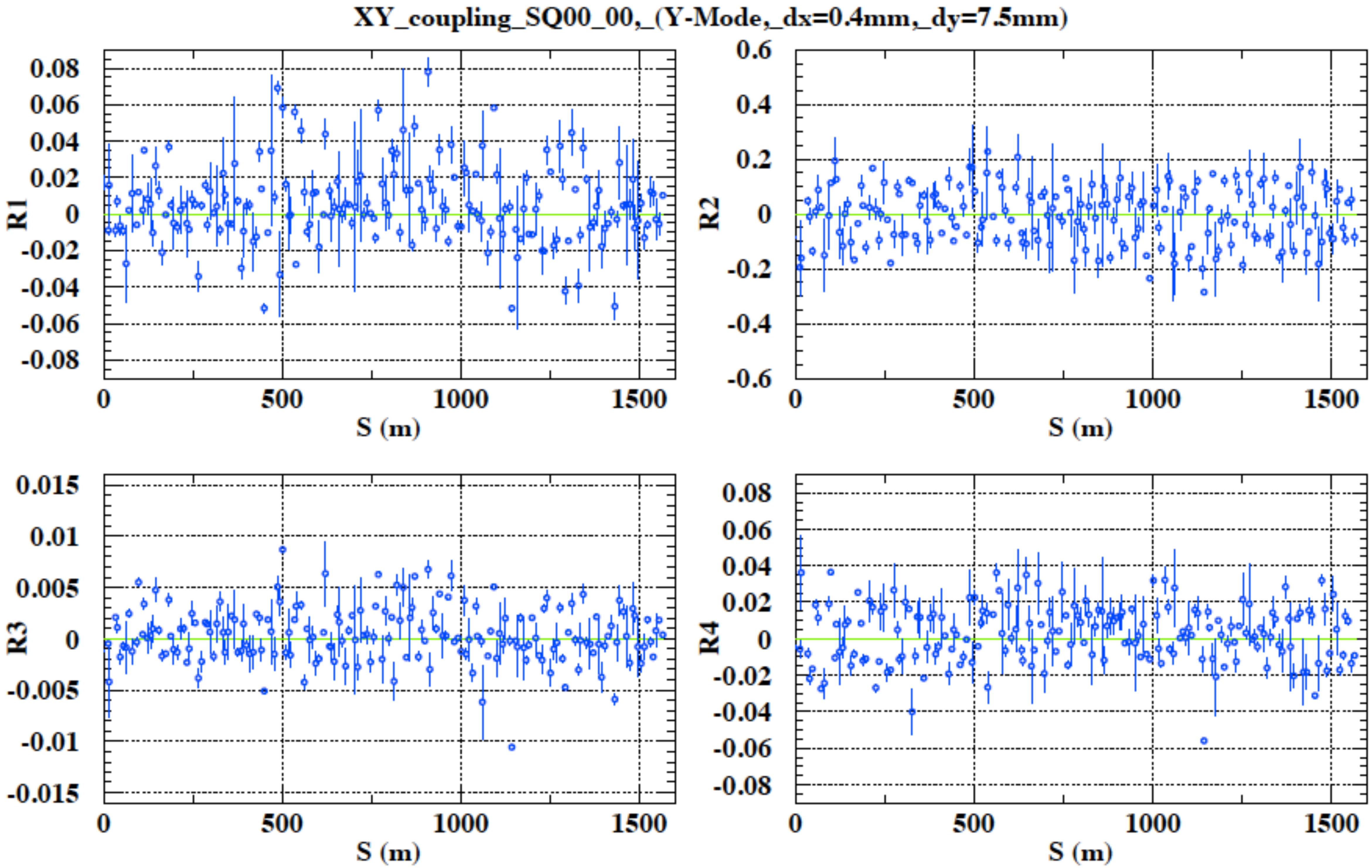




Example of measurement r_1-r_4, x mode excitation



Example of measurement r_1-r_4, y mode excitation



Comment on the measurement

- Measurement of beta function is done by x(y) signal for X(Y) mode oscillation, while that of x-y coupling is done by y signal for X mode oscillation, vice versa.
- Reliability of the beta measurement is much better than coupling.
- Calibration of monitor for rotation is unavoidable.
- At present the reliability of x-y coupling measurement is not very good.

Linear approximation

Beam envelope using measured Twiss

- Averaging of beam particles

$$\langle \boldsymbol{x}\boldsymbol{x}^T \rangle$$

4 dimensional model

- Revolution matrix including linear space charge force.

$$M(s) = \prod_{i=0}^{N-1} M_0(s_{i+1}, s_i) M_\Phi(s_i)$$

$$M_\Phi = T^{-1}(\theta) \begin{pmatrix} 1 & 0 & 0 & 0 \\ k_x & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & k_y & 1 \end{pmatrix} T(\theta) \qquad T(\theta) = \begin{pmatrix} \cos\theta & 0 & \sin\theta & 0 \\ 0 & \cos\theta & 0 & \sin\theta \\ -\sin\theta & 0 & \cos\theta & 0 \\ 0 & -\sin\theta & 0 & \cos\theta \end{pmatrix}$$

$$k_x = \frac{4r_p\lambda}{\beta^2\gamma^3}\frac{1}{a(a+b)} \qquad k_y = \frac{4r_p\lambda}{\beta^2\gamma^3}\frac{1}{b(a+b)}$$

- Transformation of the beam envelope

$$M(s)\langle \boldsymbol{x}\boldsymbol{x}^T \rangle M^T(s)$$

- Tilt and beam size

$$\tan 2\theta = \frac{\langle xy \rangle}{\langle x^2 \rangle - \langle y^2 \rangle}$$

$$a^2 = \langle x^2 \rangle \cos^2\theta + \langle xy \rangle \sin 2\theta + \langle y^2 \rangle \sin^2\theta$$
$$b^2 = \langle x^2 \rangle \sin^2\theta - \langle xy \rangle \sin 2\theta + \langle y^2 \rangle \cos^2\theta$$

- Periodic solution of the beam envelope

$$M(s) = \prod_{i=0}^{N-1} M_0(s_{i+1}, s_i) M_\Phi(s_i)$$

$$M(s)\langle \boldsymbol{x}\boldsymbol{x}^T \rangle M^T(s) = \langle \boldsymbol{x}\boldsymbol{x}^T \rangle \qquad M = V^{-1}UV$$

- Solution

$$\langle \boldsymbol{x}\boldsymbol{x}^T \rangle = V^{-1} \begin{pmatrix} \varepsilon_X & 0 & 0 & 0 \\ 0 & \varepsilon_X & 0 & 0 \\ 0 & 0 & \varepsilon_Y & 0 \\ 0 & 0 & 0 & \varepsilon_Y \end{pmatrix} (V^{-1})^t$$

Solving the periodic envelope is equivalent to solving V.

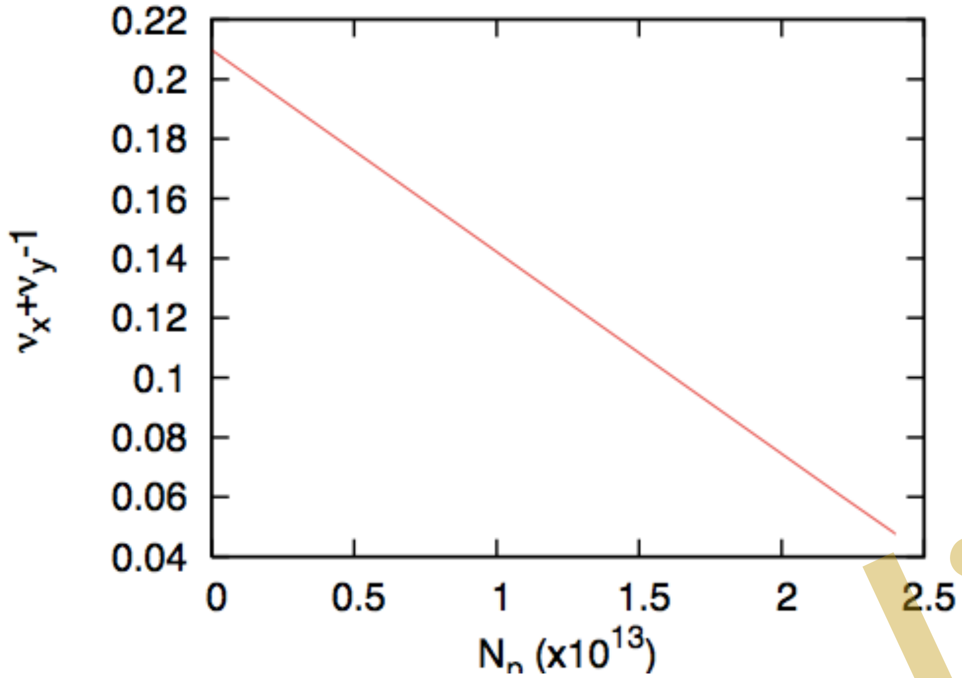
- Periodic beam envelope for zero intensity

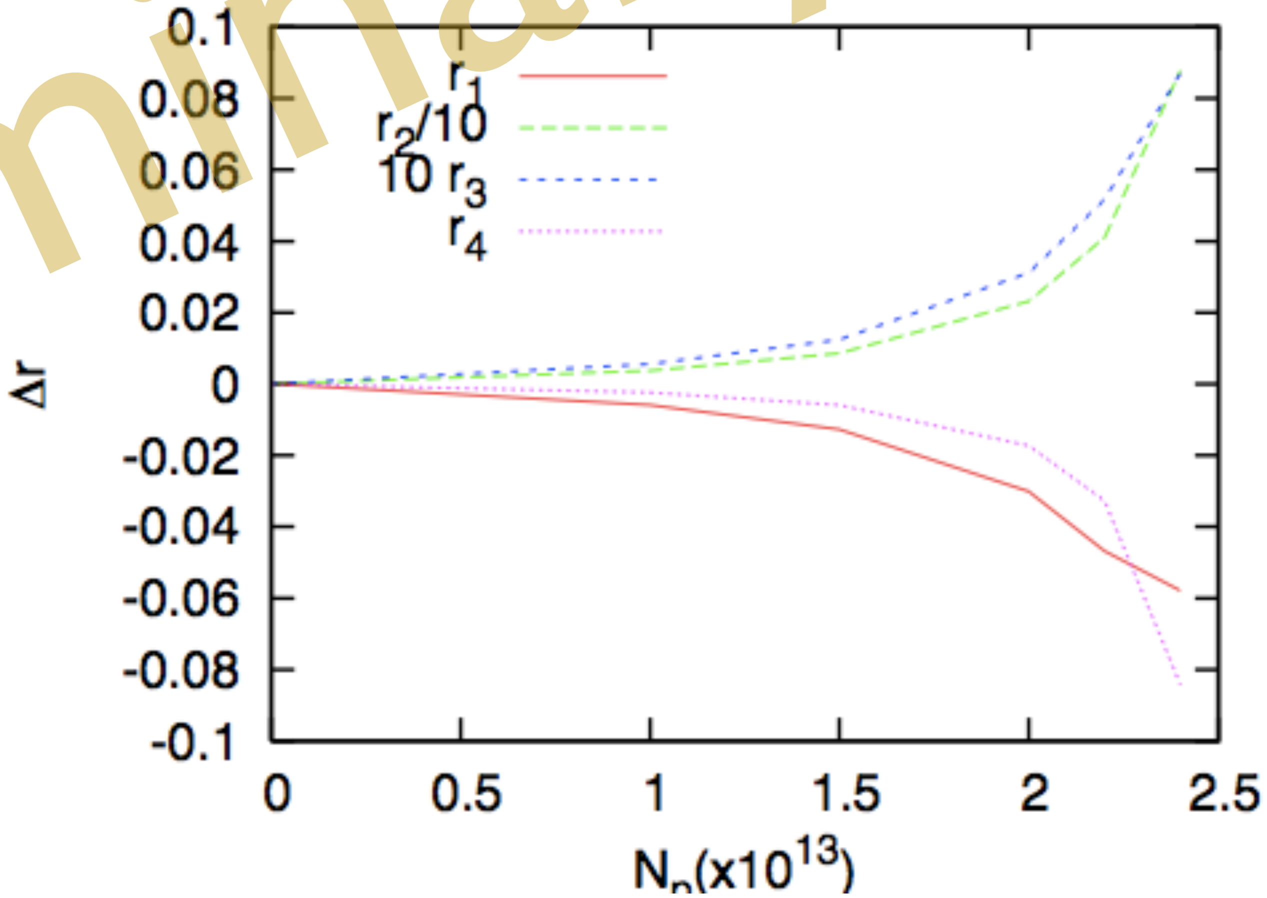
$$M_0(s)\langle \boldsymbol{x}\boldsymbol{x}^T \rangle M_0^T(s) = \langle \boldsymbol{x}\boldsymbol{x}^T \rangle \qquad M_0 = V_0^{-1} U_0 V_0$$

$$V_0 = B_0 R_0$$

Measured V_0 4 dim, $H_0=1$

Solution of envelope equation with measured Twiss





- ν_x=22.41, ν_y=20.80
- Convergence of V is limited when tune approach to the coupling resonance.

Simulation with the measured linear optics

$$\mathcal{M}(s) = \prod_{i=0}^{N-1} M_0(s_{i+1}, s_i) e^{-:H_I(s_i):}$$

$$M_0(s_{i+1}, s_i) = V_0^{-1}(s_{i+1}) U_{i+1,i} V_0(s_i)$$

$$N = N_{nl} + N_{sc} \quad H_I = \Phi \text{ or } H_{nl}$$

- Design transfer matrix M_0 is replaced by measured transfer matrix M.

$$\mathcal{M}(s) = \prod_{i=0}^{N-1} M(s_{i+1}, s_i) e^{-:H_I(s_i):}$$

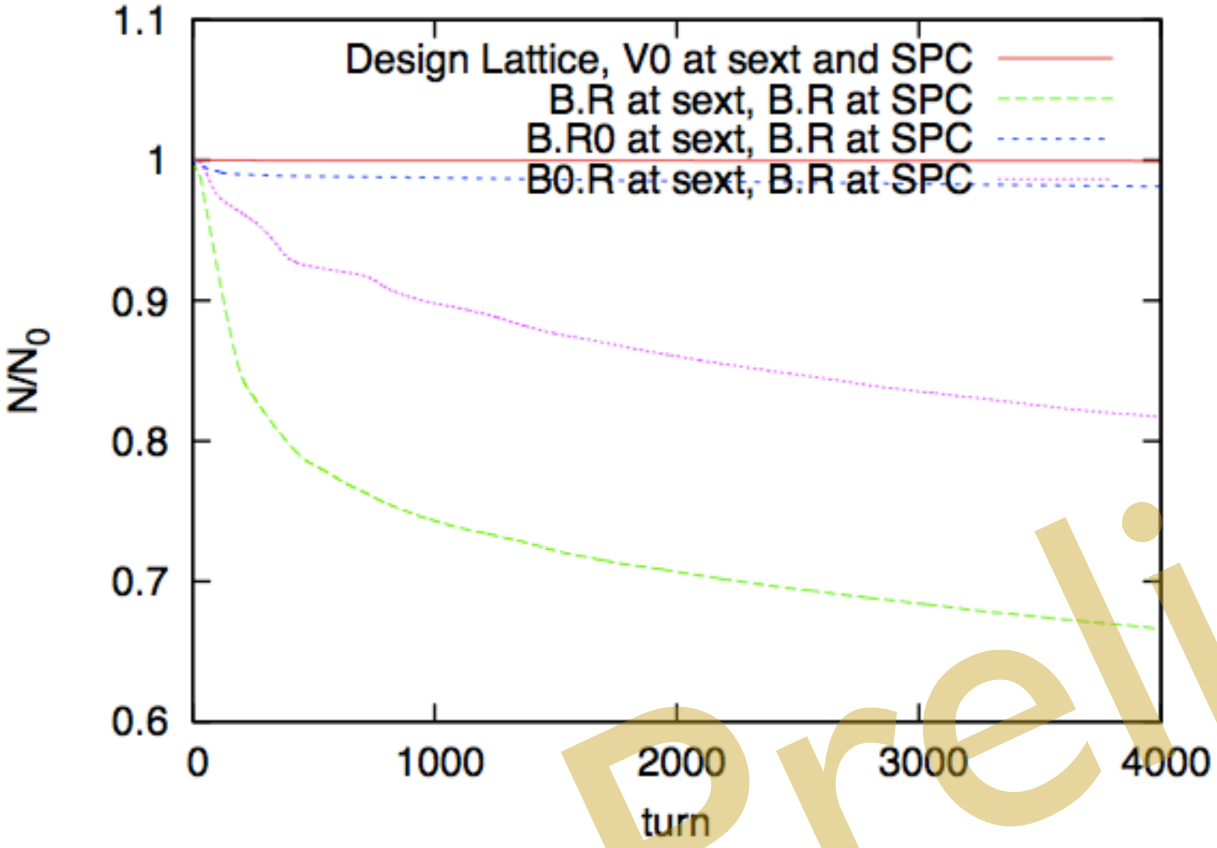
Actual coding

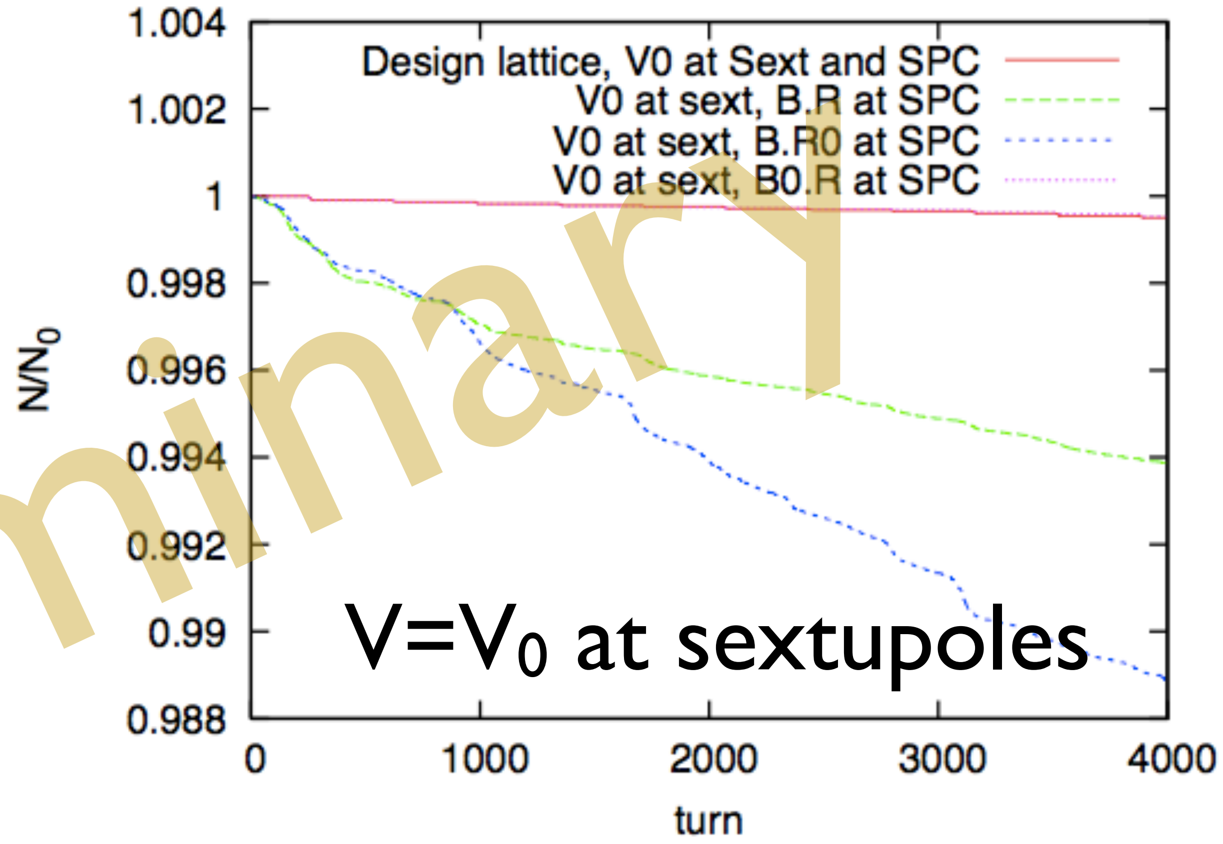
$$M(s_{i+1}, s_i) = V^{-1}(s_{i+1}) U_{i+1,i} \Delta U_i V(s_i)$$

$$= \boxed{V^{-1}(s_{i+1}) V_0(s_{i+1})} M_0(s_{i+1}, s_i) \boxed{V_0^{-1}(s_i) \Delta U_i V(s_i)}$$

insert these transformation

Simulation results using measured Twiss

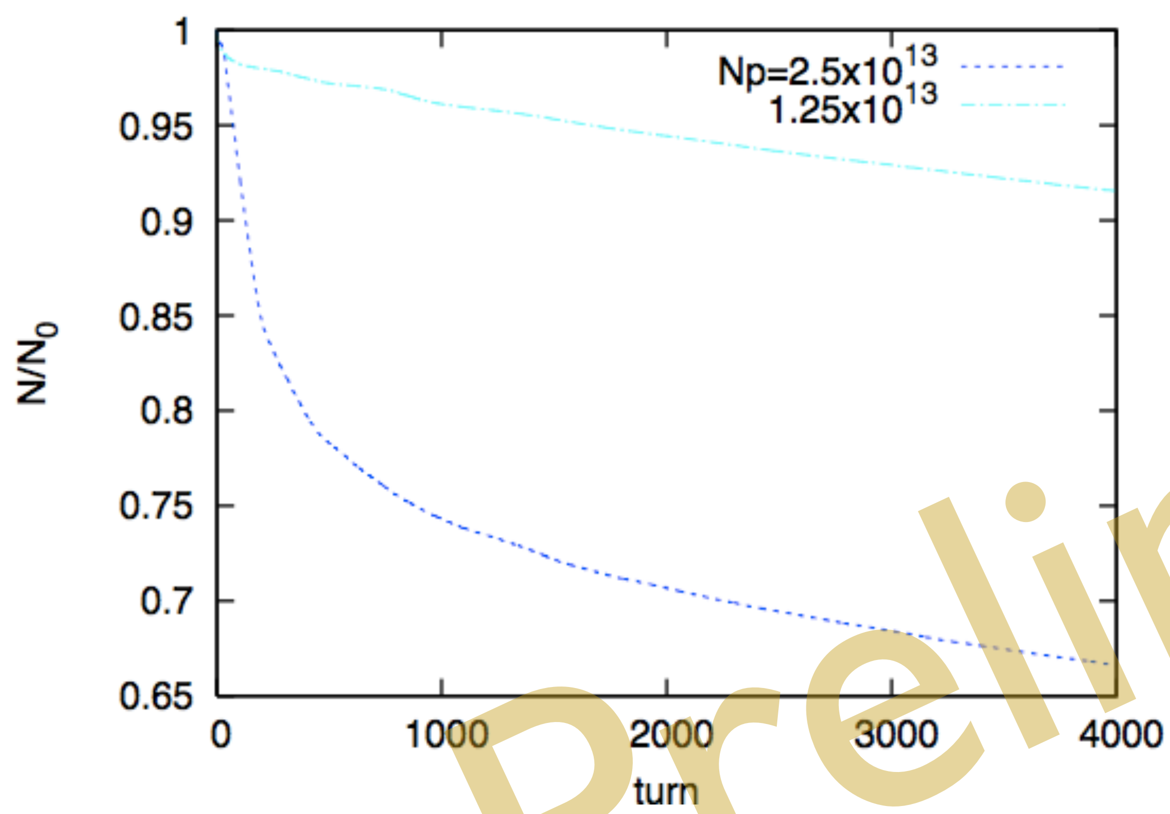




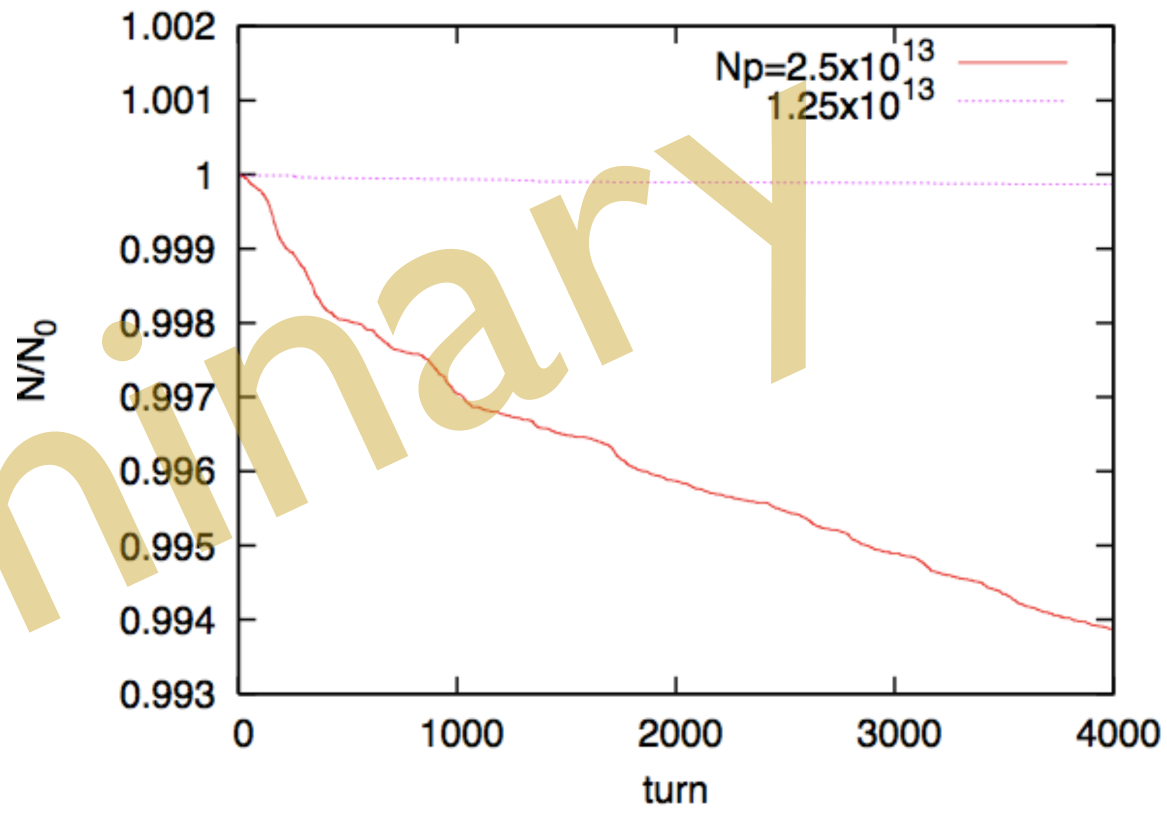
- Red:$V=V_0$ at sext and space charge
- Green:$V=V_{meas}$ at sext and space charge
- Blue:$V=B_{meas}R_0$ at sext and :$V=V_{meas}$ at spc
- Magenta:$V=B_0R_{meas}$ at sext and :$V=V_{meas}$ at spc

- Red:$V=V_0$ at sext and space charge
- Green:$V=V_{meas}$ at space charge
- Blue:$V=B_{meas}R_0$ at spc
- Magenta:$V=B_0R_{meas}$ at spc

Intensity dependence in the simulation



V=$V_{meas.}$ at sextupoles and spc



V=V_0 at sextupoles and V=$V_{meas.}$ at spc

Summary

- Extended Twiss parameters have been measured in J-PARC MR.
- Space charge simulation using the measured E-Twiss parameters has been performed.
- x-y coupling at sextupoles seems dominant for the beam loss. Space charge force may be role of tune spread source.
- Understanding the mechanism, which resonances is induced? Consistency with envelope theory?
- Establish the reliability of x-y coupling measurement. Twiss parameters α, β seems reliable.