

A METHOD TO MEASURE THE INCOHERENT SYNCHROTRON FREQUENCIES IN BUNCHES

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Abstract

The method of measuring the incoherent synchrotron frequencies in a stationary bunch is presented. It can be shown that by measuring the local current at a fixed coordinate in RF bucket the corresponding incoherent synchrotron frequency can be obtained. Test calculations were done using simulation data where longitudinal space charge effects were included. The incoherent frequencies obtained with the method are in a good agreement with theory. In real experiment, the incoherent frequencies were determined from bunch profiles recorded in the SIS18 with low intensity beam at injection energy. Bunch profiles were measured with a new Fast Current Transformer which has a relatively broad frequency range. The profiles were recorded using 8 bit resolution oscilloscope. The frequency spectra of the local current fluctuations at different longitudinal positions were obtained numerically. The strongest lines in these spectra were at positions of theoretically expected incoherent frequencies. In this paper the method is described in details, the comparison of incoherent frequencies obtained from the simulation and measurement data with theoretical predictions is shown.

INTRODUCTION

The longitudinal motion of particles in bunched beam can be described using frequency domain. The main characteristics of bunched beam in frequency domain are:

- the coherent frequencies describe the oscillations of the longitudinal bunch shape
- the incoherent frequency $\omega_s(\hat{\phi})$ as function of the particle oscillation amplitude; it is a characteristic of a single particle in the longitudinal phase space
- the particle distribution over the incoherent frequencies $\psi(\omega_s)$

Schottky noise measurements is the non-destructive technique for the measurement of the frequency characteristics in beam [1]. The main application of Schottky diagnostics is the measurement of the particle momentum distribution in coasting beams. In bunched beams, the longitudinal density fluctuations from stationary bunch contains an information on the incoherent and coherent frequencies. Using standard Schottky diagnostics, the coherent frequencies can be clearly observed in the spectrum of the bunched beam [2]. But both particle distribution and the incoherent frequencies is difficult to extract from the bunched beam Schottky noise spectrum.

In a simple way the incoherent frequencies inside RF bucket can be measured with a longitudinally mismatched bunch [3]. If initially matched bunch is shifted on the distance ϕ_x with respect to the RF bucket center then this bunch starts to perform the dipole oscillations. If the longitudinal bunch length is small then the bunch dipole frequency is equal to the incoherent frequency ω_{s_x} of a single particle with oscillation amplitude of $\hat{\phi}_x = \phi_x$. By measuring the bunch dipole frequency with different ϕ the incoherent frequency ω_s as function of single particle oscillation amplitude $\hat{\phi}$ can be found.

For long bunches the method described above will not work properly since the range of particle oscillation amplitudes inside the bunch is comparable with an initial dipole offset. In the case of long bunches a Beam Transfer Function (BTF) method can be employed [4]. The BTF method is based on the measurements of longitudinal bunch response due to tiny forced perturbations of RF bucket. Analytically, the bunch response is defined through the dispersion relation. This dispersion relation includes both the particle distribution over the incoherent frequencies and the incoherent frequency as function of the single particle oscillation amplitude. As a non-destructive method, there is "peak detection" technique where the noise produced by the peak value of the longitudinal bunch profile is measured [5]. The frequency spectrum of the peak detector signal contains information about particle distribution and the incoherent frequencies. In both methods one quantity can be extracted if the other is known.

For low intensity beams, if the RF waveform is well defined the incoherent frequencies can be easily calculated. For high intensity beams, longitudinal space charge will modify the effective RF voltage seen by the beam and the incoherent frequencies can differ from the low intensity case. This so-called potential well distortion can be calculated only if the effect of longitudinal space charge is well defined.

Using and extending the theoretical concept given in [5], we propose a method which allows to obtain the incoherent frequencies without knowledge of space charge parameters and without knowledge about the distribution function. It is based on Fourier analysis applied to recorded data.

NOISE SPECTRUM OF THE LOCAL CURRENT

In the longitudinal phase space confined by stationary RF bucket the particles moves along a certain orbits as it is shown in Fig. 1. The longitudinal phase space is repre-

sented by two coordinates:

$$\begin{aligned}\phi &= \frac{h}{R}(z - z_0) \\ \dot{\phi} &= \frac{h}{R}\eta\beta c \frac{\Delta p}{p_0},\end{aligned}\quad (1)$$

where ϕ is the longitudinal coordinate in terms of RF phase, $\dot{\phi}$ is the phase velocity, h is the RF harmonic number, R is the ring radius, η is the slip factor, β is the relativistic parameter, c is light velocity, z_0 and p_0 are the coordinate and momentum of the synchronous particle, z and $\frac{\Delta p}{p_0}$ are the coordinate and the momentum deviation of the non-synchronous particle. The angular frequency of the particle is the incoherent frequency ω_s .

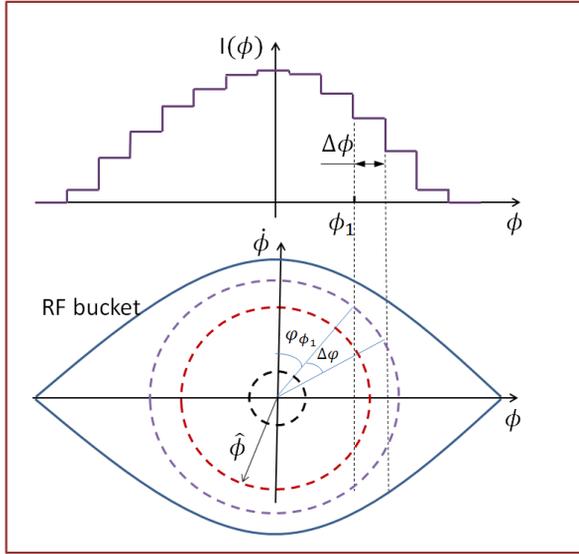


Figure 1: Longitudinal bunch profile (top) and the longitudinal phase space.

Consider a particle which oscillates in the longitudinal phase space. In the phase space created by single RF bucket the trajectory of the particle can be approximated by circle, especially for a particles with small amplitudes. The radius of trajectory or the particle oscillation amplitude $\hat{\phi}$ in this case is equal to the maximum longitudinal deviation.

Using the ansatz given in [5], the contribution of a single particle in the bunch current at the longitudinal coordinate

ϕ is:

$$\begin{aligned}I_j(t, \phi) &= \frac{e\omega_{s_j}}{2} \sum_{m=-\infty}^{\infty} \left(\delta(\omega_{s_j}t - \varphi_\phi - \theta_j - 2\pi m) \right. \\ &\quad \left. + \delta(\omega_{s_j}t - (\pi - \varphi_\phi) - \theta_j - 2\pi m) \right) = \\ &= \frac{e\omega_{s_j}}{4\pi} \sum_{m=-\infty}^{\infty} (e^{im\varphi_\phi - i\pi/2} + e^{-im\varphi_\phi + i\pi/2}) e^{i\pi/2} \\ &\quad \cdot e^{im(\omega_{s_j}t - \theta_j)} = \\ &= \frac{e\omega_{s_j}}{2\pi} \sum_{m=-\infty}^{\infty} \sin(m\varphi_\phi) e^{i\pi/2} e^{im(\omega_{s_j}t - \theta_j)},\end{aligned}\quad (2)$$

where

$$\varphi_\phi = \omega_s t_\phi = \arcsin \frac{\phi}{\hat{\phi}}.\quad (3)$$

The current from N particles measured at ϕ_1 from the bin with size $\Delta\phi$ can be found using expression:

$$\begin{aligned}I(t, \phi_1, \Delta\phi) &= \sum_{j=1}^N \int_{\varphi_{\phi_1}}^{\varphi_{\phi_1 + \Delta\phi}} I_j(t, \phi) d\varphi_\phi \sim \\ &= \sum_{j=1}^N \omega_{s_j} \sum_{m=-\infty}^{\infty} A_j e^{im(\omega_{s_j}t - \theta_j)},\end{aligned}\quad (4)$$

where A_j is the contribution of j -th particle:

$$A_j \approx m \sin m\varphi_{\phi_1} \cdot \Delta\varphi_j\quad (5)$$

for small bin size $\Delta\phi$.

There are three types of particles with respect to the measurement region can be defined:

- Particle amplitude is less than the measurement coordinate. Particle trajectory does not cross the measurement region. Obviously, there is no contribution at all from such particles
- Particle amplitude is higher or equal than the measurement coordinate and particle has maximum longitudinal deviation inside the measurement region (inside bin).
- Particle amplitude is higher than the measurement coordinate and particle has maximum longitudinal deviation outside the measurement region (outside bin).

Correspondingly for each types of particles the term $\Delta\varphi_j$ is different:

$$\Delta\varphi_j = \begin{cases} 0 & \hat{\phi}_j < \phi_1 \\ \frac{\pi}{2} - \arcsin \frac{\phi_1}{\hat{\phi}_j} & \phi_1 + \Delta\phi \geq \hat{\phi}_j \geq \phi_1 \\ \arcsin \frac{\phi_1 + \Delta\phi}{\hat{\phi}_j} - \arcsin \frac{\phi_1}{\hat{\phi}_j} & \hat{\phi}_j > \phi_1 + \Delta\phi \end{cases}\quad (6)$$

In order to get the noise power spectrum of the current $I(t, \phi_1, \Delta\phi)$ the Fourier transform of the auto-correlation function of the current Eq. 4 has to be done. Such procedure is described in many papers related to Schottky noise calculation [1],[5]. The resulting expression is:

$$P(\omega, \phi_1, \Delta\phi) \sim \sum_{j=1}^N \sum_{m=1}^{\infty} \omega_{s_j}^2 |A_j|^2 \delta(\omega - \omega_{s_j}) \quad (7)$$

Changing the sum over particles on the integration over the particles distribution $\psi(\omega_s)$ the noise power spectrum of the $m = 1$ is:

$$P_1(\omega, \phi_1, \Delta\phi) \sim \omega_s^2 A^2(\hat{\phi}(\omega_s), \phi_1, \Delta\phi) \psi(\omega_s) \Big|_{\omega_s=\omega} \quad (8)$$

where $A(\hat{\phi}(\omega_s), \phi_1, \Delta\phi)$ is the same as Eq. 5 with index j removed.

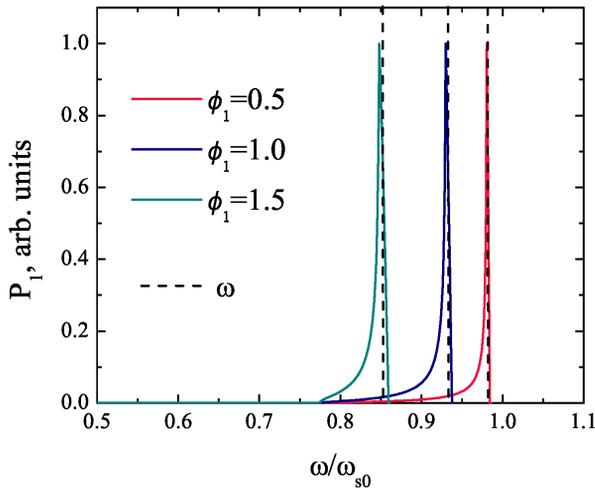


Figure 2: Noise spectrum calculated according to Eq. 8 for the bunch with elliptic phase space (see Eq. 9) in potential well created by single RF waveform (see Eq. 10). The spectra are for three different measurement coordinates given by ϕ_1 in radians. Black dashed lines are the incoherent frequencies calculated according to Eq. 10.

There two functions are needed to plot P_1 :

- Particles distribution function $\psi(\omega_s)$
- The inverse function of the incoherent frequencies $\omega_s(\hat{\phi})$

Firstly, consider the elliptic distribution in the phase space:

$$\psi(\omega_s) \sim \sqrt{\phi_m^2 - \hat{\phi}^2(\omega_s)}, \quad (9)$$

where ϕ_m is the maximum deviation in bunch or the bunch half length. Second, consider the potential well created by single RF waveform, where the incoherent frequencies are well-known:

$$\frac{\omega_s(\hat{\phi})}{\omega_{s0}} = 1 - \frac{\hat{\phi}^2}{16}, \quad (10)$$

where ω_{s0} is the bare synchrotron frequency.

With expressions 9 and 10 in 8 the resulting spectrum P_1 is shown on Fig. 2. The maximum response is produced by the particles whose maximum deviations (or amplitudes) lie inside the measurements region:

$$\phi_1 + \Delta\phi \geq \hat{\phi} \geq \phi_1. \quad (11)$$

Correspondingly the position of the maximum is the frequency of these highly response particles. Thus, the statement is:

$$POSITION [MAX[P_1(\omega, \phi_1, \Delta\phi)]] = \omega_s(\hat{\phi} = \phi_1) \quad (12)$$

or measuring the spectra P_1 at different ϕ and recording the position of the maximum the incoherent frequencies $\omega_s(\hat{\phi})$ can be reconstructed.

SPECTRUM IN SIMULATIONS

The incoherent frequencies in the absence of longitudinal space charge effects depends on the RF waveform only. The longitudinal motion of the particles in single RF bucket was simulated with Particle In Cell (PIC) code. Such code was written in Mathematica package. In this section the results of the method applied to the simulations data are presented. The bunch profiles data were taken from PIC simulations of one parabolic bunch with half-bunch length $\phi_m = 1.8$ rad. Simulations were done using $5 \cdot 10^4$ macroparticles. The bunch elliptic phase space distribution was matched to a single RF bucket. The number of bins was 100 per RF period. The time step between two consecutive profiles was equal to $T_{s0}/80$ with T_{s0} being the bare synchrotron period. The bunch profiles were collected over the time equal to $600 \cdot T_{s0}$.

The spectra of the bunch current at three different measurement coordinates are presented in Fig. 3. These spectra are compared with the theoretical predictions (see Eq. 8) with Eq. 9 and Eq. 10. The comparison shows a perfect agreement.

The next, the longitudinal space charge effects were added in the simulations in a self-consistent way. The effect produced by space charge for the bunch with elliptic distribution is well known [6, 7]:

- The phase space distribution and the longitudinal profile remains the same as in low intensity case if the RF voltage amplitude is increased:

$$\hat{V}_{rfsc} = \hat{V}_{rf0} + \hat{V}_{sc} = (1 + \Sigma) \hat{V}_{rf0}, \quad (13)$$

where \hat{V}_{rfsc} is the required RF amplitude, \hat{V}_{rf0} is the RF amplitude without space charge and \hat{V}_{sc} is the space charge voltage amplitude. Space charge parameter Σ is used in order to qualify the space charge effect.

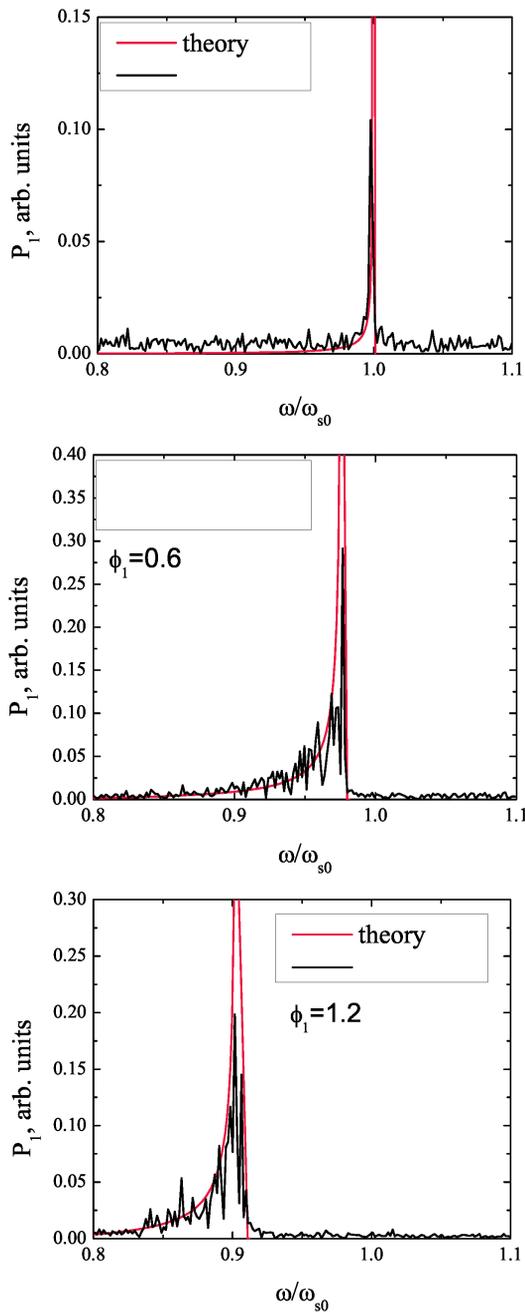


Figure 3: Comparison of simulated and theoretical spectrum for the bunch with elliptic phase space (see Eq. 9) in potential well created by single RF waveform (see Eq. 10). The amplitude of the theoretical spectra was varied in order to fit best the simulation data.

- The effect on the incoherent frequencies is:

$$\frac{\omega_s(\hat{\phi})}{\omega_{s0}} = \frac{1}{\sqrt{1 + \Sigma}} \left(1 - \frac{\hat{\phi}^2}{16}\right) \quad (14)$$

The simulation results with $\Sigma = 0.55$ are presented in Fig. 4. The expected incoherent frequency agrees with the position of the second highest peak. The first highest peak

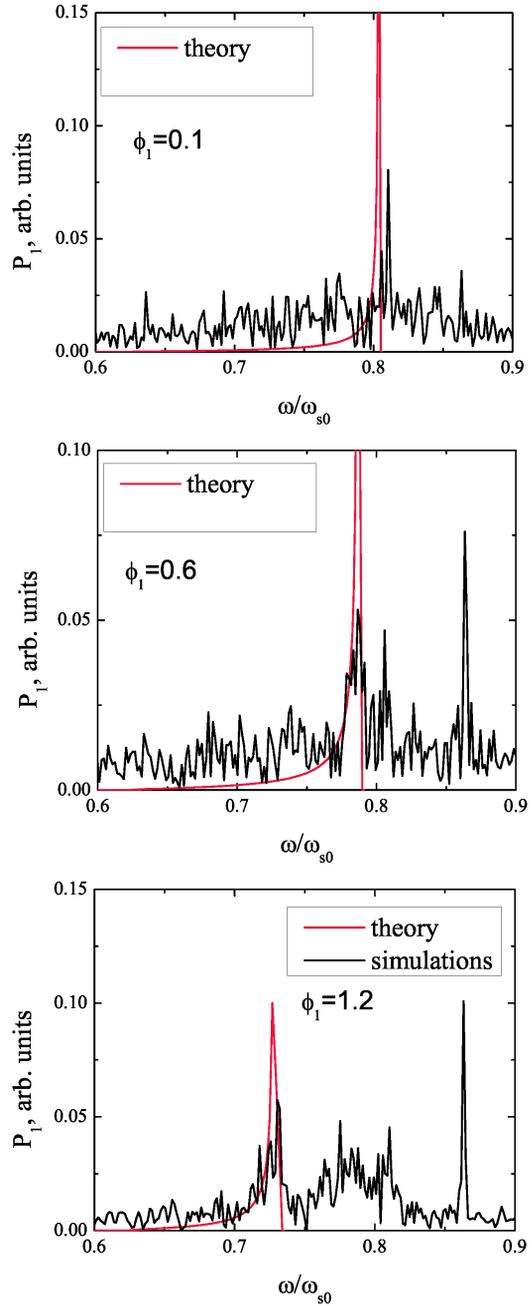


Figure 4: The same as Fig. 3, but including longitudinal space charge with $\Sigma = 0.55$

is the bunch dipole frequency since it has the same position for all spectra.

MEASUREMENT OF INCOHERENT FREQUENCIES

In order to consider the more complex case the simulation of a Gaussian bunch with space charge were performed. The space charge parameter Σ can also characterize space charge in Gaussian bunch [8]. In the simulations

Σ was set to 1. The maximum frequencies ω_{max} in spectra P_{sim} from different measurement coordinates ϕ_i were found, excluding the dipole frequency. The effect of space charge on the incoherent frequency ω_s was calculated numerically [8]. The simulation results compared with numerical results are presented in Fig. 5.

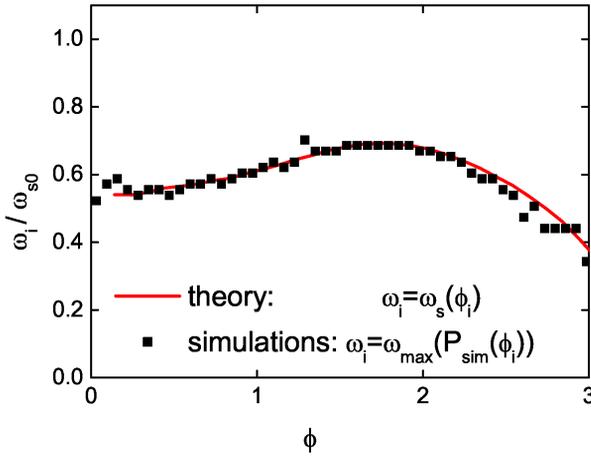


Figure 5: Maximum frequencies from simulation spectra compared with the numerically calculated incoherent frequencies.

The method was applied to a real measurements. Beam of Ar^{18+} ions was used in the experiment. The harmonic number h of RF system was equal to 4. The measurements were done at injection beam energy of 11.4 MeV which corresponds to 4.8 μs revolution period. To create stationary matched bunch the adiabatic capture of coasting beam into a single RF bucket was performed. Number of ions per ring at injection was $5 \cdot 10^9$ which can be considered for SIS18 as low intensity conditions in longitudinal plane. The measurements were done using Fast Current Transformer which has bandwidth 0.35-650 MHz. The sampling rate of the 8 bit resolution oscilloscope was set to 100 MHz.

After the particles were captured in a single RF bucket with RF amplitude of 8 kV, 48000 longitudinal profiles of the same bunch were recorded at constant injection energy during the time equal to 0.24 s which corresponds to $500 \cdot T_{s0}$. Each profile was recorded on 116 bins. The spectra P_{ms} of the signal from each bin ϕ_i was obtained, the position of the maximum frequency ω_{max} in spectra P_{ms} was found and plotted in Fig. 6. The measured incoherent frequencies are in agreement with predicted for the low intensity case (Eq. 10). The jump in incoherent frequencies above $\phi_i = 2$ is because there are no particles with $\hat{\phi} > 2$ inside RF bucket.

CONCLUSIONS

The analytical expression for the noise spectra produced by the local current fluctuations in bunch was derived. This expression perfectly agrees with the low intensity simulation results. With high intensity the form of the simula-

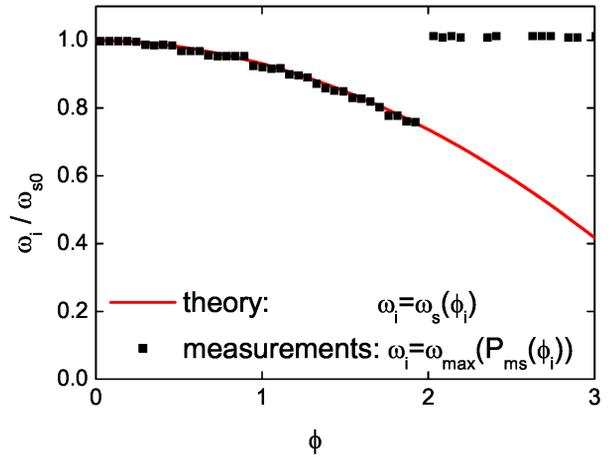


Figure 6: Maximum frequencies from measurements spectra compared with the incoherent frequencies for low intensity (Eq. 10).

tion spectra deviates from the theoretical spectra, but the frequency of the peak in simulation spectra coincide very well with the frequency of the peak in theoretical spectra. This peak frequency depends on the bunch coordinate ϕ_i where the local current was measured. And this peak frequency equal to the incoherent synchrotron frequency ω_i of the particles with amplitude $\hat{\phi} = \phi_i$. Presently the statement was proved in the real experiment for long bunches in single RF bucket at low intensity conditions.

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