SPACE CHARGE EFFECTS IN ISOCHRONOUS FFAGS AND CYCLOTRONS*

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Abstract

Effects of space charge forces on the beam dynamics of isochronous rings will be discussed. Two different kinds of phenomena will be introduced through a brief review of the literature on the topic. The first one is a consequence of the very weak vertical focusing found in the low energy region of most cyclotrons. The space charge tune shift further reduces the vertical focusing, setting an upper limit on instantaneous current. The second one arises from the fact that longitudinal phase space is frozen in isochronous rings. This leads to effects of space charge forces which are very peculiar to isochronous machines. We will present the simulation tools being developed at TRIUMF to study these effects.

LITERATURE REVIEW

In this section we present an overview of current knowledge concerning the effect of space charge in isochronous accelerators through a review of the major contributions to that field over the last forty years.

Transverse Space Charge Effects

In cyclotrons at low energy, the vertical focusing comes mostly from the azimuthal dependence of the magnetic field, through edge focusing. In compact cyclotrons, where the radius of the injection orbit is comparable to the magnet gap, the azimuthal field dependence is smoothed out, leading to a very weak vertical focusing (vertical tune $\nu_z \ll 1$). The circulating current is then ultimately limited by the defocusing effect of space charge forces. The current limit is reached when the vertical focusing nearly vanishes, leading to beam losses on the vertical apertures.

This effect is well-known in the synchrotron theory as the space charge driven incoherent tune shift. In the case of non-relativistic particles, a number of textbooks give the following formula [1]:

$$\Delta(\nu_z^2)_{SC} = -\frac{2}{\pi}\frac{NRr_p}{\beta^2\gamma^3 B_f}\left[\frac{1}{b(a+b)} + \frac{\epsilon_1}{h^2}\right], \qquad (1)$$

where N is the number of particles and B_f is the bunching factor, R is the orbit radius, r_p is the classical proton radius (1.54×10^{-18} m), β and γ are relativistic factors, a and b are the horizontal and vertical beam half-size, resp., h is the metal chamber half-height, and ϵ_1 is the Laslett image coefficient, $\simeq 0.2$ for parallel plates.

The β^{-2} factor in Eq. 1 reflects the strong energy dependence of the incoherent tune shift. One way to push further away this current limitation is to increase the injection energy. This is the reason why high-current cyclotrons use external ion sources capable of producing beams of a few hundred keV (300 keV at TRIUMF, 870 keV at PSI). Using even higher injection energy is in principle possible. One major drawback of this approach is that, for a given beam current, increasing the beam energy means increasing the beam power. For high-current machines, even a small fraction of beam loss at injection can lead to high-density power deposited onto the central region.

Another way to overcome this limitation is simply to increase the vertical focusing. This second approach, however, comes necessarily at the expense of the compactness of the machines. This is because, in the absence of radial field dependence, a strong vertical focusing is only possible if the particles experience strong azimuthal variation of the magnetic field.

Typically, cyclotrons in the 10's of MeV range are made as compact as possible to reduce cost. As a result, injection energy is a few 10's of keV and the first few turns have radius comparable to the magnet gap or smaller and there is no magnetic focusing. Use is made of rf focusing: injecting on the falling side of the waveform results in vertical focusing. Roughly, the contribution to ν_z^2 is proportional to $\cos\phi$ where the energy gain is proportional to $\sin\phi$. The result is that along the bunch, the head has $\nu_z \approx 0$ and the tail has $\nu_z \approx 0.1$ to 0.2. As current is raised, space charge begins to progressively "eat" the head of the bunch [2].

Longitudinal-transverse Effects

The absence of longitudinal focusing in isochronous accelerators, as in synchrotrons at transition, gives rise to a very peculiar effect of longitudinal space charge forces.

Detailed studies of this phenomenon were carried out by M.M. Gordon [3]. The analytical model, assumed a continuous radial beam density (*i.e.* non-separated turns) and led to the prediction of space charge induced energy spread. This study also provides the first experimental evidence of current dependent energy spread, observed at the few μA level in the MSU cyclotron. Later, W. Joho generalized the description of this phenomenon to the case of separated turns [4].

The space charge induced energy spread has current-limiting consequences when extracting a high-power beam from a cyclotron through a magnetic channel. Through dispersion, this energy spread translates into an increase of the radial size of the beam, compromising the turn separation.

A major milestone in the study of longitudinal space charge effects in the absence of phase stability is the demonstration by W.J.G.M. Kleeven that a charge distri-

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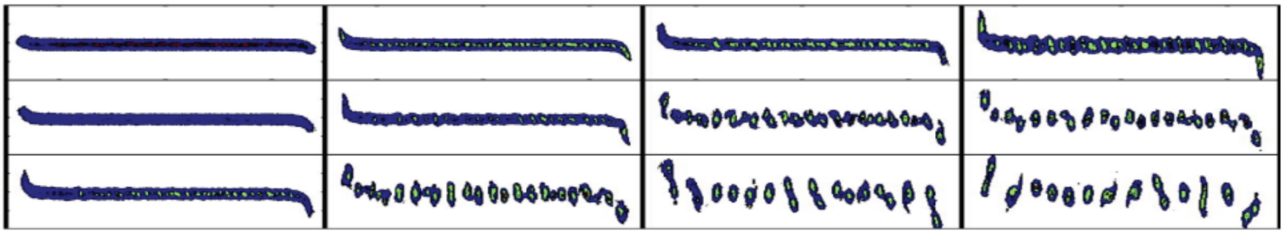

Figure 1: Simulation results obtained with CYCO – a 3D particle-in-cell (PIC) tracking code – showing space charge driven break up of a long beam. From Pozdeyev, Rodriguez, Marti, 2009 [7]. The progression is from left to right being after 4, 8, 12, and 16 turns; the top 4 frames are at 5 μA, the middle 4 at 10 μA, and the bottom 4 at 20 μA. Notice that besides the breaking up into droplets, the bunches also become S-shaped; the head (left) edge traveling to higher radius and the tail traveling to lower. This latter effect is the one predicted by Gordon and by Joho.

bution which is circularly symmetric, whatever the amount of charge in it, is stationary [5]. In other words, in the case of a short beam, any bunch charge distribution which is not rotationally symmetric in the horizontal plane will keep evolving until the bunch is circular.

In the case of a long beam, the charge distribution breaks up into several droplets. First simulation results showing such a space charge driven beam break up are found in Pozdeyev's thesis [6]. These simulations were made possible by the development of the 3D particle-in-cell (PIC) Runge-Kutta based tracking code CYCO. The first experimental evidence of beam fragmentation of a long bunch caused by longitudinal space charge was was obtained with SIR, a Small Isochronous Ring, built for beam physics studies [7]. Simulation and experimental results showing beam fragmentation, which are found in Ref. [7], are presented in Fig. 1& 2.

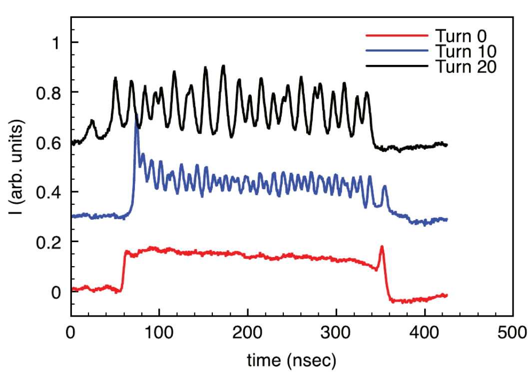

Figure 2: Experimental evidence of space charge driven beam break up obtained with the Small Isochronous Ring (SIR) at MSU. From Pozdeyev, Rodriguez, Marti, 2009 [7]

Recently, A. Adelmann has developed a 3D parallel PIC code OPAL [8], which can include more general bunch configurations, including the effects of radially neighbouring bunches such as occurs in a cw cyclotron. Effects of the neighboring turns in the PSI 590 MeV ring cyclotron were simulated with OPAL [9]. One of the interesting results of this study is that neighboring turns improve rather than worsen turn separation.

Very recently (within the past year), A. Cerfon has developed a Vlasov equation approach to the space charge calculation in isochronous machines [10]. This approach has promise for greater insights, since it is not subject to the spurious effects of the granularity inherent in macroparticle simulations.

SIMULATION TOOLS UNDER DEVELOPMENT AT TRIUMF

The TRIUMF 500 MeV cyclotron accelerates H^- ions, and uses charge exchange extraction. No turn separation is required for extraction, which allows a very large phase acceptance of this machine (about 60°) [11]. Bunches are very long, and have a very large energy spread between the head and the tail of a bunch. Each bunch therefore occupies a large volume in real space. Solving Poisson equation in a PIC code over such a large volume would require a significant computation time.

In addition, at high energy the turn separation is several times smaller than the radial beam size. It is therefore essential to take into account the effect of many neighboring turns. The multibunch calculation used in OPAL, is most appropriate when bunch length and width are comparable, but in the TRIUMF case, the bunch length can be 400 times its width.

If one makes the assumption that the beam shape evolves slowly compared to the turn to turn time scale, one can significantly reduce the computation time by using periodic boundary conditions in the radial direction. This idea was originally proposed by Pozdeyev as a possible way to improve his code CYCO [12]. In the following section we will present how this idea is being implemented at TRIUMF.

Principle of the Algorithm

We choose the radial dimension of the box inside which Poisson equation are solved equal to the turn separation. Particles of the bunch that fall out of this box are returned to the box assuming radial periodicity (see Fig. 3). In fact, these particles appear to belong to the neighboring turns.

The charge density in this 3D box is divided onto slices cut along the y direction (see Fig. 3). To take into account the image charge, we use “metallic” boundary conditions in

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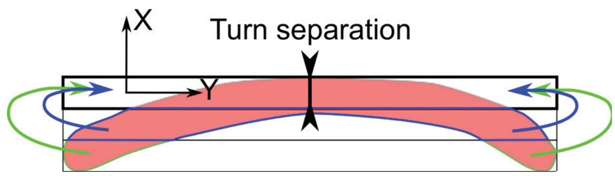

Figure 3: Cartoon illustrating how the test bunch is sliced in the radial direction assuming radial periodicity.

the vertical direction; to simulate the effect of neighboring turns, we use periodic boundary conditions in the radial direction (see Fig. 4).

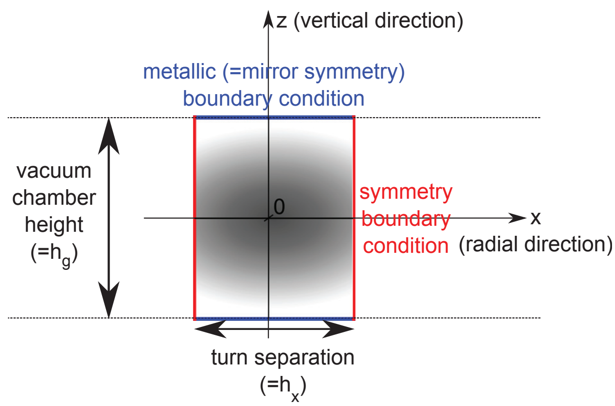

Figure 4: Schematic view of a beam "slice", with two types of boundary conditions on the edges.

For each of these slices, the charge distribution can now be expressed as a sum of Fourier harmonics:

$$\sigma(x,z) = \sum_{lm} \sigma_{lm} \exp(i2\pi x \frac{l}{h_x}) \cos(2\pi z \frac{m}{h_z}), \quad (2)$$

with $h_z = 2h_g$. To simplify the notations let's define:

$$\omega_l = 2\pi \frac{l}{h_x}, \quad (3)$$

$$\omega_m = 2\pi \frac{m}{h_z}. \quad (4)$$

$\sigma(x,z)$ becomes:

$$\sigma(x,z) = \sum_{lm} \sigma_{lm} \exp(i\omega_l x) \cos(\omega_m z). \quad (5)$$

Let's now assume that the potential produced by the (l,m) harmonic can be written in the form:

$$\phi_{lm}(x,z,y) = \exp(i\omega_l x) \cos(\omega_m z)\ f_{lm}(y) + \text{const.} \quad (6)$$

We chose $\text{const.} = 0$ (which is to say, we choose the potential along the metallic boundaries = 0).

To find $f_{lm}(y)$, let's solve the Laplace equation (in Cartesian coordinates):

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2} + \frac{\partial^2}{\partial y^2}\right) \phi = 0, \quad (7)$$

which yields for the (l,m) harmonic to:

$$\frac{\mathrm{d}^2 f_{lm}}{\mathrm{d}y^2} - \left(\omega_l^2 + \omega_m^2\right) f_{lm} = 0. \quad (8)$$

The general solution of this equation can be written as:

$$f_{lm} = C_1 \exp(\omega_{lm} y) + C_2 \exp(-\omega_{lm} y), \quad (9)$$

with C_1 and C_2 two real numbers, and $\omega_{lm} = \sqrt{\omega_l^2 + \omega_m^2}$.

Now, let's bring back some physics in this matter: $f_{m,l}$ must tend to 0 far from the plane containing the charges. This means that the solution we are looking for can be written as:

$$f_{lm} = C_{lm} \exp(-\omega_{lm}|y|). \quad (10)$$

To find the value of the constant C_{lm}, let's calculate the electric potential using the Coulomb law at $x = y = z = 0$ ($\omega_m \neq 0$):

$$\phi_{lm}(0,0,0) = C_{lm} \quad (11)$$

$$= \iint_{-\infty}^{+\infty} \frac{\sigma_{lm} \exp(i\omega_l x) \cos(\omega_m z)}{\sqrt{x^2+z^2}} \mathrm{d}x\mathrm{d}z \quad (12)$$

$$= 2\pi \frac{\sigma_{lm}}{\sqrt{\omega_l^2 + \omega_m^2}} \quad (13)$$

$$= 2\pi \frac{\sigma_{lm}}{\omega_{lm}}. \quad (14)$$

The potential created by one slice of the beam is the sum of the contribution of each harmonic:

$$\phi(x,y,z) = \sum_{lm} \exp(i\omega_l x) \cos(\omega_m z)\ C_{lm} \exp(-\omega_{lm}|y|). \quad (15)$$

The potential created by the whole beam is the sum of the potential created by each slice:

$$\phi_{tot}(x,y,z) = \sum_{lm} D_{lm} \exp(i\omega_l x) \cos(\omega_m z), \quad (16)$$

with:

$$D_{lm} = \sum_{n} C_{lmn} \exp(-\omega_{lm}|y - y_n|), \quad (17)$$

if we call y_n the y coordinate of the n^{th} slice. One now gets ϕ_{tot} in each slice from the inverse Fourier transform of D_{lm}.

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