DYNAMICAL ASPECTS OF EMITTANCE COUPLING IN INTENSE LINAC BEAMS

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Abstract

In this paper we use the TRACEWIN code to study in an idealized lattice model the dynamical behavior of non-equipartitioned bunched beams and their approaching equipartition for certain resonance conditions described by stability charts. It is shown that rms emittance transfer on these resonance stop-bands depends on times scales of tune change, whereas regions free from third or fourth order resonances are "safe" and not subject to rms emittance coupling. This provides additional information to support the validity of the stability charts, and as a practically useful design tool for high current linacs.

INTRODUCTION

One of the important criteria in high intensity linac design is avoidance of emittance transfer between the longitudinal and transverse degrees of freedom. It is commonly accepted that nonlinear space charge is the driving force of emittance coupling, if in addition a resonance condition is satisfied. In order to identify the extended regions in tune space, where coupling could occur, stability charts have been introduced to assist linac design and linac beam dynamics studies [1]. These charts are the result of an analytical self-consistent perturbational analysis of two-dimensional anisotropic beams using Vlasov's equations [2]. Numerous particle-in-cell simulations have been published to support the validity of the charts also for realistic distributions, first in 4D, then in 6D, and by using different simulation codes [3, 4, 5]. An experimental confirmation of the emittance exchange at the "main resonance" $k_z/k_x = 1$ was obtained at the GSI UNILAC [6].

The emphasis of the present study is to illustrate the emittance behavior when tunes cross dynamically regions of tune space, which are characteristic for high current linac design. These simulations with the Tracewin code can be considered as additional check of the validity of the stability charts under tune variation.

GENERAL REMARKS ON THE STABILITY CHARTS

The definition of the stability charts is to distinguish regions in a suitable representation of tune space, where nonlinear space charge modes are stable, from those where unstable emittance exchange is possible. An example for $\epsilon_z/\epsilon_x = 3$ is shown in Fig. 1. Note that the condition for equipartition (EP) is thus $k_z/k_x = 1/3$, which is indicated by a dotted line. For proper interpretation of the charts a number of observations should be made first.

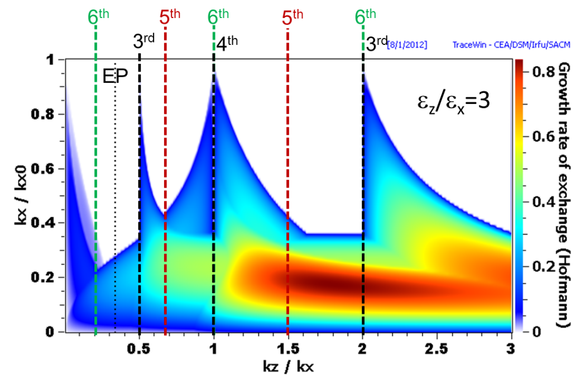

Figure 1: Stability chart for $\epsilon_z/\epsilon_x = 3$ showing stop-bands for 3^{rd} and 4^{th} order resonances as well as the location of potential 5^{th} and 6^{th} order resonances (dashed lines).

- *The charts differ essentially from the commonly used tune diagrams of circular accelerators.* The latter only visualize possible resonances with the focusing lattice. They contain no information about real driving terms (strengths) of resonances, and space charge only enters as incoherent "tune footprint". The stability charts, on the contrary, go far beyond and display these driving terms derived from a self-consistent theory (expressed as growth rates and stop-band widths). The choice of a chart with tune depression versus tune ratio is optimum for these internal space charge resonances as the latter characterizes the order of the mode, and the former the width of the associated stop-bands. A third parameter is needed, which is the ratio of emittances. For a dynamically changing emittance ratio it might be expedient to recalculate the charts.
- *The analytical Vlasov theory behind the charts* was developed as theory of resonant instabilities for an initial 2D KV-distribution, but comparison with 3D PIC-simulation has shown that it can be applied equally to realistic beams. For a uniform initial real space distribution all possible modes are present at noise level, from which they may grow at an exponential rate (indicated by the color code). A parabolic initial density profile, instead, contains already a significant space charge octupole and a predicted resonant instability evolves on a shorter time scale.
- *Only resonances up to fourth order* were considered in the Vlasov analysis due to the increasing complexity with order. Theoretically possible resonance lines could be at $k_z/k_x = m/n$, with $m+n$ the order of the resonance. The theoretical location of resonances of

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order 5 and 6 is indicated in Fig. 1 for orientation only. However, simulation over hundreds of cells has not indicated any evidence that modes with $m+n>4$ play a role. This is not surprising if one keeps in mind that even in synchrotrons resonances above fifth or sixth order are practically ignorable.

- All quantities in the charts are understood as rms quantities. This is consistent with the understanding that emittance coupling is predominantly a beam core effect. A possible effect of emittance coupling on beam halo will be left to future study.
- The equipartition (EP) condition is used in the common form $T=1$, with

$$T \equiv \frac{\epsilon_z k_z}{\epsilon_x k_x}, \tag{1}$$

where all quantities are understood as rms quantities. Obviously, such an rms equipartition definition cannot give the full picture in 6D phase space. It is, however, the most suitable quantity to study the issue of rms emittance transfer. Note that in the remainder of this paper emittance is always understood as rms emittance.

The condition $k_z/k_x = 1$ is of interest in linac design and therefore often referred to as "main resonance". Its stop-band width is the larger the more the emittance ratio $\epsilon_r \equiv \epsilon_z/\epsilon_x$ differs from unity. According to Ref. [7] there is an analytical approximation according to which this width shrinks to zero $\propto (\sqrt{\epsilon_r}-1)$ (provided that the tune depression is not too strong) for approaching emittances, which means EP in this case.

TRACEWIN SIMULATIONS

The simulation examples presented here are all based on a periodic FODO channel (cell length arbitrarily chosen as 0.4 m) with constant or linearly varying phase advance and - for simplicity - a constant RF kick per cell. The current is chosen such as to yield a tune depression typical for high current linacs. Acceleration is ignored here, because the only parameters that matter are the ratio of phase advances per cell, the tune depression and the ratios of normalized emittances. Since we always assume that the initial emittances and phase advances in x and y are equal, we only indicate ratios of z and x quantities at start. The initial distribution follows the TRACEWIN option of randomly distributed particles in transverse phase space (waterbag) and in longitudinal phase space, with usually 10^5 particles and a 3D Poisson solver (PICNIC).

Crossing of the "Main Resonance"

The tune footprint of a simulation with $\epsilon_z/\epsilon_x = 3$ is shown in the top of Fig. 2, where $k_{z0} = 69^o$ and k_{x0} kept 85^o over 10 cells (4 m), then ramped to 65^o over 5 cells only. There is practically no emittance exchange - besides

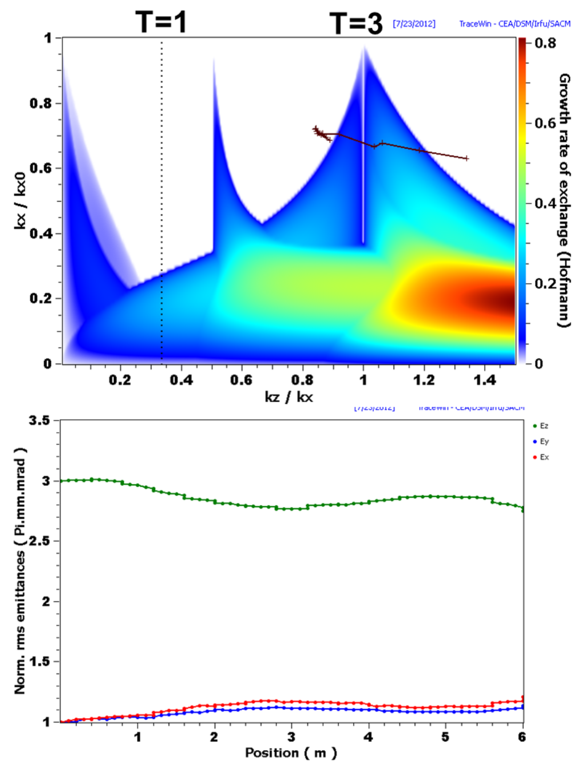

Figure 2: Top: Stability chart for $\epsilon_z/\epsilon_x = 3$ and crossing over main resonance in 5 cells; bottom: evolution of emittances (ϵ_x: red, ϵ_y: blue, ϵ_z: green line).

a small oscillation due to the proximity of the initial k_z/k_x to the stop-band as shown in Fig. 2 (bottom). Stretching the tune ramp to 20 cells leads to full exchange, with even some overshoot, as shown in Fig. 3. Note that the more complete exchange in this case leads to less swing in k_z/k_x, which is calculated self-consistently in TRACEWIN. According to Ref. [7] there is a scaling expression for partial emittance exchange: $\propto \frac{((\sqrt{\epsilon_r}-1)\Delta k_z)^2}{\dot{k_r}}$. Here $k_r \equiv k_z/k_x$, Δk_z the tune depression by space charge, and $\dot{k_r}$ the rate of change.

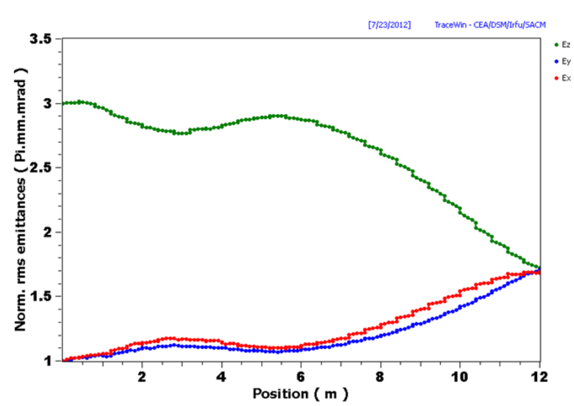

Figure 3: Chart (top) and emittance evolution (bottom) for crossing in 20 cells (ϵ_x: red, ϵ_y: blue, ϵ_z: green line).

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Crossing in the opposite direction (k_z/k_x from right to left), where k_{x0} is first kept constant at 65^o, then ramped to 85^o, yields similar results. For the tune ramp over 20 cells this is shown in the top of Fig. 4. A striking feature is the gradual splitting of the two transverse emittances in spite of the initial equipartition between x and y. It can be assumed that the selection of $\epsilon_x > \epsilon_y$ follows from a small initial matching imbalance between x and y, which possibly gets enhanced by a "three-mode" coupling resonance. The splitting effect is even more pronounced for slower crossing in 50 cells as shown in Fig. 4, where ϵ_x overshoots EP and ϵ_z seems to be attracted by ϵ_x. This phenomenon shows that the approach to EP can be quite complex in full 3D, which warrants further study.

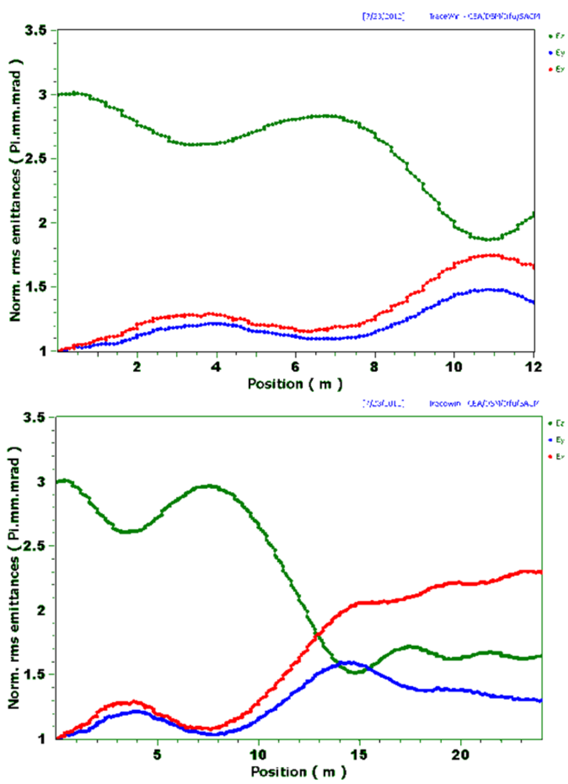

Figure 4: Emittances for reversed crossing in 20 cells (top) and 50 cells (bottom) with splitting phenomenon (ϵ_x: red, ϵ_y: blue, ϵ_z: green line).

Beyond "Main Resonance"

It may be of interest to sweep the tune ratio over the "white" space beyond the main resonance, which in real linacs requires stronger longitudinal focusing as enabled by superconducting RF. In our model we first keep k_{x0} constant at 65^o for 10 cells, then ramp it slowly over 200 cells to 50^o, which yields the results shown in Fig. 5. The initial oscillatory exchange results from the proximity of the starting tune ratio to the main stop-band, but there is no further exchange over most of the distance. The onset of weak coupling towards the end can be explained by the approach

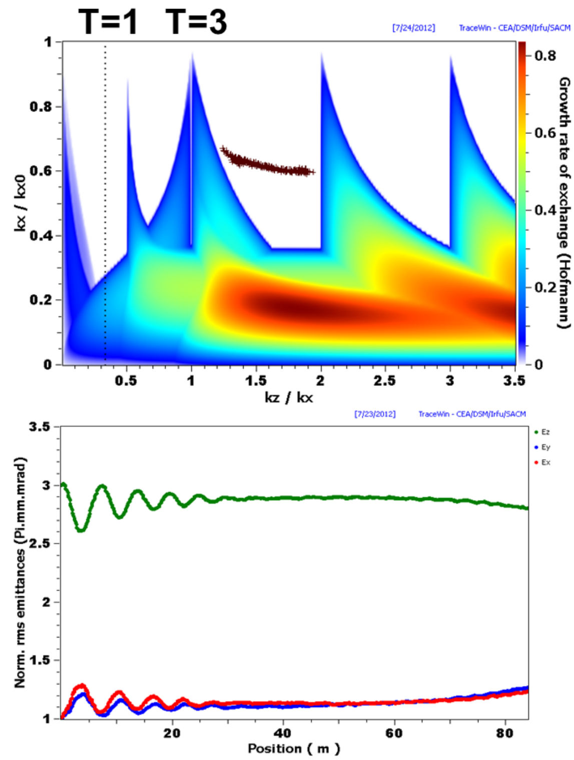

Figure 5: Chart and emittance evolution for tune crossing from left to right over 200 cells (ϵ_x: red, ϵ_y: blue, ϵ_z: green line).

to the third order stop-band at $k_z/k_x = 2$. Besides these "end effects" the stable emittances confirm the absence of effective resonances in the white region.

Absence of the "Main Resonance"

In this section we consider the case $\epsilon_z/\epsilon_x = 1$ initially, which eliminates the "main resonance" in the stability chart as shown in Fig. 6 due to $T = 1$ at the value $k_z/k_x = 1$. As previously in Fig. 4, k_{x0} is first kept constant at 65^o, then ramped to 85^o over 100 cells (with k_x varying from 35^o to 62^o). There is no net emittance transfer, which confirms that the rms definition of equipartition as $T = 1$ and Eq. 1 are reliable on the ground of rms behaviour. The use of this free region has been suggested in an earlier equipartitioned design of the IFMIF-RFQ [8].

Appearance of a "Structure Resonance"

An additional feature appears, if the ramp shown in Fig. 6 is stretched to 200 cells, with all other parameters unchanged. As shown in Fig. 7 the tune footprint in the chart is the same as in Fig. 6 over two thirds of its extent, and bends upwards as result of the substantial transverse emittance growth. This can be explained only as "Structure Resonance" of a third order mode with the periodic focusing. According to Ref. [9] this third order mode only appears for $k_{x0} > 60^o$ and has exponential growth in a

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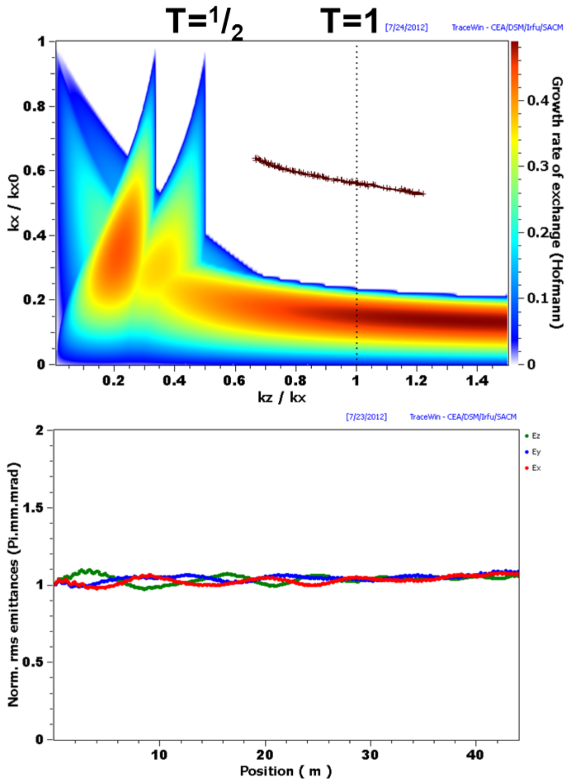

Figure 6: Chart for $\epsilon_z/\epsilon_x = 1$ and emittance evolution for tune sweep from right to left over 100 cells (ϵ_x: red, ϵ_y: blue, ϵ_z: green line).

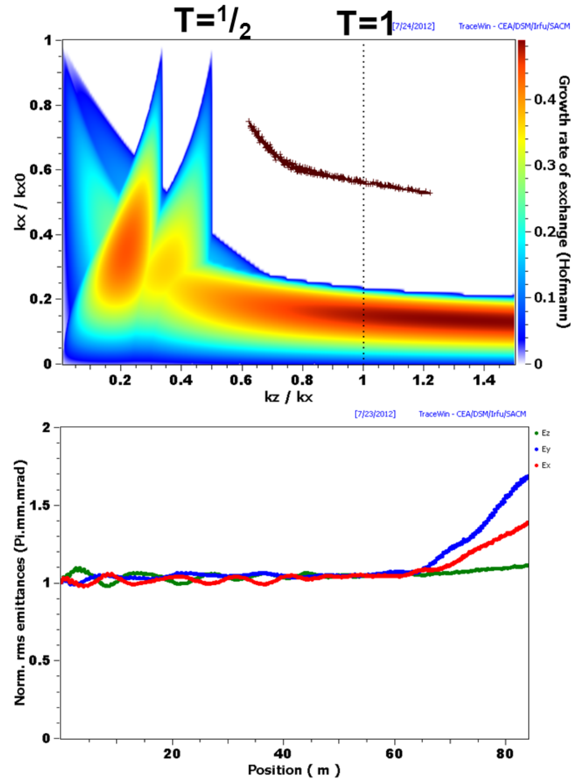

Figure 7: Same as Fig. 6, but sweep over 200 cells.

limited stop-band under $k_x = 60^o$, which is supported by a three-fold symmetry in the phase space plots of our simulation. Note that this structure resonance is complementary to the fourth order structure resonance for $k_{x0} > 90^o$ also suggested in Ref. [9]. A major difference is that the third order mode exists only on the noise level initially and takes longer time to develop, whereas the fourth order mode is already strongly present for an initial waterbag or Gaussian distribution and has a fast emittance degrading effect. This is the reason, why the requirement $k_{x0} < 90^o$ is mandatory for high-current linac design (see also Ref. [10] for an experimental evidence of this fourth order structure resonance). It should be noted that the third order structure resonance is not likely to play a role in most linac designs due to its relatively slow evolution.

Crossing of the "$k_z/k_x = 1/2$ Resonance"

Crossing of the next lower stop-band initiating at $k_z/k_x = 1/2$, and with $T = 1/2$ in our case, requires a lower longitudinal phase advance. This is realized in Fig. 8 for $k_{z0} = 47^o$ and the ramp of k_{x0} from 65^o to 85^o over 100 cells. The longitudinal emittance - initially equal to the transverse emittances - grows by 50%; the shrinking of the two transverse emittances doesn't fully account for the longitudinal growth, however. This might be associated with a transverse structure resonance and requires further study.

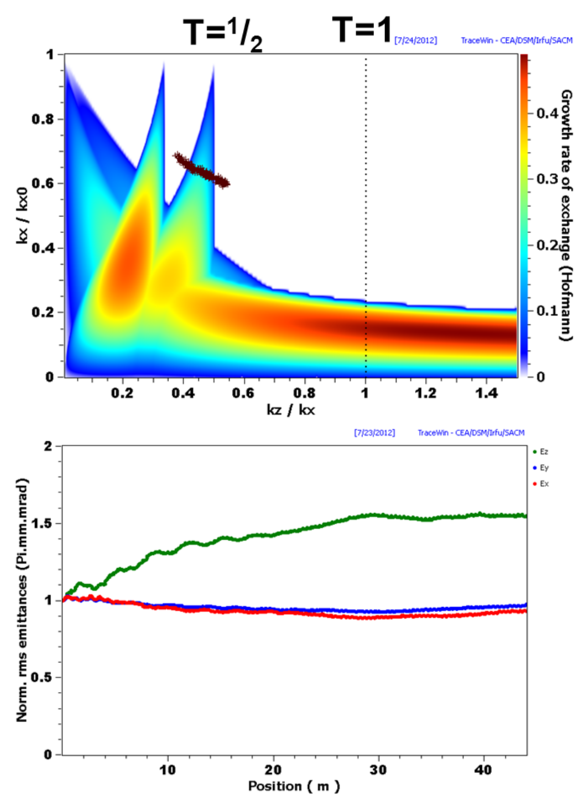

Figure 8: Chart and emittances for same case as in Fig. 6, but k_{x0} lowered to 47^o (ϵ_x: red, ϵ_y: blue, ϵ_z: green line).

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CONCLUDING REMARKS

Our simulations with continuously varying tunes at various rates have confirmed the absence of rms emittance coupling in "white regions" of the stability charts. In "coloured" regions, instead, emittance coupling occurs provided that the crossing of the stop-bands is sufficiently slow. This is consistent with the analytical perturbational theory, upon which the stability charts are based. For the fourth order "main resonance" exchange occurs on a faster time scale than for the third order resonances. This is due to the presence of a space charge pseudo-octupole in the initial beam, whereas the corresponding pseudo-sextupole has to grow from noise. We have also found that some of the resonances may lead to an emittance splitting of the initially (nearly) equipartitioned transverse degrees of freedom, which deserves further study.

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