Elettra
Sincrotrone
Trieste

Estimate of FEL Gain Length in the Presence of e-beam Collective Effects

S. Di Mitri[1], S. Spampinati[2]

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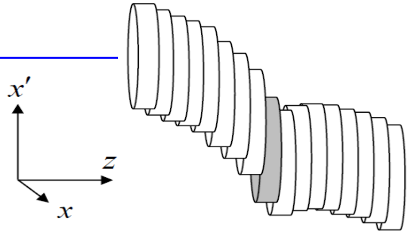
Problem

Picture courtesy D.Douglas
[1] Guetg et al. PRSTAB 18, 2015

- ☐ **Collective effects** (**C**oherent **S**ynchrotron **R**adiation, **G**eometric **T**ransverse **W**akefield) "misalign" bunch slices in the transverse phase space: ε_{proj} **is increased** albeit ε_{slice} may be not, whereas $L_{slice} \approx L_{coop} << L_{bunch}$.



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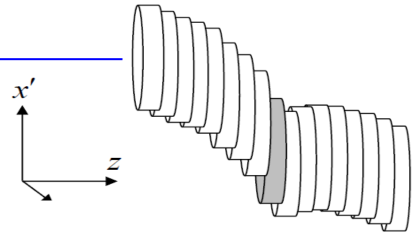

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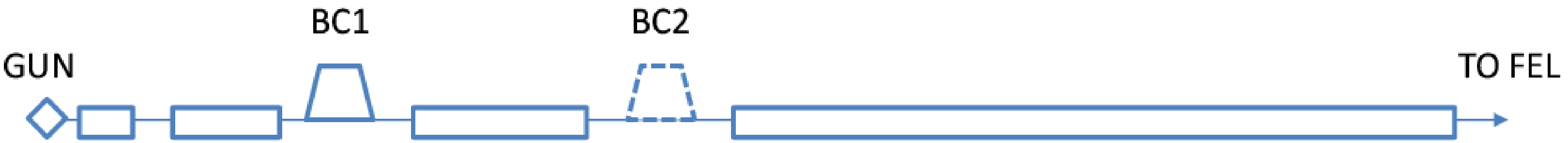

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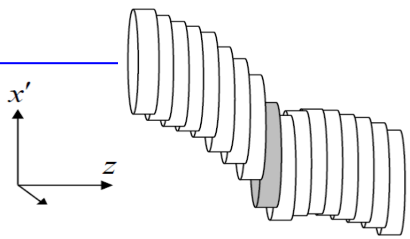

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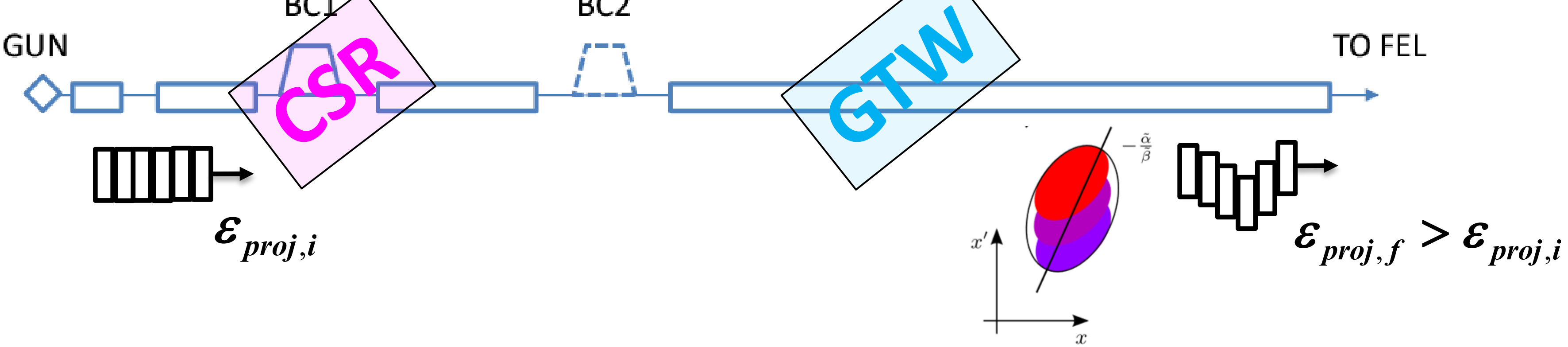

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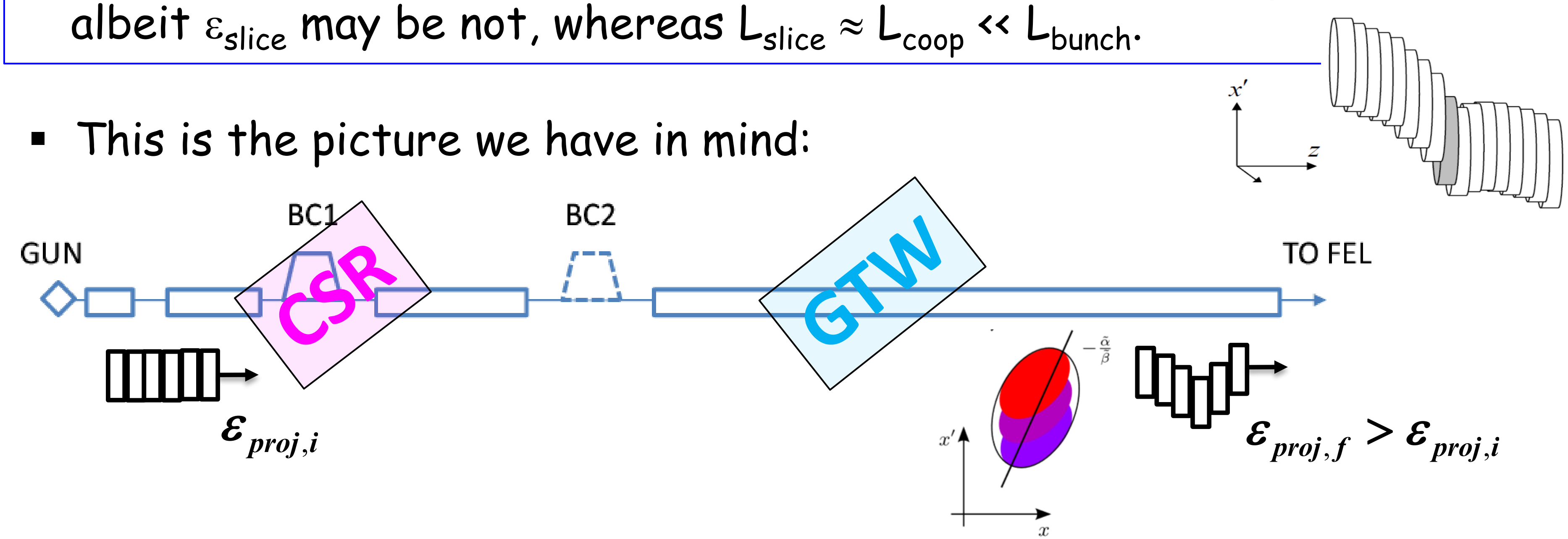
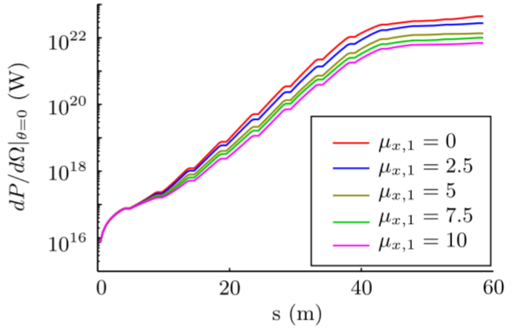
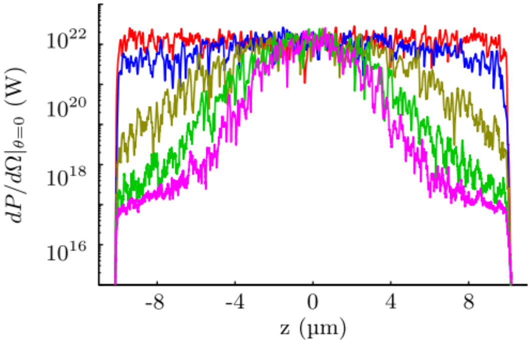


- Simulations show an impact on SASE FEL output[1]:





Can we analytically relate L_G to ε_{proj} ?

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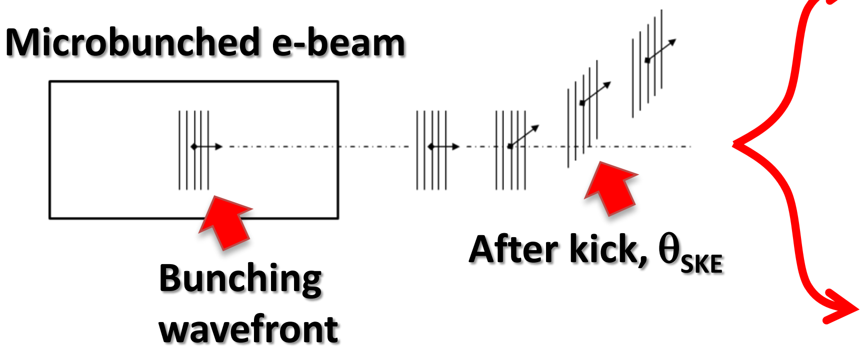
Single Kick Error (Dipole-like)

[2] Tanaka et al. NIMA 2004
[3] Chae et al., FEL 2004

❑ Consider a bunch subjected to ***dipole-like kicks*** in the ***undulator***[2], e.g. due to steering magnets or misaligned quadrupoles.



1. **Lack of photons/electrons overlap:** the undulator spontaneous radiation does not sustain efficiently the coherent emission,

$$L_{G,SKE} \approx \frac{L_G}{1-\theta_{SKE}^2/\theta_c^2}, \quad \theta_c \equiv \sqrt{\lambda/L_G}$$

2. **Smearing of microbunching:** the kick enhances the arrival time difference of electrons belonging to the same wavefront, so spoiling the phase coherence,

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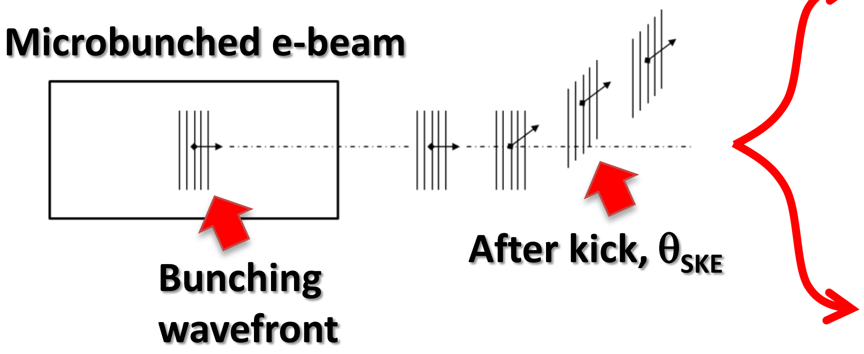

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The effect of angular divergence ("*de-bunching*") is far more important than the lack of transverse overlap ("*sustainment*").

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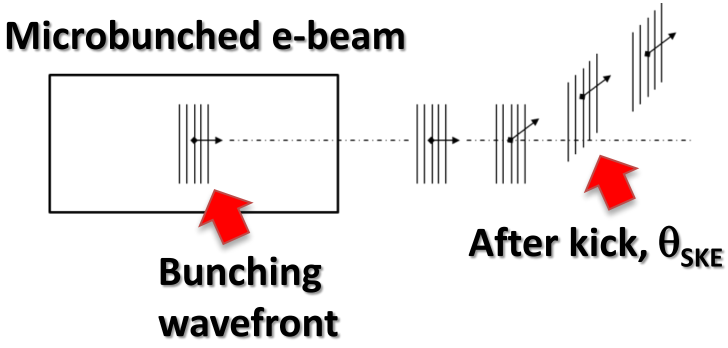

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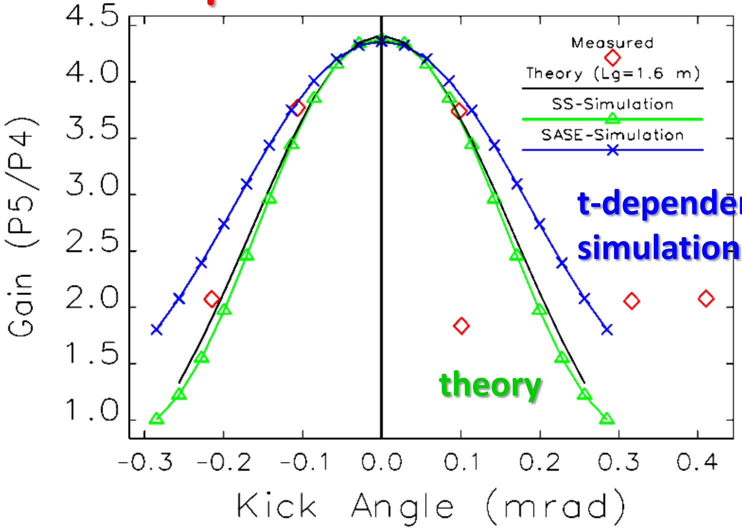
Experiment at LEUTL[3]:



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Projected Emittance Growth

☐ We now consider ***error kicks*** that affect ***individual slices***, e.g. from CSR in a dipole, and from GTW in an RF cavity.

- The "Σ-matrix" provides an ***RMS estimate of*** $\Delta\varepsilon_{proj}$ induced by those perturbations. For a single angular error (~Δx'):

$$\varepsilon_{x,1} \cong \sqrt{\det \varepsilon_{x,0} \begin{pmatrix} \beta_x & -\alpha_x \\ -\alpha_x & \gamma_x + \left\langle \Delta x'^2 \right\rangle / \varepsilon_{x,0} \end{pmatrix}} = \varepsilon_{x,0} \sqrt{1 + \frac{\beta_x \left\langle \Delta x'^2 \right\rangle}{\varepsilon_{x,0}}}$$

Twiss functions are at the location of the perturbation

- Consider the ***uncorrelated sum*** of m-consecutive ***CSR kicks*** in magnetic compressors and ***GTW kicks*** in the linac. The resultant projected emittance, e.g. at the undulator, turns out to be:

$$\varepsilon_{n,f} \approx \varepsilon_{n,0} \sqrt{\prod_{i=1}^{m} \left(1 + P^i\left(\varepsilon_{n,i-1}\right)\right)} = \ldots$$

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RMS kick averaged over all the slices.

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We will evaluate this later.

Collective Angle

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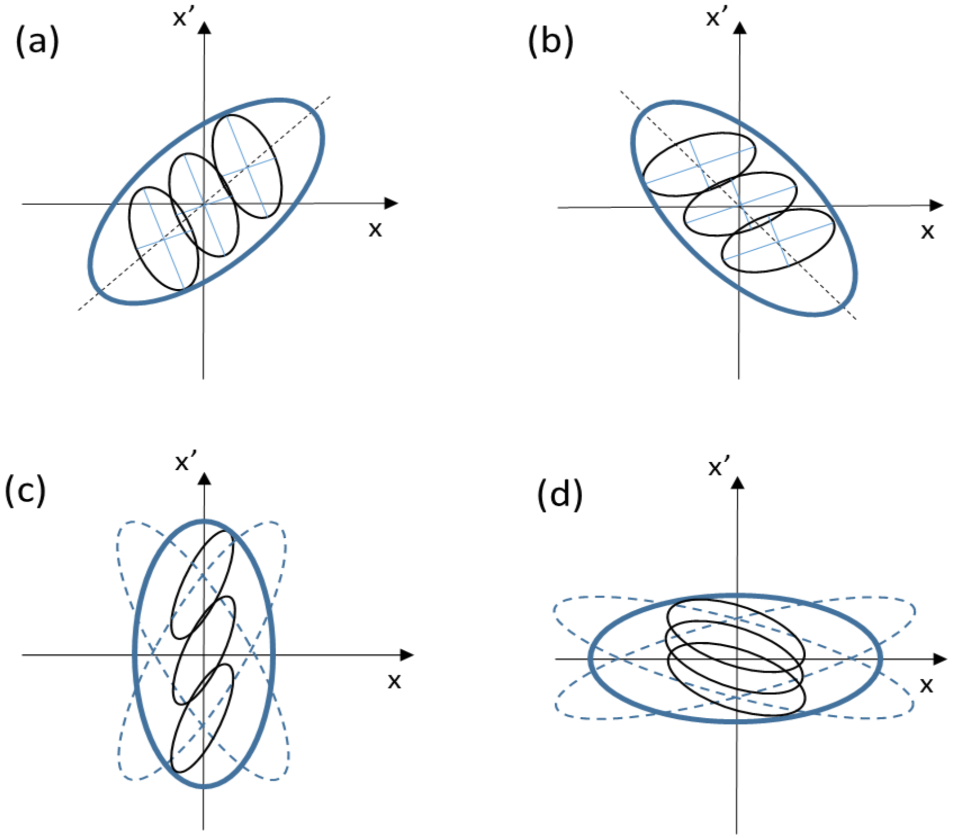

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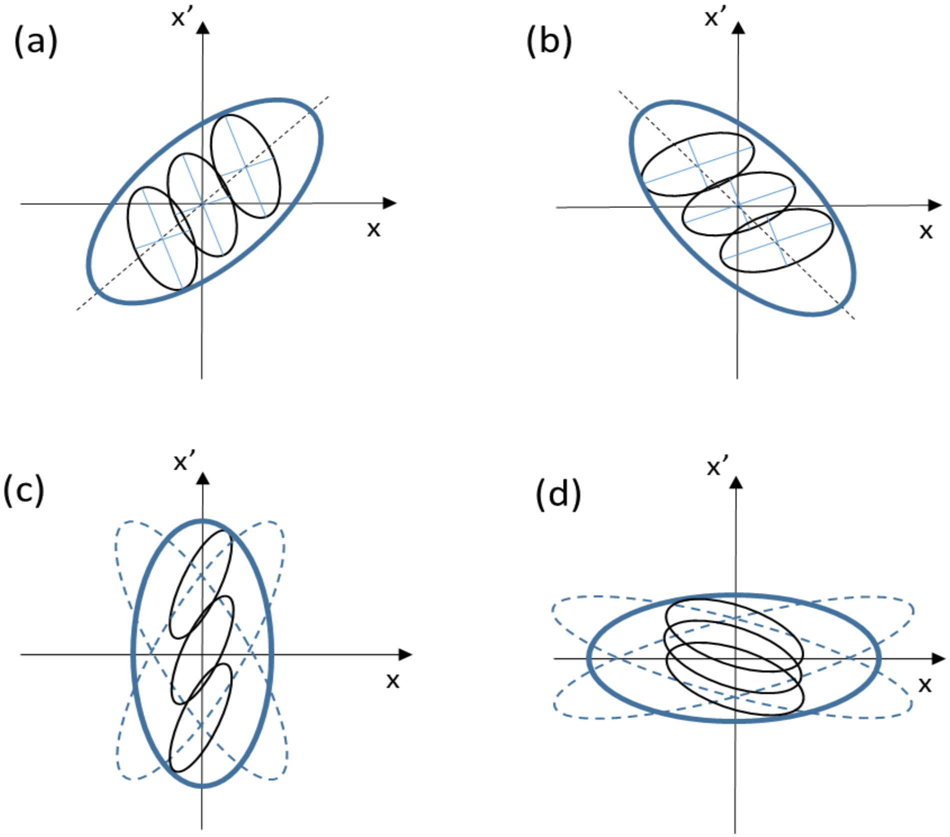

Collective Angle

This is solely determined by the linac dynamics.

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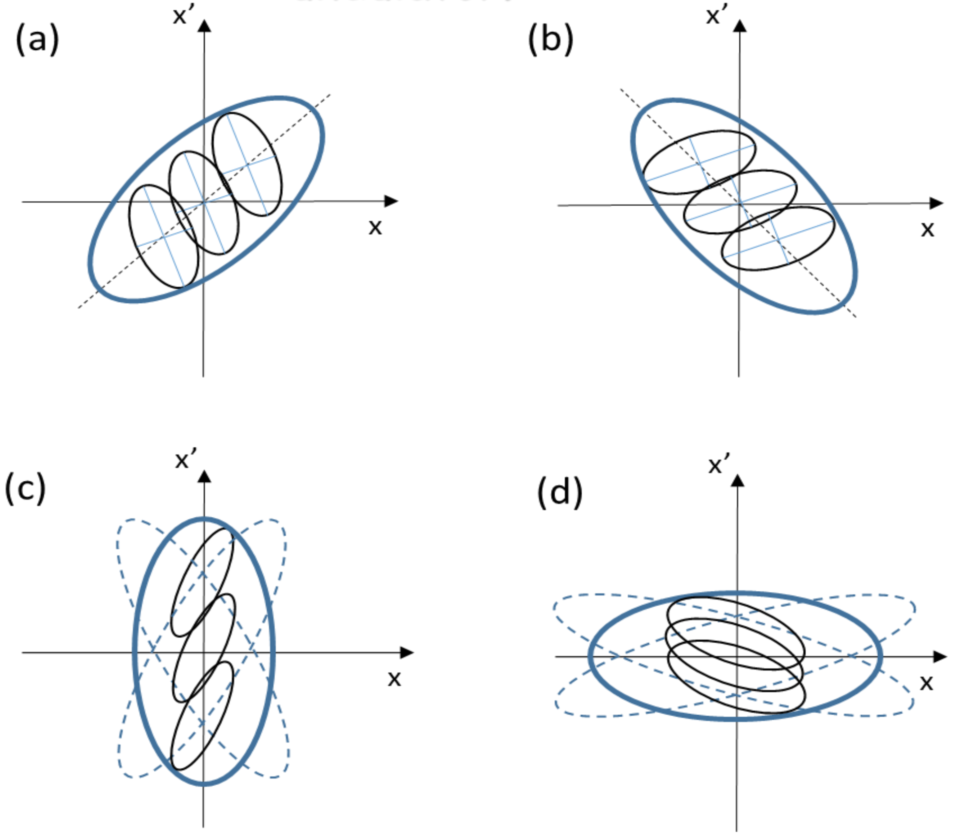
This is the resultant angular spread of the bunch slices' centroids.

This is <β> in the undulator.



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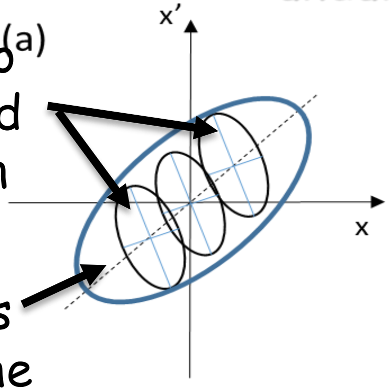
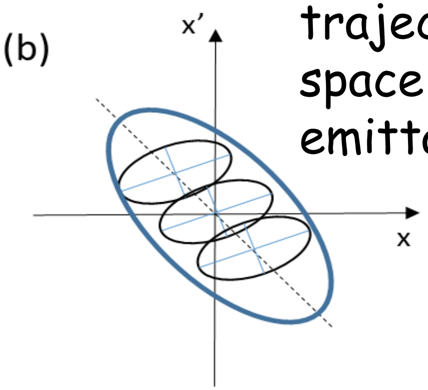
In the LINAC, two slices are displaced along the direction of the kick.



The slice emittance is unperturbed, while the projected is enlarged.

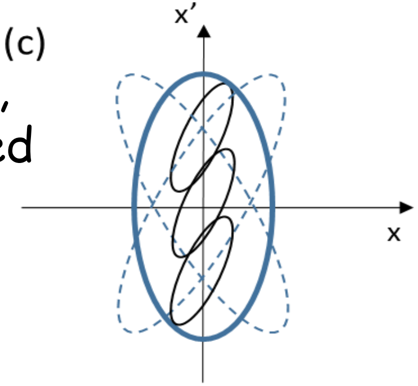


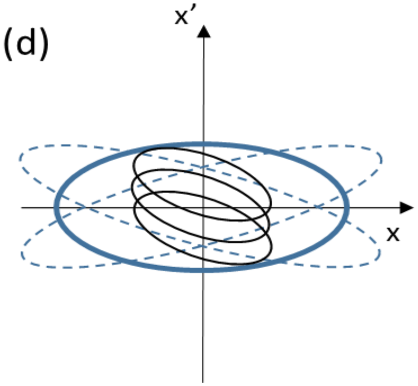
AFTER the kick(s), the slices follow different trajectories in phase space, but the projected emittance is preserved:

$$\beta\left\langle\theta_{coll}^2\right\rangle = cte$$

In the UNDULATOR, if the beam is matched to a **SMALL β_u**, the **slices are largely dispersed in angle:**

$$\left\langle\theta_{coll}^2\right\rangle \neq 0$$





If the beam is matched to a **LARGE β_u**, the **slices overlap in angle:** $\left\langle\theta_{coll}^2\right\rangle \rightarrow 0$

We expect **a BIG impact on FEL gain**

We expect **a SMALL impact on FEL gain**

3-D Gain Length

[7] Di Mitri, Spampinati, PRSTAB 17, 2014

$$L_{G,3D} = L_G[1 + \Lambda(\varepsilon_{x,y}, \sigma_\delta)]$$

Single-slice dynamics

$$L_{G,SKE} \approx \frac{L_G}{1 - \pi\theta^2_{SKE} / \theta^2_c}$$

De-bunching

$$\varepsilon_{n,f} \approx \varepsilon_{n,i}\sqrt{1 + \frac{\gamma\beta_u \left\langle \theta^2_{coll} \right\rangle}{\varepsilon_{n,i}}}$$

Projected dynamics

We propose[7]*:*

$$L_{G,coll} \approx \frac{L_{G,3D}}{1 - \pi\left\langle \theta^2_{coll} \right\rangle / \theta^2_{th}}, \qquad \theta_{th} \equiv \sqrt{\lambda / L_{G,3D}}$$

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This depicts the SLICE dynamics

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Projected dynamics

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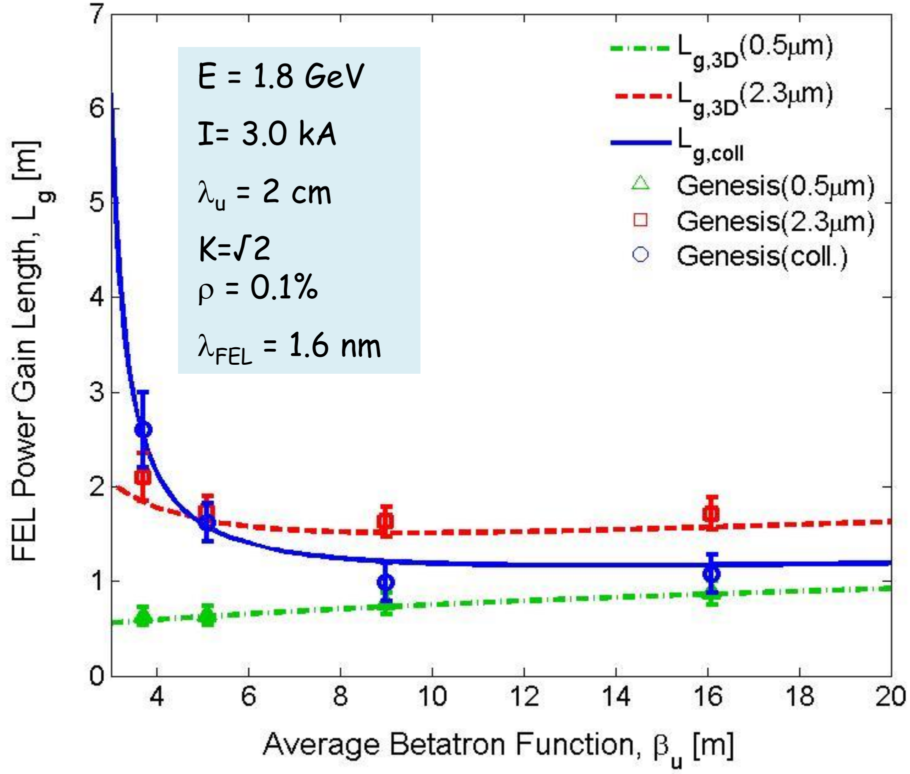
This depicts the SLICE dynamics

This depicts the PROJECTED dynamics

❑ Theory vs. GENESIS simulations:

1) - - - $\varepsilon_{n,proj} = \varepsilon_{n,slice} = 0.5$ μm
2) - - - $\varepsilon_{n,proj} = \varepsilon_{n,slice} = 2.3$ μm
3) - - - $\varepsilon_{n,proj} = 2.3$ μm > $\varepsilon_{n,slice} = 0.5$ μm

- Intuitively, we expect L_G of 3) in between that of 1) and 2); confirmed by simulations.
- $\beta_u := (\langle\beta_x\rangle\langle\beta_y\rangle)^{1/2}$; the scan spans different scenarios of radiation diffraction.

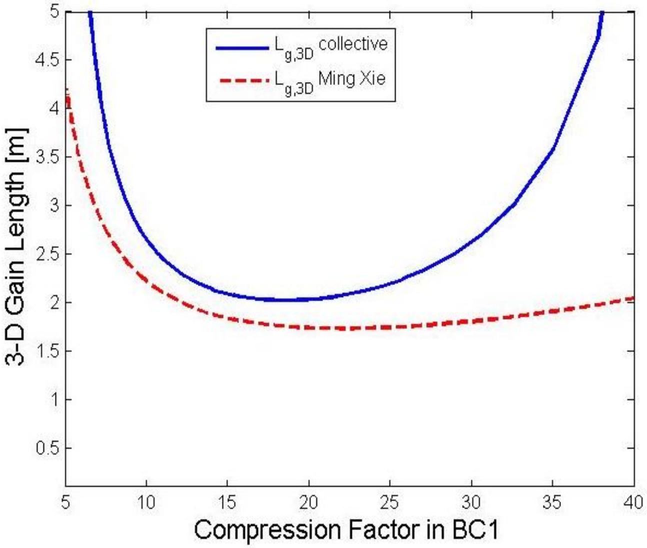


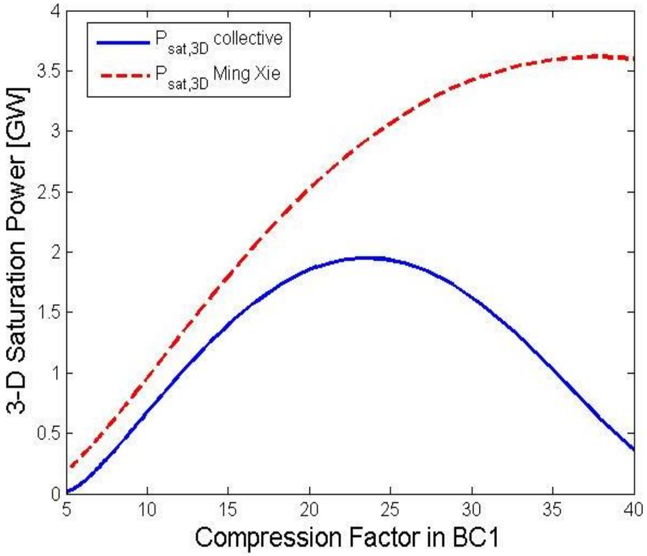
[7] Di Mitri, Spampinati, PRSTAB 17, 2014

Comparison with M.Xie-L_G

- ☐ FERMI-*like* linac (S-band, 1.4 GeV, 0.5 kA, 10 nm), one-stage compression.

Choice of the Linac working point





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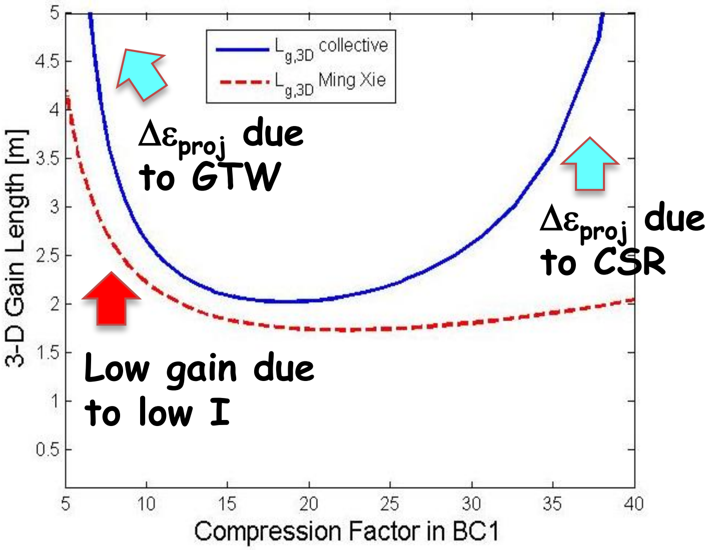
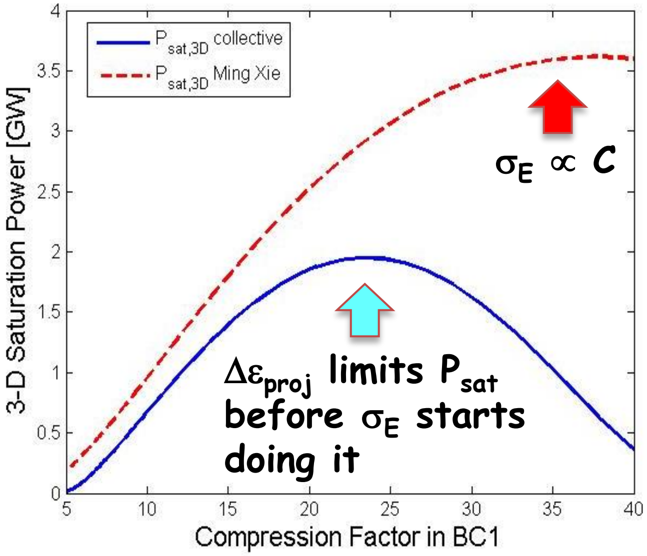


$\Delta\varepsilon_{proj}$ due to GTW

$\Delta\varepsilon_{proj}$ due to CSR

Low gain due to low I



$\sigma_E \propto C$

$\Delta\varepsilon_{proj}$ limits P_{sat} before σ_E starts doing it

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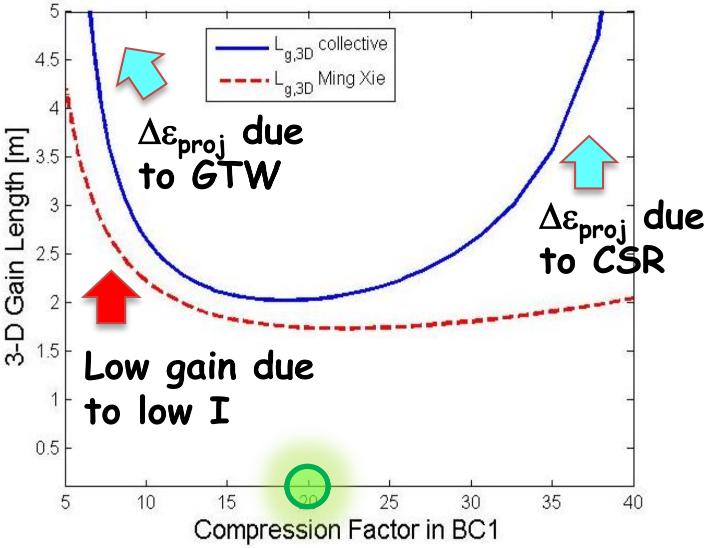

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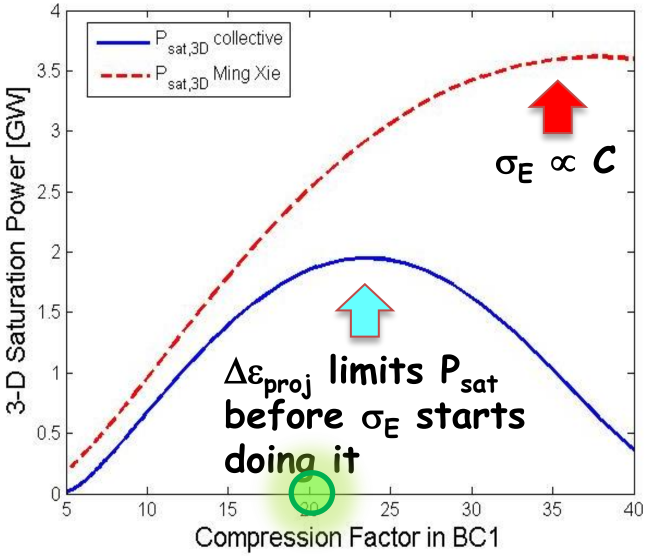

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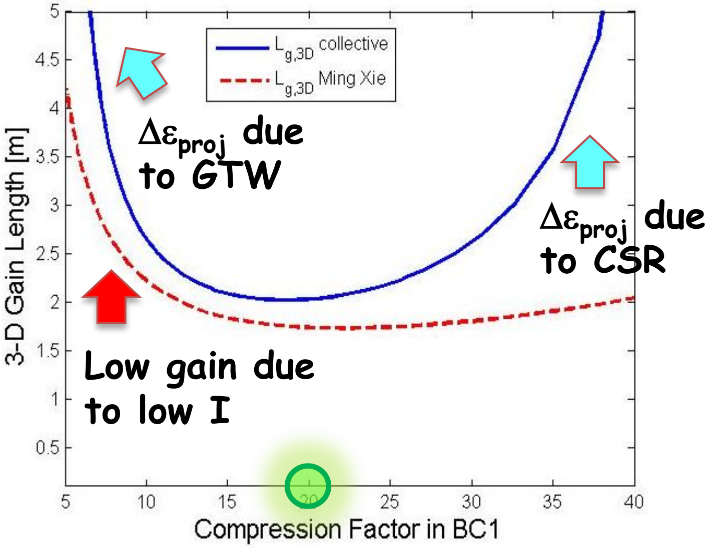
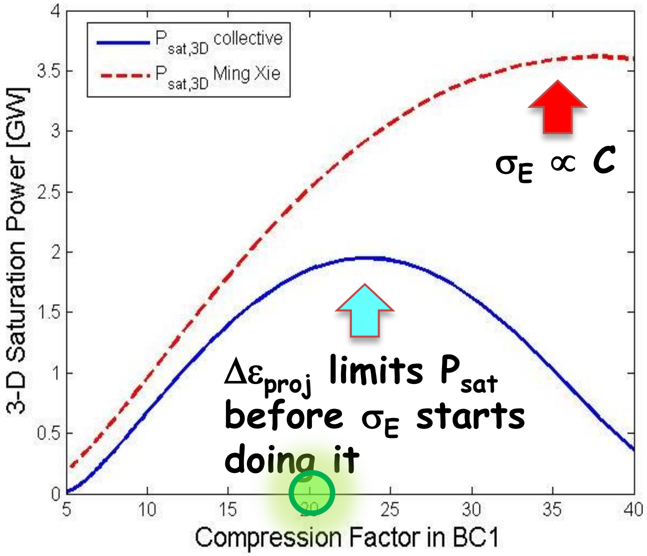
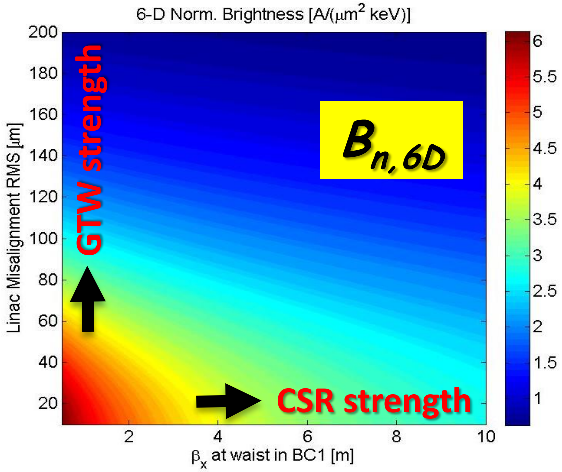
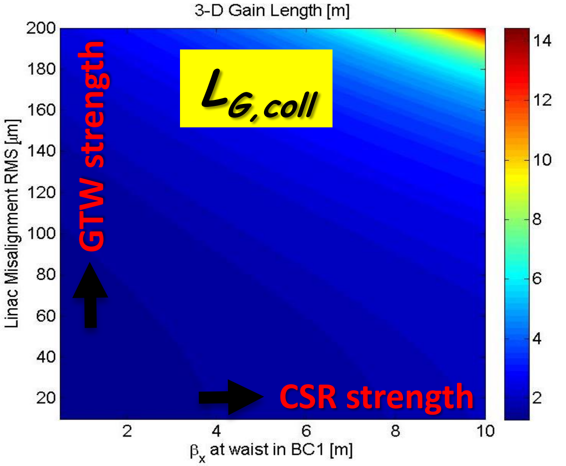
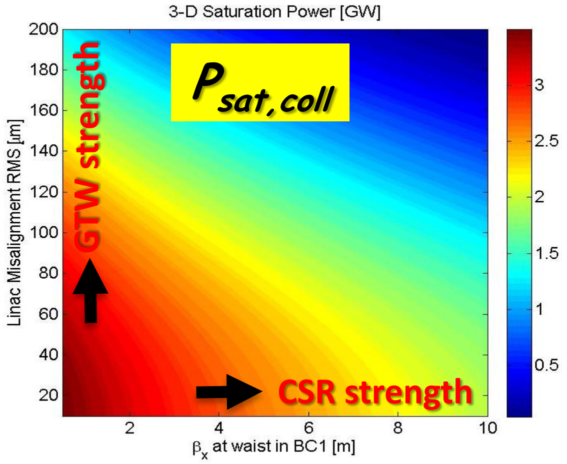




FEL Sensitivity to Linac setting







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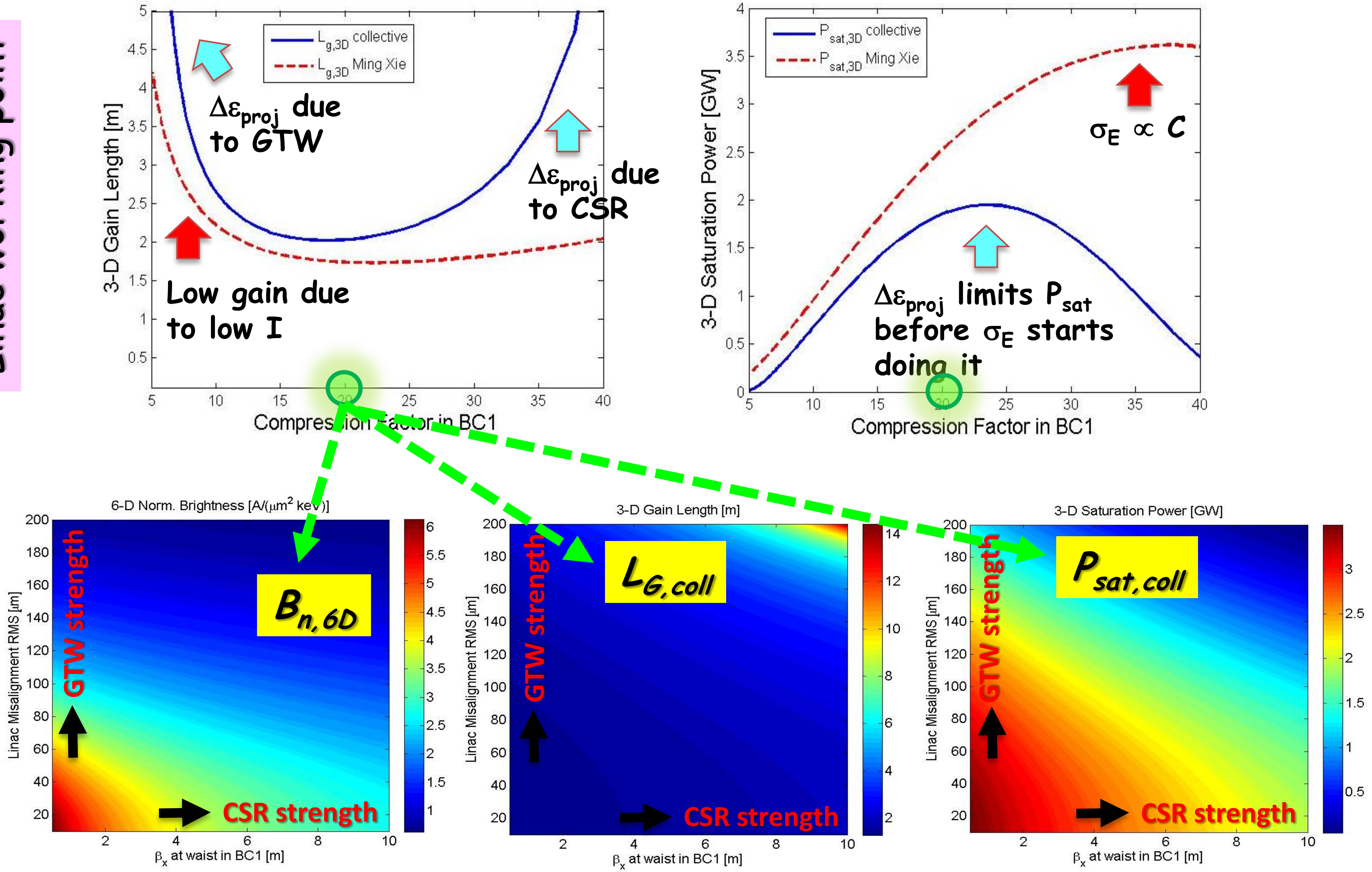
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$B_{n,6D}$

$L_{G,coll}$

$P_{sat,coll}$

GTW strength

CSR strength

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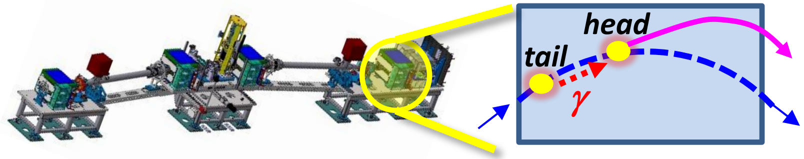
Collective Effects

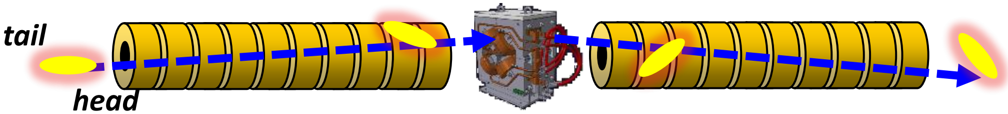
[4] Dohlus, Emma, Limberg ICFA 38, 2005
[5] Raubenheimer PRSTAB 3, 2000
[6] Di Mitri, Cornacchia Phys. Rep. 539, 2014

CSR in a 4-Dipole Compressor[4]:

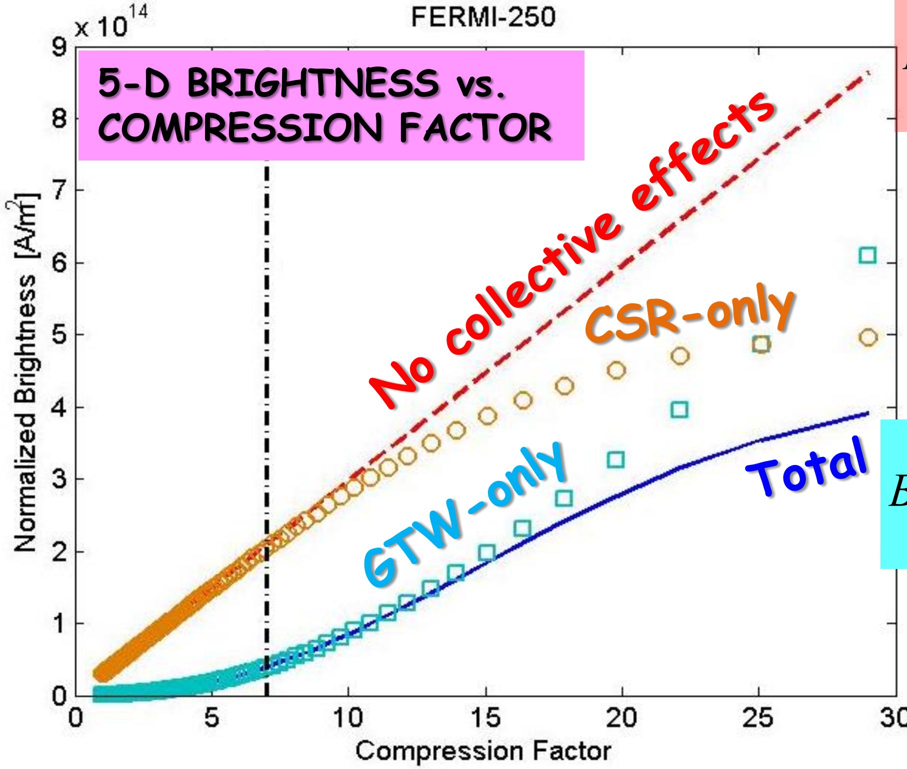


$$\varepsilon_{x,CSR} \cong \varepsilon_x \sqrt{1+\frac{\breve{\beta}\theta^2\sigma^2_{\delta,CSR}}{\varepsilon_x}} \propto \frac{1}{\sigma_z^{4/3}}$$

GTW in RF cavities[5]:



$$\varepsilon_{x,GTW} \cong \varepsilon_x \sqrt{1+2\left(\frac{\pi r_e}{Z_0 c e}\right)^2 \frac{Q^2\overline{W}_\perp^{\,2}(2\sigma_z)L_{FODO}L_{tot}\bar{\beta}}{\varepsilon_x\sqrt{\gamma_0\gamma_f}}\Delta^2} \propto \sigma_z^{1/2}$$



$$B^{5D}_{n,f} \equiv \frac{I}{\varepsilon_{nx,f}\varepsilon_{ny,f}} = CB^{5D}_{n,i}$$

The effect of CSR and GTW on ε_{proj} has opposite dependence on σ_z.

The beam brightness[6] may show a local maximum as a function of the final σ_z.

$$B^{5D}_{n,f} \equiv \frac{I}{\varepsilon_{nx,f}\varepsilon_{ny,f}} \equiv \frac{CB^{5D}_{n,i}}{\zeta_x\zeta_y}$$

$$\varepsilon_{n,f} \cong \varepsilon_{n,i}\sqrt{\left(1+P^{L1}_{GTW}\right)\left(1+P^{BC1}_{CSR}\right)\ldots} \approx \varepsilon_{n,i}\sqrt{1+\sum_{i=1}^{n}P^{i}_{CSR}+\sum_{j=1}^{m}P^{j}_{GTW}}$$

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Conclusions

- ☐ $L_{G,coll}$ aims to include the beam projected dynamics. A **deviation ~10%** was found **vs. 3-D time-dependent simulations**, over a wide range of β_u.

- ☐ **When ε_{proj} > ε_{slice}**, a considerable deviation from M.Xie-L_G appears. This suggests a **larger β_u for optimum FEL** performance.

- ☐ The model can be used either for **tuning of the accelerator** in order to maximize the FEL performance, or for specifying the **FEL tolerance** on the beam **projected emittance, vs. the undulator optics**.

➢ Further numerical studies will assess the **limits** of the proposed **model**:

- $\langle\theta^2_{coll}\rangle$ **neglects correlations** between consecutive CSR and GTW kicks.
- the ***"π-factor"*** in $L_{G,coll}$ might actually depend on e-beam parameters.
- **P_{sat} and L_{sat}** were re-scaled to the 1-D model.

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Thank You for Your attention

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