Abstract

The minimum transverse emittances achieved in a beam line are determined by the two transverse eigen-emittances of the beam. Without coupling, they are equal to the transverse rms-emittances. Eigen-emittances are constants of motion for all symplectic beam line elements. To allow for rms-emittance transfer, the eigen-emittances are changed by a non-symplectic action to the beam, preferably preserving the four-dimensional rms-emittance.

Unlike emittance swapping, the presented concept will allow the transformation of a beam of equal rms-emittances into a beam of different rms-emittances while preserving the four-dimensional rms-emittance. This contribution will introduce the concept for eigen-emittance shaping and rms-emittance transfer at an ion beam line. The actual work status towards the experimental demonstration of the concept at the GSI UNILAC is presented.

INTRODUCTION

For injection of beams into circular machines with different horizontal and vertical emittance acceptances, the injection efficiency can be increased if these beams are flat. However, beams provided from the linear accelerator are generally round, and the horizontal and vertical emittances are quite equal.

Round-to-flat transformation requires a change of the beam eigen-emittances by a non-symplectic transformation [1]. Such a transformation can be performed by placing a charge state stripper foil inside a longitudinal field region as proposed in [2]. Inside such a solenoidal stripper, the transverse inter-plane correlations are created non-symplectically. Afterwards they are removed symplectically with a coupling correction section. The new set-up providing round-to-flat transformation is shown in Fig. 1. Such an emittance transfer section is proposed to be integrated into the existing beam line between the UNILAC [3] and the SIS synchrotron.

EMITTANCE

The four-dimensional symmetric beam matrix $C$ contains ten unique elements, four of which describe the coupling. The rms-emittances, $\varepsilon_x$ and $\varepsilon_y$, are defined as the square roots of the determinants of the on-diagonal submatrices. If one or more of the elements of the off-diagonal submatrix is non-zero, the beam is $x$-$y$ coupled. Diagonalization of the beam matrix yields the beam eigen-emittances, $\varepsilon_1$ and $\varepsilon_2$, and the values are calculated as:

$$\varepsilon_1 = \frac{1}{2} \sqrt{-tr(CJ)^2 - \sqrt{tr^2(CJ)^2 - 16|C|}}$$  
$$\varepsilon_2 = \frac{1}{2} \sqrt{-tr(CJ)^2 + \sqrt{tr^2(CJ)^2 - 16|C|}}$$

The four-dimensional matrix $J$ is the skew-symmetric matrix with non-zero entries on the block diagonal of form. Eigen-emittances are invariant under symplectic transformations, and the eigen-emittances are equal to rms-emittances when the inter-plane correlations are zero.

BEAM TO BE STRIPPED

Multi-particle beam dynamics simulations have been done using the TRACK code [4]. The uncoupled particle distribution at the entrance of this beam line is concluded from beam experiments and plotted in Fig. 2.
the solenoid is calculated by the OPERA-3D code [6], and the stripper foil is placed at the center. The oscillations of the rms-emittances and eigen-emittances along the longitudinal field with and without stripper are shown in Fig. 3.

![Figure 3: The rms-emittances and eigen-emittances along the longitudinal field without and with stripper foil.](image)

Without the stripper, the eigen-emittance variation which is created by the entrance fringe field would be canceled by the exit fringe field and the beam experiences a symplectic transformation. The input and output rms-emittances are same. If the stripper is considered, the exit fringe is passed by the beam with reduced rigidity, thus overcompensating the eigen-emittances variation, and at the exit of longitudinal field region, a coupled output beam is achieved.

The foil stripper itself is modeled by increasing the spread of the angular distribution through scattering. Because of scattering the rms-emittances increase during the stripping process, and the output rms-emittances are larger than the input values. The particle distributions before and after the stripper are illustrated in Fig. 4.

![Figure 4: The particle distributions before and after foil.](image)

**COUPLING CORRECTION SECTION**

The coupling correction section starts at the exit of the longitudinal field, which is then followed by a normal triplet and a skew triplet separated by appropriate drift space. A simple realization of such a system is sufficient to remove inter-plane correlations exhaustively. Along the subsequent elements after the solenoid, the gradients of normal and skew triplet are determined numerically to correct four \( \langle xy \rangle, \langle x'y' \rangle, \langle x'y \rangle, \text{ and } \langle xy' \rangle \) beam correlations, respectively. This scheme allows total correction of any arbitrary linearly coupled beam with correction range limited only by the available quadrupole strength.

Our design process is to produce inter-plane correlations non-symplectically, thus changing the eigen-emittances in the longitudinal field, then remove them symplectically by the following quadrupole correction section. Fig. 5 shows the simulation of this process, the optics of the quadrupole correction section has been designed to bring the horizontal rms-emittance down to the lower eigen-emittance.

After the quadrupole correction section the output uncoupled beam is divergent, and since there is still a long drift space between the quadrupole correction section and the emittance measurement, another normal triplet is required. The particle distributions at the position of emittance measurement are shown in Fig. 6. The inter-plane correlations between the horizontal and vertical phase spaces are removed \( \varepsilon_x = \varepsilon_1 \).

**MACHINE-RELATED ERRORS**

There are many sources of magnet imperfection, which are mainly generated by gradient fluctuations, rotational angle errors, and misalignment errors. The misalignment errors of the quadrupole cause the beam trajectory to deviate from and oscillate around the designed path in the beam line, but the values of rms-emittances and eigen-emittances are not affected. The gradient fluctuation errors and the rotational angle errors are the potential sources of inter-plane correlations not be removed exhaustively. We use a statistical method in which the error values of each magnet are randomly generated under given maximum values. The influences of these errors on the rms-emittances and eigen-emittances are shown in Fig. 7.

In the presence of the gradient errors, the fluctuations of the rms-emittances and eigen-emittances are not obvious, and the values of rms-emittances and eigen-emittances are almost the same and the cross-plane coupling beam after stripper can be corrected to a large degree.
In the presence of the rotational angle errors, the values of lower eigen-emittances are almost constant but the values of horizontal rms-emittances are varied. This bias may lead to attempting to correct the implied coupling, which will actually introduce coupling rather than correct it.

Based on the manufacturing experiences of the UNILAC, the maximum values of gradient fluctuation, rotational angle and misalignment errors are 0.1% and 0.3 degree.

**CONCLUSION AND OUTLOOK**

The longitudinal field strength and beam sizes at the stripper foil are free parameters, the amount of emittance transfer scales with the field strength and beam rms sizes. This freedom makes the difference to a emittance swapping scheme. The simulations illustrate that the combination of a normal triplet and a skew triplet has ability to correct a coupled beam, and the beam line errors degrade the accuracy of correction.

GSI aims at experimental verification of emittance transfer by assembling the beam line presented in this paper, and the corresponding hardwire devices are under manufacturing. The beam experiment is planned for early 2014.

**REFERENCES**