COHERENT SYNCHROBETATRON RESONANCE AT THE FNAL BOOSTER*

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Abstract

At reduced chromaticity, transverse coherent oscillations rapidly develop in the Fermilab Booster at the end of bunching process [1]. The growing mode was seen with almost zero fractional tune, and the growth time was extremely short and hardly sensitive to the beam intensity. This instability can be explained as an excitation of a head-tail mode when its frequency crosses a nearest multiple of the revolution frequency. This coherent synchro-betatron resonance (CSBR) can be driven by non-zero dispersion and/or its derivative inside cavities. Model of the CSBR is developed; calculations for the Booster yield growth rate close to observations. Methods to suppress CSBR are discussed.

INTRODUCTION

Booster [1] is a fast cycling proton synchrotron operating at 15 Hz; its main parameters are presented in Table 1.

Table 1: Booster parameters

Energy	0.4 – 8 GeV
Transition energy	5.1 GeV
Total number of particles	$4.5{\cdot}10^{12}$
Circumference	474.2 m
Harmonic number, *q*	84
Betatron tunes, Q_x/Q_y	6.82 /6.81
RF voltage	0.7- 0.9 MV
Injection type	H^-, 11 turns

Large chromaticities are used to avoid transverse instability. That negatively affects the dynamic aperture and the beam lifetime. For studies, both chromaticities were reduced, and data were taken from 4 channel digital scope with sampling time 0.4 ns and total recording time ~700 turns. As a result, a coherent instability at injection was recorded in details. Off-line processing of these data included marking boundaries for each bunch, subtracting closed orbit offsets from the differential signal, and obtaining density and dipole moment distributions. Analysis revealed the following features:

- The instability starts at the end of beam bunching, when the RF voltage achieves about half of its maximum. It continues during about 30-100 turns.
- The amplitude of the coherent oscillations grows to few mm.
- Coherent betatron motion in all bunches has the same structure and betatron phase.
- Turn-by-turn betatron phase advances are almost zero, as if the tunes are equal to integers (but they were far from that).
- Amplitude growth is almost insensitive to the beam intensity.

An example of this coherent growth is presented at Fig. 1.

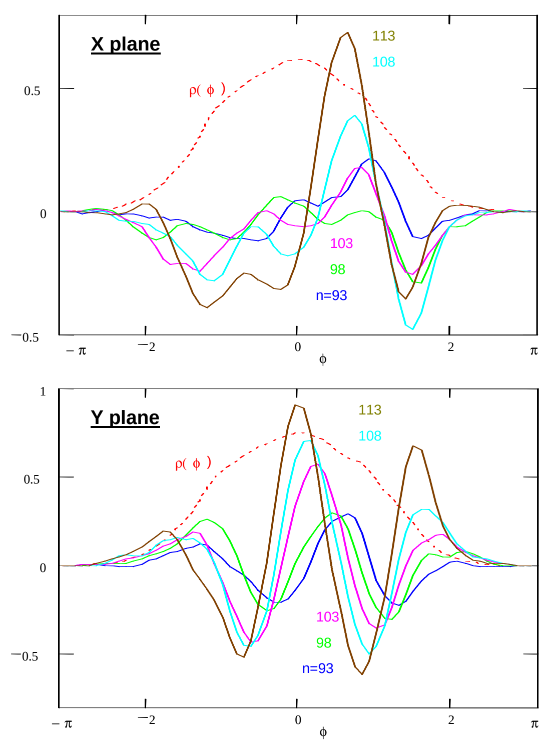

Figure 1: Measured dipole moment distributions for *x*-(top) and *y*-planes (bottom) over bunch length for every 5-th turn (turns from 93 to 113); beam intensity $4.5{\cdot}10^{12}$.

The described above features compel to seek some collective synchro-betatron resonance as an explanation of this coherence. Indeed, with a growing RF, head-tail modes cross resonances, when their tunes $\nu_l = \nu_b + l\nu_s$ cross integers. Since high space charge requires bare tunes ν_b stay slightly below integers ($\Delta\nu$~0.1) and synchrotron tune is large ($\Delta\nu_s$~0.05), this is possible at not so high resonance orders ***l***. A model of CSBR is developed below, with a driving force generated by non-zero dispersion in cavities.

RESONANCE DYNAMICS

CSBRs were first mentioned and studied by Sundelin [2], assuming the resonance is driven by transverse wake in RF cavities. The localized wake-driven CSBRs were later analyzed in Ref. [3,4]. It was claimed in Ref. [5] that CSBRs can be driven by non-zero dispersion inside the cavities, although no quantitative model was suggested. Single-particle SBR driven by dispersion in RF cavities was treated in Ref. [6,7]. A quantitative model showing how this dispersion drives coherent SBR was presented in our paper [8], and is mostly followed below.

When a mode is at resonance, $\nu_l = \nu_b + l\nu_s = n$, any dipole perturbation leads to its growth. To understand main features of the phenomenon, a longitudinal distribution is taken here as the air-bag one (hollow beam); *i.e.* all the particles have the same synchrotron amplitude r_0, they are homogeneously distributed over the synchrotron phases φ, the longitudinal offset is $z = r_0\cos\varphi$. The Vlasov equation for the distribution function ψ can be conventionally written as

$$\frac{\partial\psi}{\partial s} + \frac{\omega_b}{\mathrm{v}}\frac{\partial\psi}{\partial\theta} + \frac{\omega_s}{\mathrm{v}}\frac{\partial\psi}{\partial\varphi} + \tilde{x}'\frac{\partial\psi}{\partial x} + \tilde{p}'\frac{\partial\psi}{\partial p} = 0 .$$

Here s is time in units of length, $x=q\cos\theta$ and $p=-(q/\beta)\sin\theta$ are the betatron coordinate and momentum (angle), similar values with tilde are their perturbations, and v is the beam velocity. It can be assumed here, that the perturbation kicks are localized at a single point $s = s_j$; the final result can be obtained by summation over j. Such periodic perturbations can be presented as

$$\tilde{x}' = \Delta x\,\delta_P(s - s_j); \quad \tilde{p}' = \Delta p\,\delta_P(s - s_j);$$

$$\delta_P(s) \equiv C^{-1}\sum_{m=-\infty}^{\infty}\exp(-2\pi i m s / C).$$

Betatron kicks $\Delta x, \Delta p$ are generated by non-zero dispersion D and its derivative D' in a cavity, every time a particle passes through it:

$$\Delta x = -D\sin(kz)\Delta p_{\max} / p_0;$$

$$\Delta p = -(D' + \alpha D/\beta)\sin(kz)\Delta p_{\max} / p_0.$$

Here α and β are the Twiss parameters in the cavity, k is the RF wave number and $\Delta p_{\max}/p_0$ is an amplitude of the RF kick in the cavity in terms of the relative longitudinal momentum offset. According to the conventional perturbation approach, a solution of the Vlasov equation is presented as a sum of a steady state distribution and a perturbation: $\psi = \psi_0 + \tilde{\psi}$. For the air-bag distribution:

$$\psi_0 = f_0(q)\delta(r - r_0)$$

$$\tilde{\psi} = A(s)\sqrt{\beta_x}\,f_0'(q)\delta(r - r_0)\exp(i\theta + il\varphi + i\chi z / r_0 - i\Omega_l s / \mathrm{v})$$

$$\Omega_l = \omega_b + l\omega_s \approx n\omega_0$$

$$\chi \equiv 2\pi\frac{\xi r_0}{C\eta}; \quad \xi \equiv \frac{d\nu_b}{d(\Delta p / p_0)}; \quad \eta \equiv \frac{1}{\gamma_t^2} - \frac{1}{\gamma^2}$$

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Substituting this linearized expansion in the Vlasov equation, neglecting the second-order terms, and leaving only a resonant contribution, a time derivative for the mode amplitude is obtained:

$$\frac{dA}{ds} = F; \quad F = \frac{(-i)^{l+1}}{4}\frac{\Delta p_{\max}}{p_0}\frac{J_l(\chi - kr_0) - J_l(\chi + kr_0)}{C}\Sigma;$$

$$\Sigma \equiv \sum_j \exp(2\pi i n s_j / C)\frac{D(s_j) + i\big(\beta_x(s_j)D'(s_j) + \alpha_x(s_j)D(s_j)\big)}{\sqrt{\beta_x(s_j)}} ,$$

where $J_l(x)$ is the Bessel functions, and the summation is performed over all cavities. This equation describes a linear growth of the mode amplitude driven by the external resonant force F. It was assumed, that the mode stays exactly on the resonance – that is why the force F does not depend on time. If the mode is slightly detuned from the resonance, $\Delta\Omega_l \equiv \omega_b + l\omega_s - n\omega_0 \neq 0$, the force has to be modified by an oscillating factor:

$$\frac{dA}{ds} = F\exp\Big(i\int^{s}\Delta\Omega_l(s')ds'/\mathrm{v}\Big).$$

When RF grows, the synchrotron frequency changes, leading to the time–dependent frequency offset: $\Delta\Omega_l(s) = l\dot{\omega}_s s/\mathrm{v}$. Substituting this to the above equation leads to the total amplitude growth after resonance crossing:

$$\Delta A = \int_{-\infty}^{\infty}\frac{dA}{ds}ds = (1+i)\sqrt{\frac{\pi}{l\dot{\omega}_s T^2}}FC,$$

where $T=C/\mathrm{v}$ is the revolution time.

Now, several comments can be added to this result. First, the amplitude growth appears to be independent on the beam intensity. In fact, the beam intensity has to be high enough to make CSBR possible. If intensity is so low that the space charge does not separate coherent and incoherent tunes, CSBR would not be seen due to strong Landau damping. Note also, that the betatron and synchrotron tunes here are coherent ones.

Second, the entire growth is a sum of complex contributions from individual cavities. In principle, with a proper choice of amplitudes and phases, they may cancel each other. In the Booster, there are 9 pairs of adjacent cavities. At the beginning of bunching, the voltage of every cavity is almost as high as at maximum, but the phases of adjacent cavities differ by π, so the net longitudinal focusing is zero. Then, this so called ‘paraphasing’ goes down to zero at the end of the bucket formation. Formulas above assume that all the cavities are in phase. Taking into account the paraphasing $\pm\psi$ in the adjacent cavities can be done by a substitution

$$J_l(\chi - kr_0) - J_l(\chi + kr_0) \to J_l(\chi - kr_0)e^{i\psi} - J_l(\chi + kr_0)e^{-i\psi}.$$

Third, all the calculations above assume no x-y coupling in the beam optics. This assumption is not valid for the Booster, which stays close to the coupling resonance, and where coupling is strong. In this case, correct treatment should be based on the coupled 4D phase space Twiss parameters. Below, we still apply the described uncoupled formalism to the Booster as a first approximation, and to see the significance of the phenomenon. To do that, we are using design optics with no coupling.

Making calculations for such idealized Booster lattice, we found that the result is very sensitive to the degree of mutual compensation from the cavity pairs. Contributions of all the 9 cavity pairs are presented at Fig. 2. A resulting force appears close to a contribution of a single pair of cavities due to the assumed optical symmetry. Introduction of ~10% optical perturbation can increase the sum by a factor of 2.

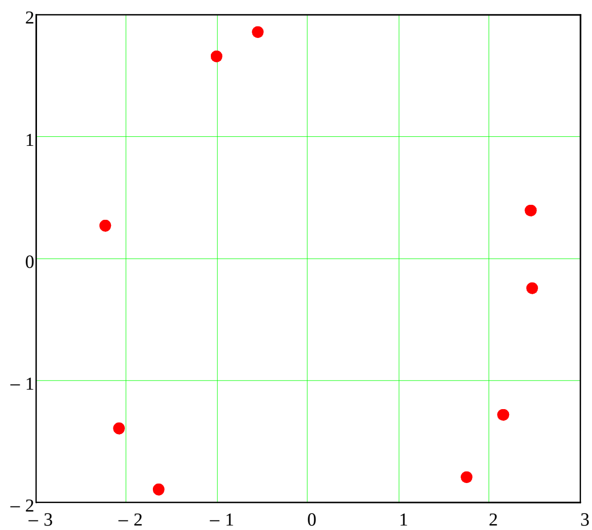

Figure 2: Calculated complex contributions of 9 cavity pairs into the net driving force (arbitrary units). Every red dot shows contribution of a single pair in the complex plane. The net result is close to a single pair contribution.

Calculations for the idealized Booster optics results in the amplitude growth of $\Delta A\sqrt{\beta_{\rm bpm}} \approx 1{\rm mm}$ comparable to the experimental observations of about 1-3 mm.

Discussing CSBR with the authors, R. Baartman noted that RF asymmetry in the cavities can drive CSBR similar to dispersion. He found the RF asymmetry ~1% would matter as much as dispersion for the Booster [9]. We are estimating an upper limit on asymmetry for the Booster cavities as a much smaller number, so we exclude that as a source of CSBR for the Booster.

SUPPRESSION OF CSBR

A possible danger of CSBR is an excitation of high starting value for then impedance-driven instability. However, normally this does not happen in the Booster In this case, there is no reason to care about the Landau damping of CSBR since the Landau damping just transfers coherent oscillations into incoherent, what eventually occurs anyway. Even if exponential growth does not happen, CSBR is still detrimental, because of increase in emittance and possible particle loss. Several ways to suppress CSBR can be suggested.

- The chromaticity ξ can be increased, reducing the driving force $\propto 1/\sqrt{\xi}$ and the consequent emittance growth $\propto 1/\xi$.
- The bare tunes and chromaticities can be optimized to reduce the net emittance growth caused by all the crossed CSBRs.
- To cross CSBR faster. This could be achieved, in particular, with introduction of 3rd RF harmonic, leading to increase of non-linearity in the synchrotron motion $d\omega_s / dr_0$.
- If there is some excess in RF power, it can be re-distributed between all the cavities in more optimal way, so that contributions from different cavities in the driving force Σ cancel each other.

The first item is presently used in Fermilab Booster to suppress the resonance. The second and third ones require more study. At the moment, the last item is looking as the most promising.

SUMMARY

- A model of CSBR is developed.
- It is in agreement with observations in the Booster.
- CSBR can lead to emittance growth and particle loss
- Methods to suppress CSBR are discussed.

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