

# MEASUREMENTS AND INTERPRETATION OF THE BETATRON TUNE SPECTRA OF HIGH INTENSITY BUNCHED BEAM AT SIS-18 \*

R. Singh, O. Boine-Frankenheim, GSI, Darmstadt, Germany and TEMF, TU Darmstadt, Germany  
 O. Chorniy, P. Forck, R. Haseitl, W. Kaufmann, P. Kowina, K. Lang, GSI, Darmstadt, Germany  
 T. Weiland, TEMF, TU Darmstadt, Germany

## ABSTRACT

Two independent tune measurement systems were installed in the GSI heavy ion synchrotron SIS-18. Using these fast and sensitive systems, tune spectra were obtained with high accuracy. Besides the machine tune, the spectra reveal information about the intensity dependent coherent tune shift and the incoherent space charge tune shift. The space charge tune shift is obtained from a fit of the observed shifted positions of the synchrotron satellites to an analytic expression for the head-tail eigenmodes with space charge. Time domain identification of the head tail modes is also performed.

## INTRODUCTION

Accurate measurements of the machine tune and of the chromaticity are of importance for the operation of fast ramping, high intensity ion synchrotrons. In such machines the tune spread  $\delta Q_{x,y}$  at injection energy due to space charge and chromaticity can be up to 0.5. In order to limit the incoherent particle tunes to the resonance free region the machine tune has to be controlled with a precision better than  $\Delta Q \approx 10^{-3}$ . In the GSI heavy-ion synchrotron SIS-18 there are currently two betatron tune measurement systems installed. The Tune, Orbit and Position measurement system (TOPOS) is primarily a digital position measurement system which calculates the tune from the measured position [1]. The Baseband Q measurement system (BBQ) conceived at CERN performs a tune measurement based on the concept of diode based bunch envelope detection [2]. The BBQ system provides a higher measurement sensitivity than the TOPOS system. For fast tune measurements using standard pick-ups, external excitation is often applied for measurement of transverse beam signals. The frequency resolution of both systems depend on the time scale of measurement, the tune fluctuations during the measurement and width of tune peak due to non-linearities in the machine. During acceleration,  $\Delta Q$  achieved is  $\approx 10^{-3}$  limited by length of measurement time window. A higher resolution of  $10^{-4}$  is achievable during injection or extraction plateaus where the main limitation is machine non-linearities and long term beam losses.

For low intensities the theory of transverse signals from bunched beams and the measurement principles are well known [3, 4]. In intense, low energy bunches the tune spec-

tra can be modified significantly by the transverse space charge force and by ring impedances. Previously the effect of space charge on head-tail modes had been the subject of several analytical and simulation studies [5, 6, 7, 8, 9]. Recently these effects were observed experimentally at SIS-18 [10, 11, 12, 13].

This contribution aims to complement the previous studies and extract the relevant intensity parameters from tune spectra measurements using the TOPOS and BBQ tune measurement systems. Section presents the space charge and image current effects on tune measurements and respective theoretical models. Section report on the experimental conditions and compares the beam excitation mechanisms and measurement systems. Section presents the experimental results in comparison with the theoretical estimates of the various high intensity effects.

## TUNE SPECTRUM FOR HIGH INTENSITY BEAM

If the transverse signal from a low intensity bunch is sampled with the revolution period  $T_s$  then the positive frequency spectrum consists of one set of equidistant lines

$$Q_k = Q_0 + \Delta Q_k, \quad (1)$$

usually defined as baseband tune spectrum, where  $Q_0$  is the fractional part of the machine tune,  $\Delta Q_k = \pm k Q_s$  are the synchrotron satellites and  $Q_s$  is the synchrotron tune. For a single particle performing betatron and synchrotron oscillations, the relative amplitudes of the satellites are [3]

$$|d_k| \sim |J_k(\chi/2)| \quad (2)$$

where  $\chi = 2\xi\phi_m/\eta_0$  is the chromatic phase,  $\xi$  is the chromaticity,  $\phi_m$  is the longitudinal oscillation amplitude of the particle and  $\eta_0$  the frequency slip factor.  $J_k$  are the Bessel functions of order  $k$ .

At high beam intensities the transverse space charge force together with the coherent force caused by the beam pipe impedance will affect the motion of the beam particles and also the tune spectrum. The space charge force induces an incoherent tune shift  $Q_0 - \Delta Q_{sc}$  for a symmetric beam profile of homogeneous density where

$$\Delta Q_{sc} = \frac{qI_p R}{4\pi\epsilon_0 c E_0 \gamma_0^2 \beta_0^3 \epsilon_x} \quad (3)$$

is the tune shift,  $I_p$  the bunch peak current,  $q$  the particle charge and  $E_0 = \gamma_0 m c^2$  the energy. The relativistic parameters are  $\gamma_0$  and  $\beta_0$ , the ring radius is  $R$  and the emittance of the rms equivalent K-V distribution is  $\epsilon_x$ .

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In the case of an elliptic transverse cross-section the emittance  $\varepsilon_x$  in Eq. 3 should be replaced by

$$\frac{1}{2} \left( \varepsilon_x + \sqrt{\varepsilon_x \varepsilon_y \frac{Q_{0x}}{Q_{0y}}} \right) \quad (4)$$

For the vertical plane the procedure is the same, with  $x$  replaced by  $y$ . The image charges induced in the beam pipe, which is assumed here to be perfectly conducting, cause a purely imaginary horizontal impedance

$$Z_x = -i \frac{Z_0}{2\pi(\beta_0 \gamma_0 b_x)^2} \quad (5)$$

and real coherent tune shift

$$\Delta Q_c = -i \frac{qIR^2 Z_x}{2Q_{x0} \beta_0 E_0} \quad (6)$$

For a round beam profile with radius  $a$  and pipe radius  $b$  the coherent tune shift is smaller by  $\Delta Q_c = \frac{a^2}{b^2} \Delta Q_{sc}$  than the space charge tune shift. Therefore the contribution of the pipe is especially important for thick beams at low or medium energies.

In the presence of incoherent space charge, represented by the tune shift  $\Delta Q_{sc}$ , and pipe effects, represented by the real coherent tune shift  $\Delta Q_c$ , the shift of the synchrotron satellites in bunches can be reproduced rather well by [6]

$$\Delta Q_k = -\frac{\Delta Q_{sc} + \Delta Q_c}{2} \pm \sqrt{(\Delta Q_{sc} - \Delta Q_c)^2/4 + (kQ_s)^2} \quad (7)$$

where the + is used for  $k > 0$ . For  $k=0$  one obtains  $\Delta Q_{k=0} = -\Delta Q_c$ . The above expression represents the head-tail eigenmodes for an airbag bunch distribution in a barrier potential [5] with the eigenfunctions

$$\bar{x}(\phi) = \cos(k\pi\phi/\phi_b) \exp(-i\chi\phi/\phi_b) \quad (8)$$

where  $\bar{x}$  is the local transverse bunch offset,  $\chi = \xi\phi_b/\eta_0$  is the chromatic phase,  $\phi_b$  is the full bunch length and  $\eta_0$  is slip factor. The head-tail mode frequencies obtained from Eq. 7 are shown in Fig. 1 for two cases, i.e. only space charge ( $\Delta Q_c = 0$ ), and the practical case when ( $\Delta Q_c = \frac{\Delta Q_{sc}}{10}$ ). In Ref. [5] the analytic solution for the eigenvalues (Eq. 7) is obtained from a simplified approach, where the transverse space charge force is assumed to be constant for all particles. This assumption is correct if there are only dipolar oscillations. In Ref. [6], Eq. 7 has been successfully compared to Schottky spectra obtained from 3D self-consistent simulations for realistic bunch distributions in rf buckets. It has also been pointed out, that the negative- $k$  eigenmodes are suppressed due to intrinsic Landau damping. Analytic and numerical solutions for Gaussian and other bunch distribution valid for  $q_{sc} \gg 1$  were presented in [7, 8].

For head-tail modes the space charge parameter is defined as a ratio of the space-charge tune shift (Eq. 3) to the small-amplitude synchrotron tune ( $Q_{s0}$ ),

$$q_{sc} = \frac{\Delta Q_{sc}}{Q_{s0}} \quad (9)$$

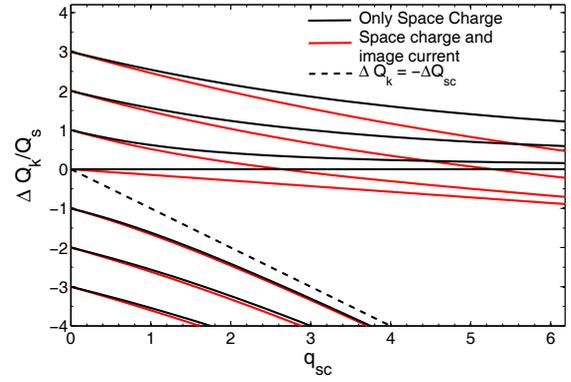


Figure 1: Head-tail mode frequencies as a function of the space charge parameter. The red curves represent the result obtained for  $q_c = q_{sc}/10$  and the coherent intensity parameter as

$$q_c = \frac{\Delta Q_c}{Q_{s0}} \quad (10)$$

## MEASUREMENT SET-UP

This section presents the types of beam excitation used in the measurements. The beam excitation method is common to both TOPOS and BBQ systems. A brief comparison of both the tune measurement systems and the experimental conditions is given.

### Beam Excitation Methods

Excitation types such as band limited noise and frequency sweep are utilized at various power levels to induce coherent oscillations 50  $\Omega$  terminated stripline exciters.

**Band limited noise:** The generation of this signal is done in the following way; RF is mixed with DDS generated fractional tune frequency, resulting in RF harmonics and their respective tune sidebands. This signal is further modulated by a pseudo random sequence resulting in a band limited noise source around the tune frequency. The procedure provides an efficient tunable noise source available during the whole acceleration ramp. The main disadvantage is the difficulty in correlation of the resultant tune spectrum with the excitation signal.

**Frequency sweep:** Tune measurements during acceleration using frequency sweep excitation is not trivial during the whole acceleration cycle. However, this method offers advantages compared to the previous method for careful interpretation of tune spectrum in storage mode, e.g., injection or extraction plateaus. Frequency sweep is used during measurements at injection plateau to compare and to understand the output of noise excitation type.

### Comparison of TOPOS and BBQ

The main difference in the BBQ and TOPOS system is the noise filtering. In BBQ, after bunch envelope detection,

Table 1: Beam Parameters During the  $U^{73+}$ -Experiment

Beam/Machine parameters	Values
Atomic mass(A), Charge state(q)	238, 73
$Q_x, Q_y$	4.31, 3.27
$\xi_x, \xi_y$	-0.94, -1.85
Transverse emittances( $\epsilon_x, \epsilon_y$ )( $2\sigma$ )	45, 22 mm-mrad
Slip factor( $\eta$ )	0.94
Bunching factor ( $B_f$ )	0.4
Synchrotron tune ( $Q_{s0}, Q_{s1}$ )	0.007, 0.0065

the signal chain has a high-order low-pass filter response with cut-off at 1 MHz. In TOPOS, the whole bunch signal traverses the wideband amplifier chain and each bunch is integrated to determine position. Integration serves as a first order low pass filter whose response is defined by the number of samples integrated per bunch (in the range of 10-80). TOPOS and BBQ have been utilized as independent and often complementary systems for these measurements. It also provides a cross-check on the reliability of the results while ruling out any system dependent errors. Detailed description of both systems can be found in [1, 2, 11].

### Beam Parameters During the Measurements

Experiments were done using  $N^{7+}$  and  $U^{73+}$  ion beams at the SIS-18 injection energy of  $\approx 11.4$  MeV/u. The measurements were taken during 600 ms long plateaus. At injection energy space charge effects are usually strongest. Four bunches are formed from the initially coasting beam during adiabatic RF capture. The experiment was repeated for different injection currents. At each intensity level several measurements were performed with different types and levels of beam excitation in both planes. Tune measurements were done simultaneously using the TOPOS and BBQ systems. The beam current and the transverse beam profile are measured using the beam current transformer [14] and the ionization profile monitor (IPM) [15] respectively. The measured dipole synchrotron tune  $Q_{s1}$  has been used as an effective synchrotron tune for all experimental results and will be referred as  $Q_s$  from hereon. Important beam parameters during a typical experiment ( $U^{73+}$ ) are given in Table 1.

## EXPERIMENTAL OBSERVATIONS

Selected experimental results are shown in this section. For more details, see [13].

### Modification of the Tune Spectrum with Intensity

Figure 2 shows the horizontal tune spectra (obtained with the BBQ system using band-width limited noise) at different intensities. The plot at the bottom shows the horizontal tune spectrum at low intensity. Here the  $k =$

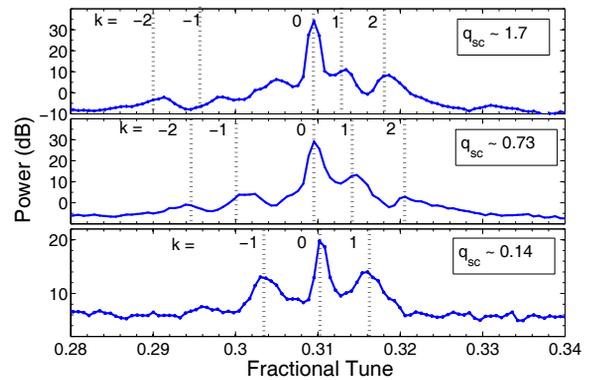


Figure 2: Horizontal tune spectra with  $U^{73+}$  ions for beam conditions given in Table 1. The space charge parameters ( $q_{sc}$ ) are  $\approx 0.15, 0.7$  and  $1.7$  from bottom to top (see text). The dashed lines indicate the head-tail tune shifts from Eq. 7(see text).

$1, 0, -1$  lines are almost equidistant, which is expected for low intensity bunches. The vertical lines indicate the positions of the synchrotron satellites obtained from 7 (with  $Q_s = Q_{s1}$  and  $\Delta Q_{sc}$  predicted by Eq. 3). The space charge parameter obtained from this procedure is  $q_{sc} \approx 0.14$ . Figure 2 (middle) shows the tune spectrum at medium intensity ( $q_{sc} \approx 0.7$ ). The  $k = 2, -2$  lines can both still be identified. The upper plot in Fig. 2 shows the tune spectra at high intensity ( $q_{sc} \approx 1.7$ ). The  $k = 0, 1, 2$  lines can be identified very well, whereas the amplitudes of the peaks for negative  $k$  start to decrease (see Section ).

The width of the mode lines during the measurement is determined by the cumulative effect of non-linear synchrotron motion, non-linearities of the optical elements, closed orbit distortion, tune fluctuation during the measurement interval as well as due to space charge induced "intrinsic" Landau damping.

### Determination of Coherent and Incoherent Tune Shifts

The coherent tune shift  $\Delta Q_c$  can be obtained by measuring shift of the  $k = 0$  line as a function of the peak bunch current as shown in Fig.3. The transverse impedances is obtained by a linear least square error fit of the measured shifts in both planes to Eq. 5 and Eq. 6. The impedance values obtained in horizontal and vertical planes at injection energy are  $0.23 \text{ M}\Omega/m^2$  and  $1.8 \text{ M}\Omega/m^2$  respectively, which agrees very well with the expected values for the average beam pipe radii. The error bars in the horizontal plane are given by uncertainties in current measurements, and in the vertical plane by the width of  $k = 0$  mode. The FFT resolution is  $\approx 5 \cdot 10^{-4}$  which is smaller than the mode width.

Figures 4 and 5 show the measured positions of the lines for different intensities together with the analytical curves from Eq. 7 using the  $\Delta Q_{sc}$  estimated from the beam pa-

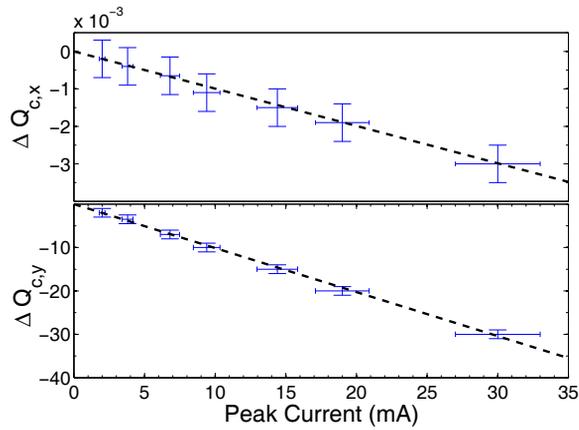


Figure 3: Coherent tune shift obtained from the measurement for the horizontal and for the vertical planes as a function of the peak beam current. The dashed lines is the result of a least square error fit on the measured points.

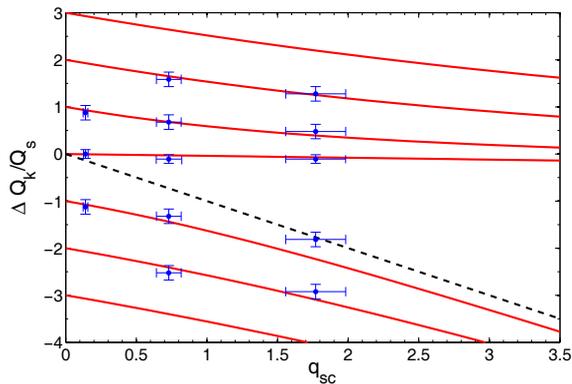


Figure 4: The measured positions of the lines in the horizontal tune spectra for different  $U^{73+}$  beam intensities overlaid on the analytical curves from Eq. 7 using the space charge tune shift estimated from the beam parameters in Table 1. The dashed line corresponds to the analytical incoherent tune shift.

rameters in Table 1. The error bar on vertical plane correspond to the width of measured lines. The horizontal error bars are estimated by propagation of uncertainties of each measured parameter. The incoherent space charge tune shift can be determined directly from the tune spectra by comparing the separation between the  $k = 0$  and  $k = 1$  lines to Eq. 7. The space charge tune shifts measured from the tune spectra using the mentioned procedure are found to be systematically lower by a factor  $\approx 0.75$  than the predicted shifts in the measured range of  $q_{sc} \lesssim 3.5$ . This might be due to simplified airbag model used in the analytical calculations for Eq. 7.

### Time Domain Identification of Head-tail Modes

The frequency sweep allows to resolve the different modes both spectrally and temporally as shown in Fig. 6.

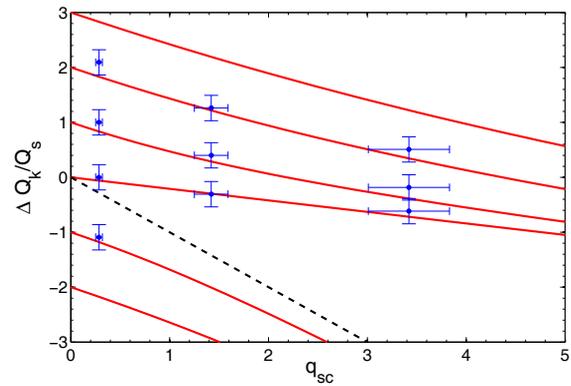


Figure 5: The measured positions of the lines in the vertical tune spectra for different  $U^{73+}$  beam intensities overlaid on the analytical curves from Eq. 7 (see Table 1).

The various modes get individually excited as the sweep frequency co-incides with the mode frequency. The  $k = 0, 1, 2$  modes are marked with dashed lines. Symmetric sidebands due to coherent synchrotron oscillations are visible around the sweep frequency spaced with  $Q_s$  and are clearly distinguished from the head-tail modes. Figure 7 shows the turn by turn transverse center of mass along the bunch for each mode  $k = 0, 1, 2$  for 10 consecutive turns as they are individually excited (corresponding to Fig. 6). This allows the possibility of identifying each head-tail mode temporally and confirm the spectral information obtained. This is an exclusive feature of TOPOS system.

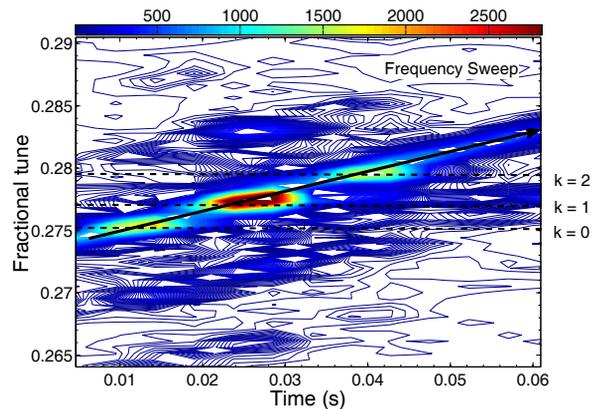


Figure 6: Contour plot for vertical tune spectra with frequency sweep over time at  $1.5 \cdot 10^9$  stored  $N^{7+}$  particles. The arrow marks the frequency sweep, and the dashed lines indicate the mode number of excited head-tail mode.

## APPLICATIONS

Another important point is the tune measurement during acceleration. The space charge parameter for heavy ions in the SIS-18 from injection to extraction reduces only by  $\approx 20\%$ . The dynamic shift of head-tail modes during acceleration is shown in Fig. 8 and the asymmetry of  $k = 1, -1$  modes around  $k = 0$  mode can only be un-

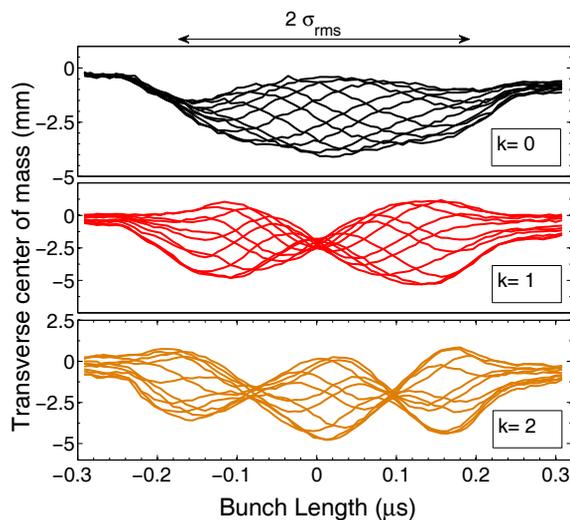


Figure 7: Turn by turn transverse center of mass along the bunch for  $k = 0, 1, 2$  modes for 10 consecutive turns corresponding to Fig. 6. Frequency sweep is used to excite the beam which enables to resolve the distinct head modes temporally.

derstood in view of the space charge effects predicted by Eq. 7. The head-tail modes are getting closer since the synchrotron tune reduces with acceleration, but the asymmetry of the  $k = 1$  and  $k = -1$  modes around  $k = 0$  which is maintained throughout the ramp depends primarily on the space charge parameter ( $q_{sc}$ ). Thus a correct estimate of this parameter plays an important role in understanding the tune spectra not only during dedicated experiments on injection plateau but also during regular operations. Further applications are discussed in [13].

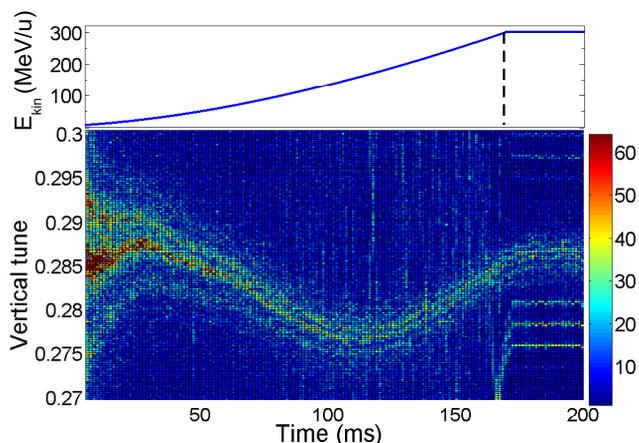


Figure 8: The tune spectra showing movement of head-tail modes during acceleration for  $1 \cdot 10^{10} A_r^{18+}$  ions using band limited noise excitation.

## SUMMARY AND OUTLOOK

The methods used for tune measurement at GSI heavy ion synchrotron SIS-18 are discussed. The predicted mod-

ification of tune spectra due to space charge effects based on analytical models were studied. Coherent and incoherent tune shift for bunched beams at GSI SIS-18 at injection energies were directly measured experimentally using frequency shift of head-tail modes. These measurements bring better understanding of tune spectra during acceleration and opens a new domain in precise beam diagnostics based on excitation of stable head-tail modes.

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