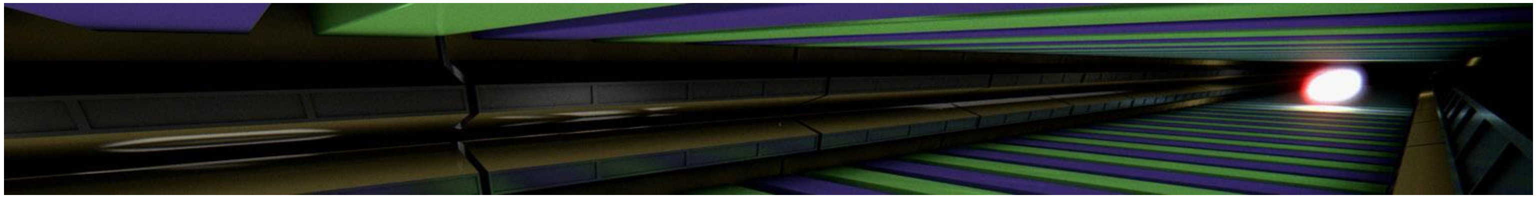

Optimization of a High Efficiency Free Electron Laser Amplifier

E.A. Schneidmiller and M.V. Yurkov

FEL 2015 Conference, Deajeon, Korea, August 23-28, 2015

ID: MOC02

- Tapering of seeded FEL amplifier in the presence of diffraction effects.
- Optimal tapering of SASE FEL and the radiation properties in the post-saturation regime.

Practical use of undulator tapering

Undulator tapering is a useful mechanism for many practical applications (FEL prize talk by Bill Fawley this morning) :

- Positive tapering - undulator K decreases along the undulator length:

  - Compensation of the beam energy loss due to spontaneous undulator radiation;
  - Compensation of the energy chirp in the electron beam;
  - Increase power of a high-gain FEL after saturation (post-saturation taper).

- Negative tapering - undulator K increases along the undulator length:

  - Compensation of the energy chirp in the electron beam;
  - Suppression of the radiation from the main undulator for organization of effective operation of afterburners (e.g., circular polarization).
  - Application in the scheme of attosecond SASE FEL.
  - Increase power of FEL oscillator.

A concept of post-saturation undulator tapering

- Undulator tapering: originally proposed by [N.M. Kroll, P.L. Morton, and M.N. Rosenbluth, IEEE J. Quantum Electronics, QE-17, 1436 (1981)] for increasing the radiation power in the post-saturation regime preserving resonance condition:

$$\lambda \simeq \lambda_{\mathrm{w}}(z)\frac{1+K^2(z)}{2\gamma^2(z)}.$$

FEL prize talk by Bill Fawley

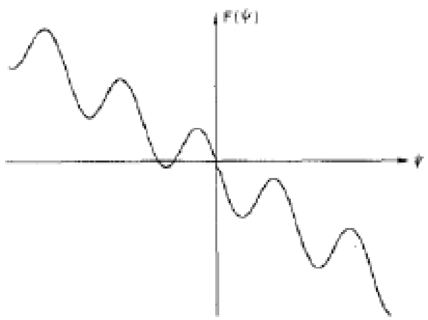

Fig. 1. The poderomotive potential $F(\psi)$. The case shown is for positive ψ_r corresponding to the case in which energy is extracted from the electrons.

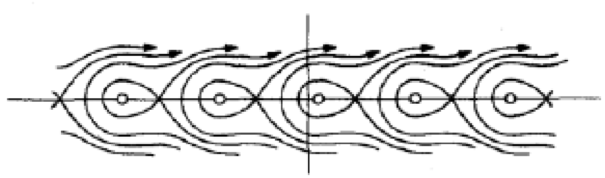

Fig. 2. Trajectories in the ψ, $\delta\gamma$ phase plane for $\psi_r > 0$.

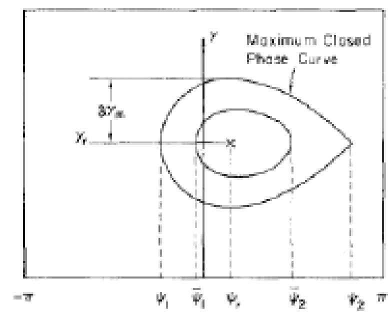

Fig. 3. Stable phase plane trajectories.

change in parameters is small. For small oscillations about ψ_r, one can expand $F(\psi)$ about ψ_r. The motion for these orbits is harmonic with period of oscillation

$$Z = \frac{\pi\mu}{(k_w + \delta k_s)\sqrt{a_s a_w \cos\psi_r}} \approx \frac{\mu\lambda_w}{2\sqrt{a_s a_w \cos\psi_r}}$$

$(\lambda_w \equiv 2\pi/k_w)$. (2.50)

Undulator tapering in the presence of diffraction effects

- The problem of optimum undulator tapering in the presence of diffraction effects is now a “hot” topic due to practical applications for X-ray FELs and potential industrial applications.

- **Empirical tapering dependencies for known so far from the literature are physically inconsistent with the asymptotical behavior of the radiation power produced in the tapered section.**

- Here we present our view of the problem based on the recent findings:

– *Optimization of a high efficiency free electron laser amplifier, Phys. Rev. ST AB, 18, 030705 (2015);*

– *The universal method for optimization of undulator tapering in FEL amplifiers, Proc. of SPIE Vol. 9512 951219-1 (2015);*

– *Statistical properties of the radiation from SASE FEL operating in a post-saturation regime with and without undulator tapering, J. Modern Optics, DOI:10.1080/09500340.2015.1035349 (2015).*

Undulator tapering in the presence of diffraction effects

Nuclear Instruments and Methods in Physics Research A 375 (1996) 550–562

NUCLEAR INSTRUMENTS & METHODS IN PHYSICS RESEARCH Section A

N·H ELSEVIER

"Optical guiding" limits on extraction efficiencies of single-pass, tapered wiggler amplifiers ☆

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Abstract

Single-pass, tapered wiggler amplifiers have an attactive feature of being able, in theory at least, of extracting a large portion of the electron beam energy into light. In circumstances where an optical FEL wiggler length is significantly longer than the Rayleigh length z_R corresponding to the electron beam radius, diffraction losses must be controlled via the phenomenon of optical guiding. Since the strength of the guiding depends upon the effective refractive index n exceeding one, and since $(n-1)$ is inversely proportional to the optical electric field, there is a natural limiting mechanism to the on-axis field strength and thus the rate at which energy may be extracted from the electron beam. In particular, the extraction efficiency for a prebunched beam asymptotically grows linearly with z rather than quadratically. We present analytical and numerical simulation results concerning this behavior and discuss its applicability to various FEL designs including oscillator/amplifier-radiator configurations.

It is well known from early studies that:

- Radiation power grows linearly with the undulator length for the asymptote of thin electron beam (i.e., long undulator)
- Radiation power grows quadratically with the undulator length for the asymptote of wide electron beam (i.e., short undulator / initial stage of tapered regime).

$$S_{\mathrm{rad}} \simeq r_{\mathrm{b}}^2 \qquad \text{for} \qquad r_{\mathrm{dif}}^2 \ll r_{\mathrm{b}}^2 .$$

Assuming that a significant fraction of the particles is trapped in the regime of coherent deceleration, we can estimate the power loss by the electron beam in the tapered section as:

$$W \simeq I_0 |\tilde{E}| \theta_{\mathrm{s}} z_{\mathrm{tap}} .$$

In the latter expression we used the following estimation for the radiation field:

$$\int_0^{z_{\mathrm{tap}}} \tilde{E}(z) \mathrm{d}z \simeq \tilde{E}(z_{\mathrm{tap}}) z_{\mathrm{tap}} .$$

Thus, we have the following power balance:

1. Thin electron beam ($r_{\mathrm{b}}^2 \ll c z_{\mathrm{tap}}/\omega$):

$$S_{\mathrm{rad}} \simeq c z_{\mathrm{tap}}/\omega ,$$

$$W \simeq I_0 |\tilde{E}| \theta_{\mathrm{s}} z_{\mathrm{tap}} \simeq |\tilde{E}|^2 c^2 z_{\mathrm{tap}}/\omega \rightarrow$$

$$W \simeq I_0^2 \theta_{\mathrm{s}}^2 z_{\mathrm{tap}} \omega / c^2 , \quad |\tilde{E}| \simeq I_0 \theta_{\mathrm{s}} \omega / c^2 .$$

2. Wide electron beam ($r_{\mathrm{b}}^2 \gg c z_{\mathrm{tap}}/\omega$):

$$S_{\mathrm{rad}} \simeq r_{\mathrm{b}}^2 ;$$

$$W \simeq I_0 |\tilde{E}| \theta_{\mathrm{s}} z_{\mathrm{tap}} \simeq |\tilde{E}|^2 c r_{\mathrm{b}}^2 \rightarrow$$

$$W \simeq I_0^2 \theta_{\mathrm{s}}^2 z_{\mathrm{tap}}^2 / (c r_{\mathrm{b}}^2) , \quad |\tilde{E}| \simeq I_0 \theta_{\mathrm{s}} z_{\mathrm{tap}} / (c r_{\mathrm{b}}^2) .$$

Our estimates show that in the case of a thin electron beam, the radiation field, acting on the electrons, is almost constant along the undulator axis. The radiation power grows linearly with the length of the tapered section, z_{tap}. Thus, we can conclude that the regime of coherent deceleration of the particles should take place only for a linear law of undulator tapering, i.e. the detuning $\hat{C}(\hat{z})$ should change linearly with the $\hat{z}$ coordinate.

Let us consider the case of a large value of the diffraction parameter,

$$B = \Gamma \omega r_{\mathrm{b}}^2 / c \gg 1 .$$

At the beginning of the tapering section, when $\Gamma z_{\mathrm{tap}} \lesssim B$, we deal with the case of a wide electron beam and most of the radiation overlaps with the electron beam. When the length of the tapered section increases, the radiation expands out of the electron beam. When $\Gamma z_{\mathrm{tap}} \gg B$ we always fall in the region when diffraction effects are important, i.e. the electron beam becomes thin with respect to the radiation beam. Thus, we come to the conclusion that the asymptotically stable regime of coherent deceleration should occur only for a linear law of undulator tapering.

E. Saldin, E. Schneidmiller, M. Yurkov, The Physics of Free Electron lasers, 2000 /

Undulator tapering in the presence of diffraction effects

The key element for understanding the physics of the undulator tapering are the properties of the radiation from modulated electron beam.

Indeed, in the case of tapered FEL the beam bunching is frosen (particles are trapped in the regime of coherent deceleration).

Thus, we deal with the electron beam modulated at the resonance wavelength.

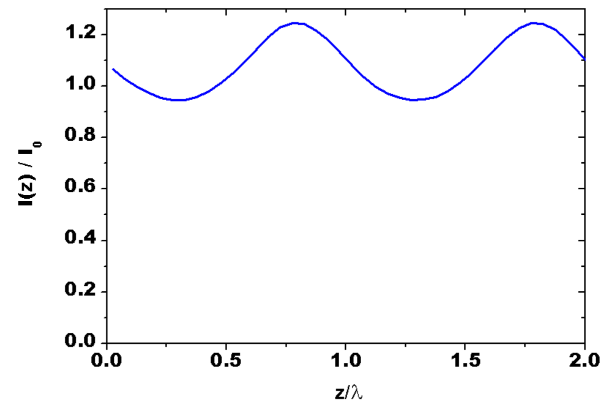
If we know the radiation power of the modulated electron beam as function of the undulator length, we know the law of the undulator tapering.

The problem of the radiation of modulated electron beam has been solved ten years ago (Nucl. Instrum. and Methods A 539, 499 (2005)), and recently connected with the problem of the undulator tapering (Phys. Rev. ST AB 18 (2015) 030705)).

Radiation of modulated electron beam

- Radiation power of modulated electron beam:

$$W = \frac{2\pi^2 I_0^2 a_{\mathrm{in}}^2 \sigma^2}{c\lambda\lambda_{\mathrm{u}}} \frac{K^2 A_{\mathrm{JJ}}^2}{1+K^2} f(\tilde{z})\tilde{z} \, , \qquad f(\tilde{z}) = \arctan(\tilde{z}/2) + \tilde{z}^{-1} \ln\left(\frac{4}{\tilde{z}^2+4}\right)$$

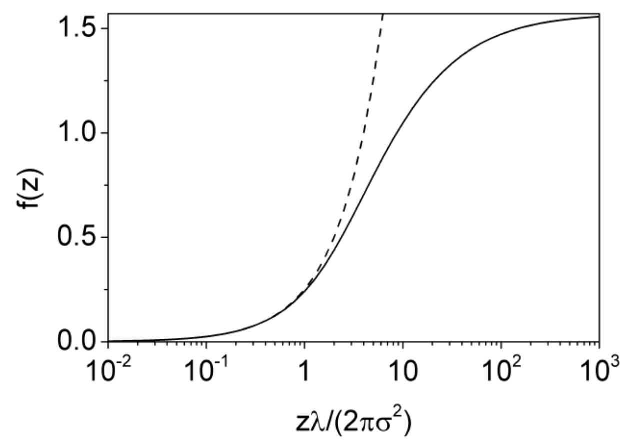


- Thin and wide beam asymptote:

$$\begin{aligned} f(\tilde{z}) &\to \pi/2 \qquad \text{for} \qquad \tilde{z} \gg 1 \qquad (N \ll 1) \, , \\ f(\tilde{z}) &= \tilde{z}/4 \qquad \text{for} \qquad \tilde{z} \ll 1 \qquad (N \gg 1) \, . \end{aligned}$$

The Fresnel number: $N = 2\pi\sigma^2/(\lambda z)$.

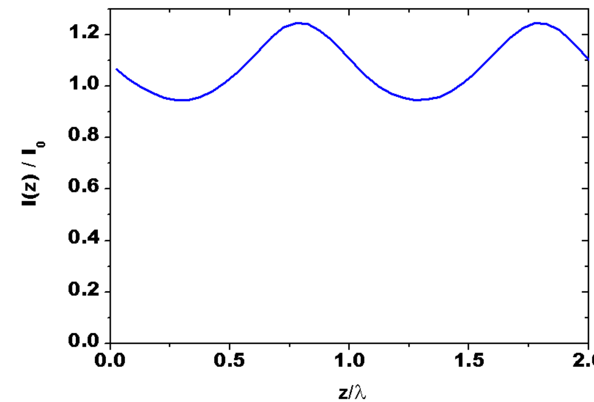
- Both asymptotes (of wide and thin electron beam) discussed in earlier papers are well described by this expression.
- Asymptote of a wide electron beam corresponds to large values of Fresnel number N, and the radiation power scales quadratically with the undulator length, $W \propto z^2$.
- Asymptote of a thin electron beam corresponds to small values of the Fresnel Number N, and the radiation power grows linearly with the undulator length, $W \propto z$.
- Undulator tapering should adjust detuning according to the energy loss by electrons, and we find that the tapering law should be quadratic for the case of wide electron beam, $C \propto W \propto z^2$, and linear for the case of thin electron beam, $C \propto W \propto z$.



Radiation of modulated electron beam

- Radiation power of modulated electron beam:

$$W = \frac{2\pi^2 I_0^2 a_{\rm in}^2 \sigma^2}{c\lambda\lambda_{\rm u}} \frac{K^2 A_{\rm JJ}^2}{1+K^2} f(\tilde{z})\tilde{z} \,, \qquad f(\tilde{z}) = \arctan\left(\tilde{z}/2\right) + \tilde{z}^{-1} \ln\left(\frac{4}{\tilde{z}^2+4}\right) \,.$$

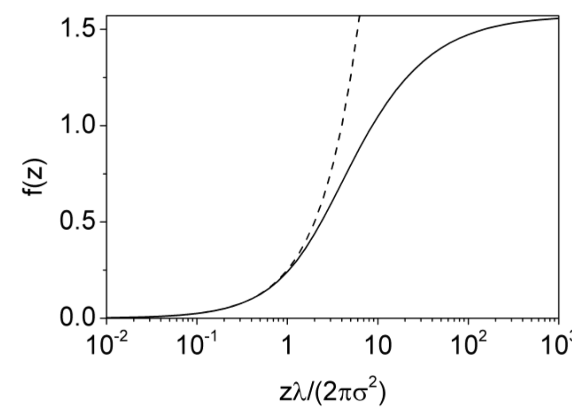


Thin and wide beam asymptote:

$$\begin{aligned} f(\tilde{z}) &\to \pi/2 \qquad \text{for} \qquad \tilde{z} \gg 1 \qquad (N \ll 1) \,, \\ f(\tilde{z}) &= \tilde{z}/4 \qquad \text{for} \qquad \tilde{z} \ll 1 \qquad (N \gg 1) \,. \end{aligned}$$

The Fresnel number: $N = 2\pi\sigma^2/(\lambda z)$.

- Asymptote of the wide electron beam works reasonably well for the values of the Fresnel number $N \gtrsim 1$. Asymptote of the thin electron beam converges pretty slowly, and reasonable accuracy is achieved for very small $N \lesssim 0.01$.
- Example for LCLS operating at the radiation wavelength of 0.15 nm and 1.5 nm. Transverse size of the electron beam is about 25 μm in both cases.
- The wide beam asymptote is applicable up to $z \simeq z_{\rm wb} \simeq 26$ m for wavelength 0.15 nm, and $z \simeq z_{\rm wb} \simeq 2.6$ m for operation at 1.5 nm wavelength. Here we see general feature illustrating shortening with the radiation wavelength of the applicability region of the wide beam asymptote.
- The thin beam asymptote becomes to be applicable at LCLS for $z \gtrsim 2500$ m (for wavelength 0.15 nm) and 260 m (for wavelength 1.5 nm). Note, that for both practical examples the limit of thin electron beam is achieved only for very long undulator, and exact formula should be used for calculation of the radiation power for undulator length $z > z_{\rm wb}$.

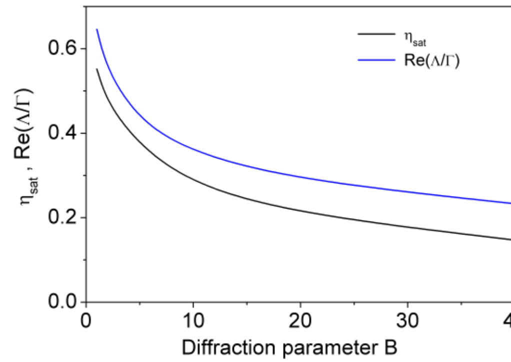


Application of similarity techniques

- In the framework of the three-dimensional theory the operation of the FEL amplifier is described by the diffraction parameter B, the energy spread parameter $\hat{\Lambda}_{\mathrm{T}}^2$, the betatron motion parameter $\hat{k}_\beta$ and detuning parameter $\hat{C}$:

$$B = 2\Gamma\sigma^2\omega/c\ , \qquad \hat{C} = C/\Gamma\ , \qquad \hat{k}_\beta = 1/(\beta\Gamma)\ , \qquad \hat{\Lambda}_{\mathrm{T}}^2 = (\sigma_{\mathrm{E}}/\mathcal{E})^2/\rho^2\ ,$$

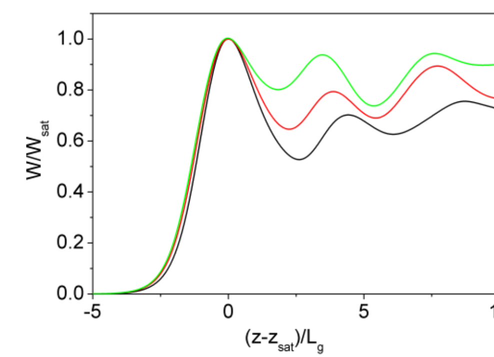
with the gain parameter $\Gamma = 4\pi\rho/\lambda_w$. For the case of "cold" electron beam, $\hat{\Lambda}_{\mathrm{T}}^2 \to 0$, $\hat{k}_\beta \to 0$, the operation of the FEL amplifier is described by the diffraction parameter B and the detuning parameter $\hat{C}$.



- FEL equations:

$$\frac{d\Psi}{d\hat{z}} = \hat{C} + \hat{P}, \qquad \frac{d\hat{P}}{d\hat{z}} = U\cos(\phi_U + \Psi)\ ,$$

where $\hat{P} = (E - E_0)/(\rho E_0)$, $\hat{z} = \Gamma z$, and U and ϕ_U are the amplitude and the phase of the effective potential.



- We normalize the radiation power to the saturation power, and undulator length to the field gain length. Then we find that the radiation power before saturation exhibits similar behavior for all values of the diffraction parameter $B > 1$.

- In view of: i) universal scaling of the FEL characteristics on the diffraction parameter B; ii) The Fresnel number and the diffraction parameter has the same physical meaning, we find that optimum undulator tapering should be:

$$\hat{C} = \alpha_{tap}(\hat{z} - \hat{z}_0)\left[\arctan\left(\frac{1}{2N}\right) + N\ln\left(\frac{4N^2}{4N^2+1}\right)\right]\ , \qquad N = \frac{\beta_{tap}}{\hat{z} - \hat{z}_0}\ .$$

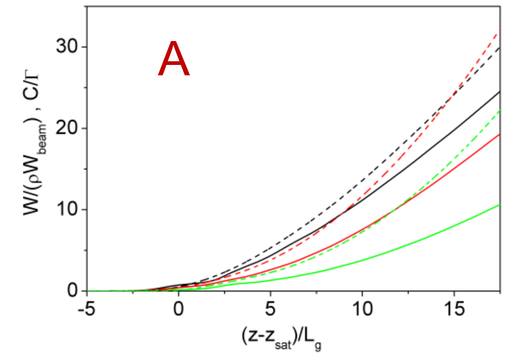
European XFEL

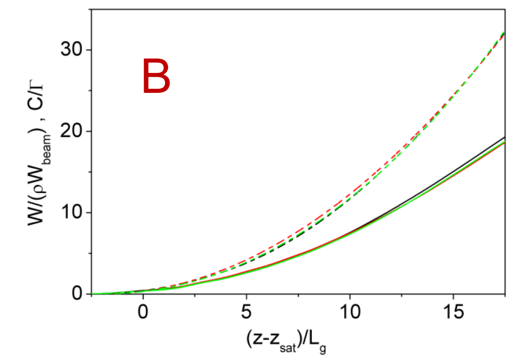
Global numerical optimization versus the universal law of the undulator tapering

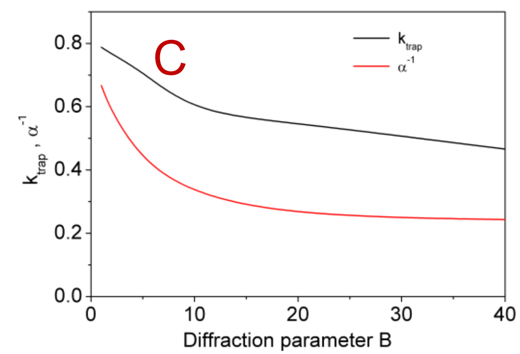
- First, we perform straightforward global optimization with three-dimensional, time-dependent FEL simulation code FAST.
- Target of the optimization is maximum of the output power at 15 gain lengths after saturation. We divide undulator into many pieces and change detuning of all pieces independently. Number of sections is controlled to provide the result independent on the number of sections.
- (A) We choose the tapering law $C(B, z)$ corresponding to the maximum power at the exit of the whole undulator.
- (B) Then we fit parameters of the universal tapering law:

$$\hat{C} = \alpha_{tap}(\hat{z} - \hat{z}_0) \left[\arctan\left(\frac{1}{2N}\right) + N \ln\left(\frac{4N^2}{4N^2 + 1}\right) \right] , \qquad N = \frac{\beta_{tap}}{(\hat{z} - \hat{z}_0)}$$

- Start of the undulator tapering z_0 is fixed by the global optimization procedure, $z_0 = z_{sat} - 2L_g$.
- Another parameter of the problem, β_{tap}, is rather well approximated with the linear dependency on the diffraction parameter, $\beta_{tap} = 8.5 \times B$.
- (C) Remaining parameter, α_{tap}, is plotted in Figure. It is a slow varying function of the diffraction parameter B, and scales approximately to $B^{1/3}$ as all other important FEL parameters including capture efficiency.
- Thus, application of similarity techniques gives us an elegant way for the general parametrical fit.







Global numerical optimization versus the universal law of the undulator tapering and the rational fit

- Universal tapering law:

$$\hat{C} = \alpha_{tap}(\hat{z} - \hat{z}_0) \left[\arctan\left(\frac{1}{2N}\right) + N \ln\left(\frac{4N^2}{4N^2+1}\right) \right] ,$$

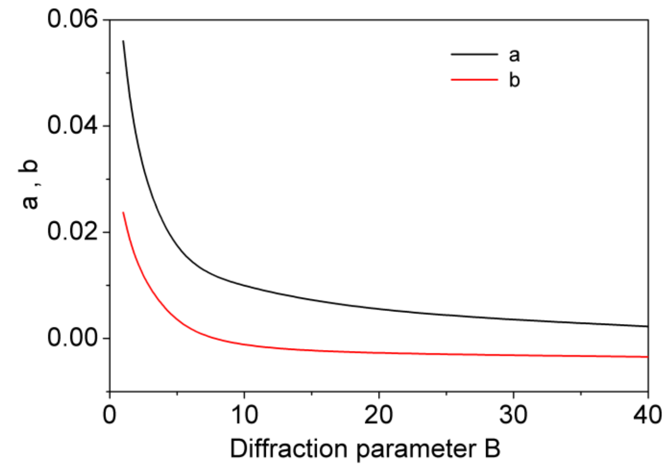
with Fresnel number N fitted by $N = \beta_{tap}/(\hat{z} - \hat{z}_0)$. Start of the undulator tapering is $z_0 = z_{sat} - 2L_g$. Parameter β_{tap}, is $\beta_{tap} = 8.5 \times B$.

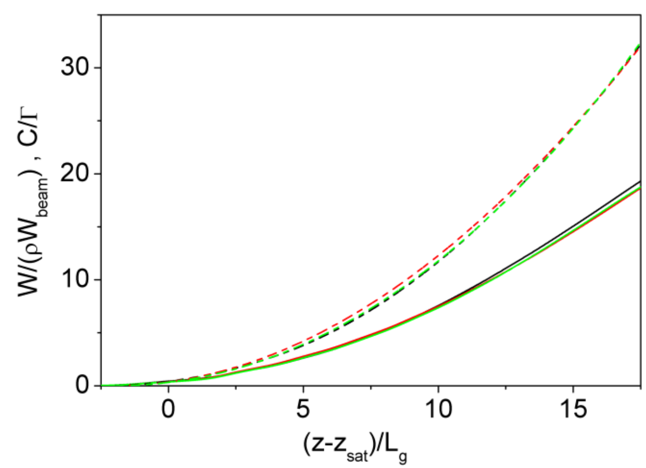
- Expression for the universal tapering law has quadratic dependence in z for small values of z (limit of the wide electron beam), and linear dependence in z for large values of z (limit of the thin electron beam). It is natural to try a fit with a rational function which satisfies both asymptotes. The simplest rational fit is:

$$\hat{C} = \frac{a(\hat{z} - \hat{z}_0)^2}{1 + b(\hat{z} - \hat{z}_0)} .$$

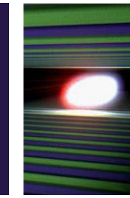
- The coefficients a and b are the functions of the diffraction parameter B, and are plotted in the Figure. Start of the undulator tapering is set to the value $z_0 = z_{sat} - 2L_g$ suggested by the global optimization procedure.

- Lower Figure: evolution along the undulator of the reduced radiation power $\hat{\eta} = W/(\rho W_{\mathrm{beam}})$ (solid curves) and of the detuning parameter $\hat{C} = C/\Gamma$ (dashed curves). Color codes: black - FEL with global optimization of undulator tapering, red - fit with the universal tapering law, green - fit with the rational function. The value of the diffraction parameter is $B = 10$.



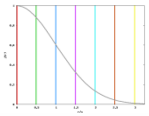


Trapping process for optimum tapering: how it works



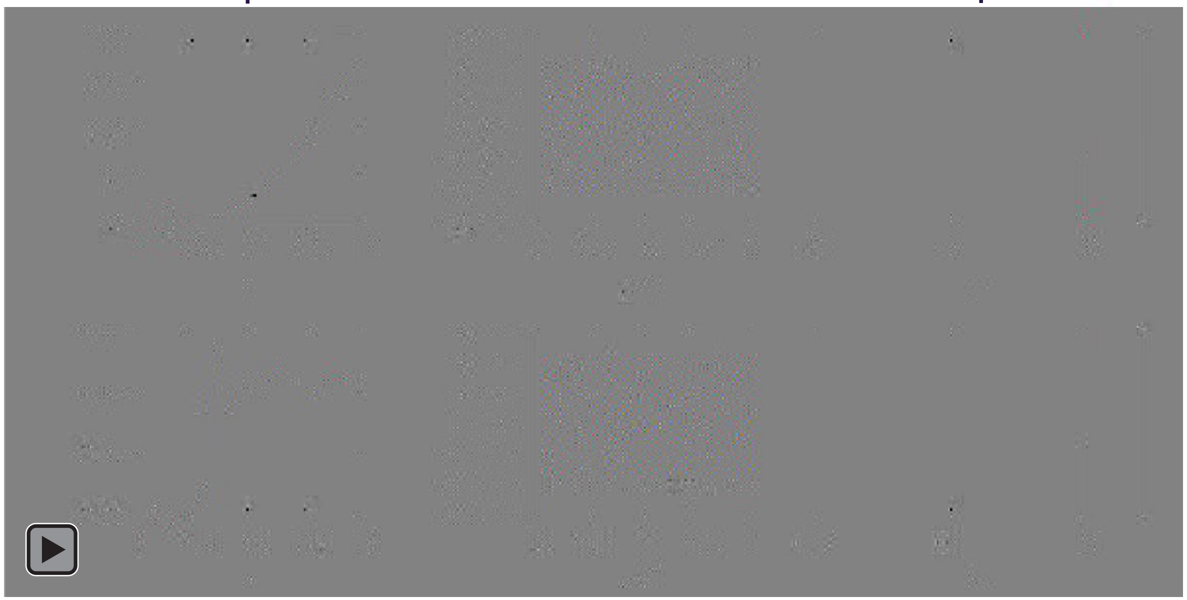
Top: tapered

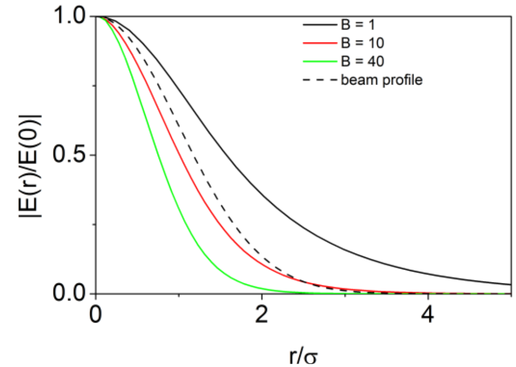
Bottom: untapered

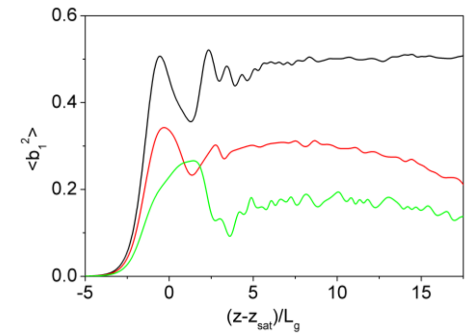


Color codes:

radial position

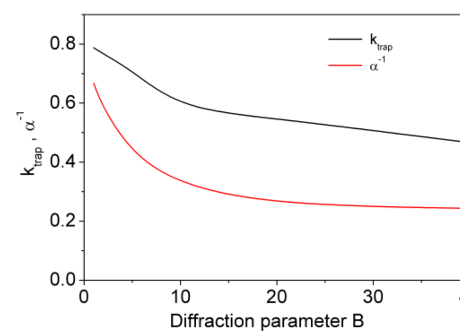




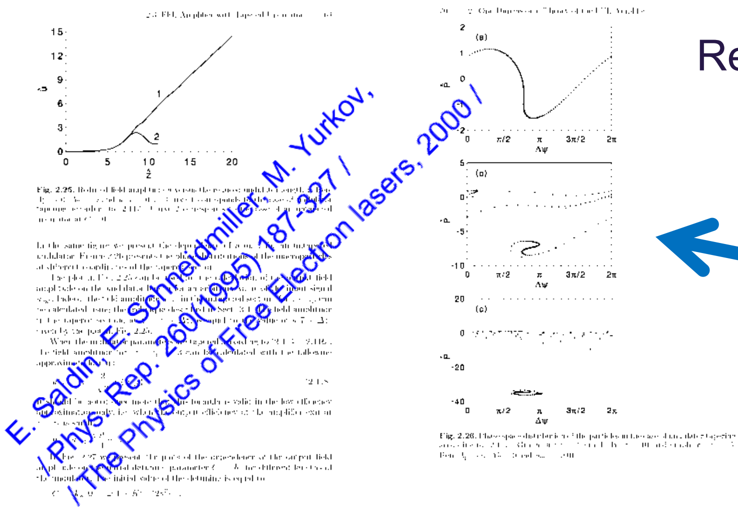


- The particles in the core of the beam red, green, blue color) are trapped most effectively. Nearly all particles located at the edge of the electron beam (braun, yellow color) leave the stability region very soon. The trapping process lasts for a several field gain lengths when the trapped particles become to be isolated in the trapped energy band for which the undulator tapering is optimized further. Non-trapped particles continue to populate low energy tail of the energy distribution.

- Experimental observation at LCLS: energy distribution of non-trapped particles is not uniform, but represent a kind of energy bands. Our simulations give a hint on the origin of energy bands which are formed by non-trapped particles. This is the consequence of nonlinear dynamics of electrons leaving the region of stability. Note that a similar effect can be seen in the early one-dimensional studies.



Energy bands in early 1D simulations

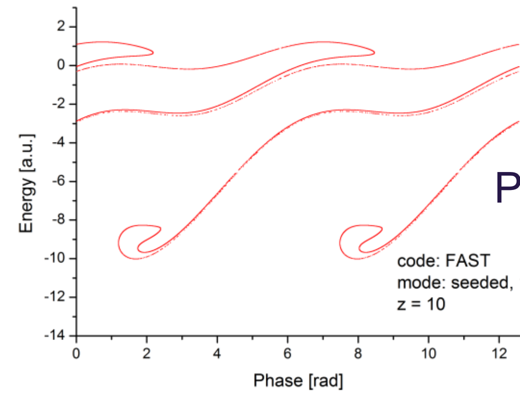


E. Saldin, E. Schneidmiller, M. Yurkov, / Phys. Rep. 260(1995) 187-327 / / The Physics of Free Electron lasers, 2000 /

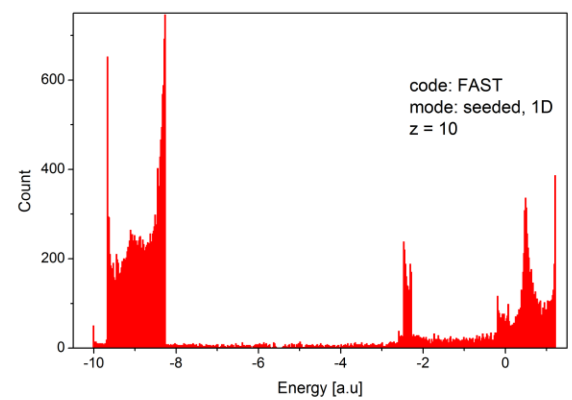
Tutorial example from a book (1D FEL theory)

Revisited: the same code (FAST), just more macroparticles

Phase space

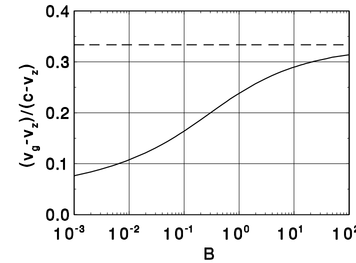


Energy spectrum

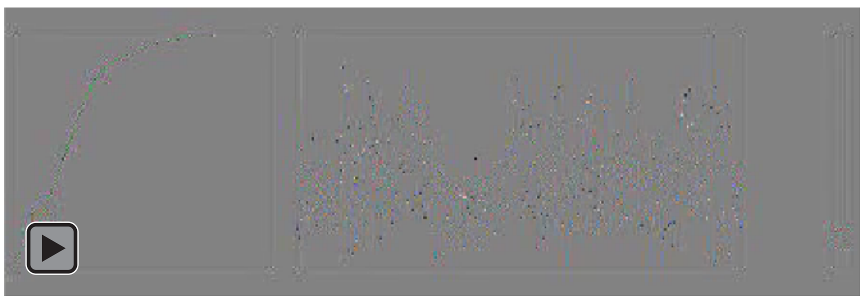


SASE FEL: Optimum tapering

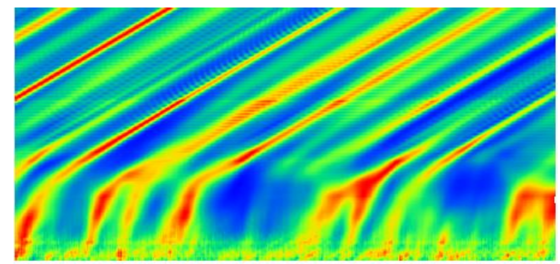
- Radiation of SASE FEL consists of wavepackets (spikes). In the exponential regime of amplifications wavepackets interact strongly with the electron beam, and their group velocity $\mathrm{d}\omega/\mathrm{d}k$ visibly differs from the velocity of light, and the slippage of the radiation with respect to the electron beam is by several times less than kinematic slippage. I.e., wavepackets are closely connected with the modulations of the electron beam current.



$< P(z) >$ $\quad P(\hat{s})$, $P_e(\hat{s})$ $\quad z$ $\quad P(\hat{s}, z)$



$\hat{s} = \rho\omega(z/v_z - t)$



$\hat{s} = \rho\omega(z/v_z - t)$

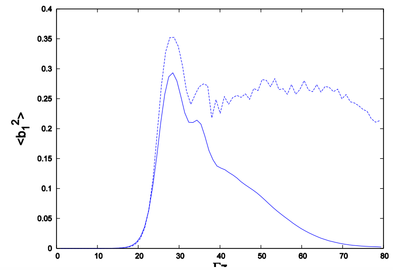
- When the amplification process enters nonlinear (tapering) stage, the group velocity of the wavepackets approaches to the velocity of light, and the relative slippage approaches to the kinematic one. When a wavepacket advances such that it reaches the next area of the beam disturbed by another wavepacket, we can easily predict that the trapping process will be destroyed, since the phases of the beam bunching and of the electromagnetic wave are uncorrelated in this case.

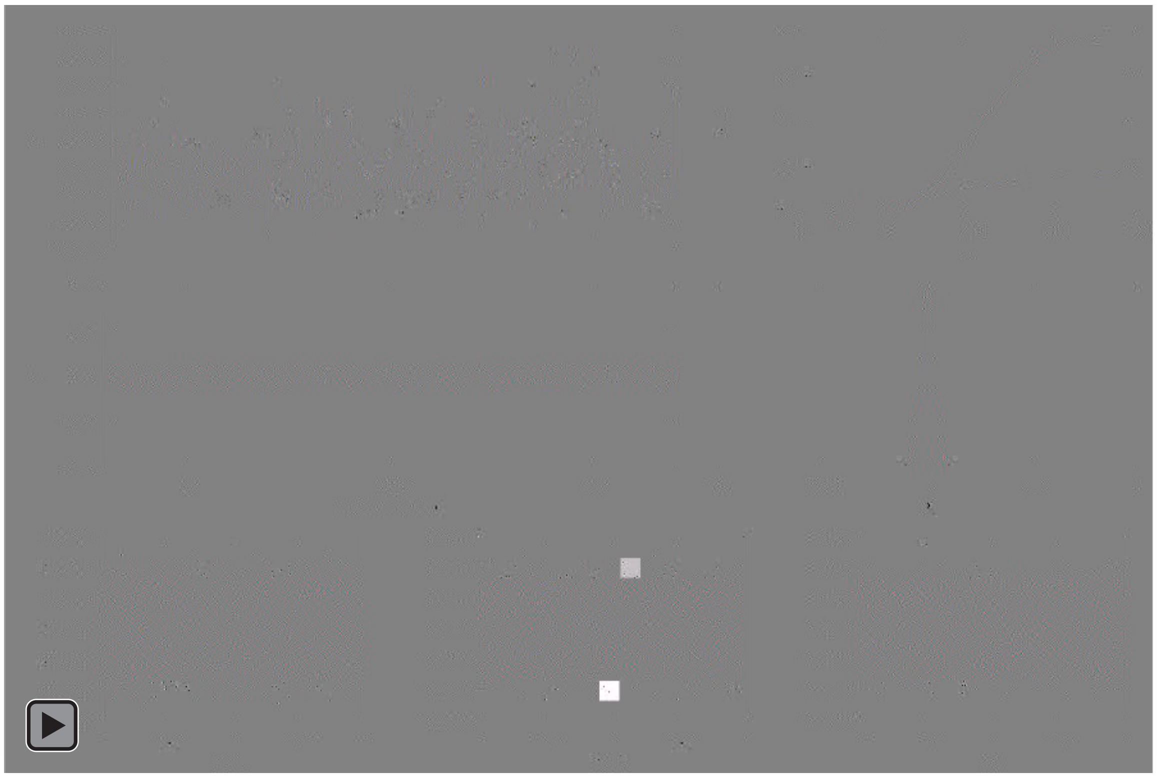
- Typical scale for the destruction of the tapering regime is coherence length, and the only physical mechanism we can use is to decrease the group velocity of wavepackets. This happens optimally when we trap maximum of the particles in the regime of coherent deceleration, and force these particles to interact as strong as possible with the electron beam. Thus, the strategy is exactly the same as we used for optimization of seeded FEL.

- Conditions of the optimum tapering for SASE are similar to those of the seeded case. Start of the tapering is by two field gain lengths before the saturation. Parameter β_{tap} is the same, $8.5 \times B$. The only difference is the reduction of the parameter α_{tap} by 20% which is natural if one remember statistical nature of the wavepackets.

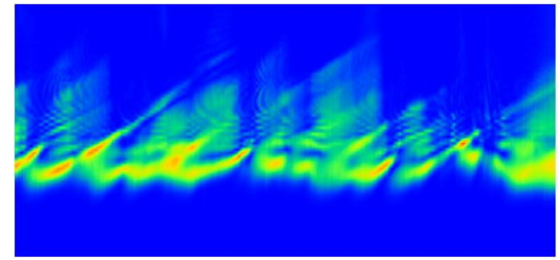
SASE FEL, optimum tapering: how it works

- Left: slice radiation power and energy loss; phase space
- Right: bunching, average power, particle energy spectrum



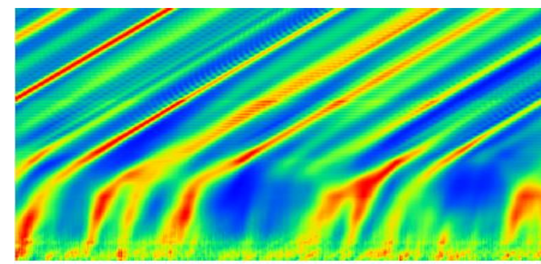


Bunching

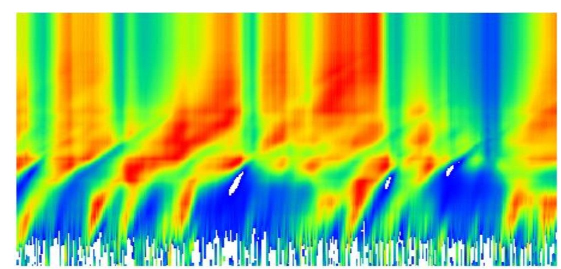


z

Radiation power



Energy loss



z

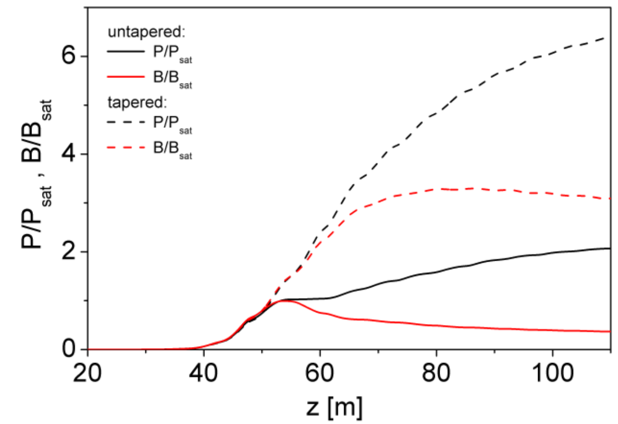
$$\hat{C} = \alpha_{tap}(\hat{z} - \hat{z}_0) \left[\arctan\left(\frac{1}{2N}\right) + N \ln\left(\frac{4N^2}{4N^2+1}\right) \right] , \qquad N = \frac{\beta_{tap}}{(\hat{z} - \hat{z}_0)}$$

$$\hat{s} = \rho\omega(z/v_z - t)$$

Properties of the radiation: Optimum tapered versus untapered case

- Practical example: European XFEL, SASE3, radiation wavelength 1.6 nm.
- General feature of tapered regime is that both, spatial and temporal coherence degrade in the nonlinear regime, but a bit slowly than for untapered case.
- Peak brilliance grows due to the growth of the radiation power, and reaches maximum value in the middle of tapered section. Benefit in the peak brilliance is about factor of 3 with respect to untapered case.

Power and brilliance



- Spatial corellations and degree of transverse coherence:

$$\gamma_1(\vec{r}_\perp, \vec{r'}_\perp, z, t) = \frac{\langle \tilde{E}(\vec{r}_\perp, z, t)\tilde{E}^*(\vec{r'}_\perp, z, t)\rangle}{\left[\langle|\tilde{E}(\vec{r}_\perp, z, t)|^2\rangle\langle|\tilde{E}(\vec{r'}_\perp, z, t)|^2\rangle\right]^{1/2}} ,$$

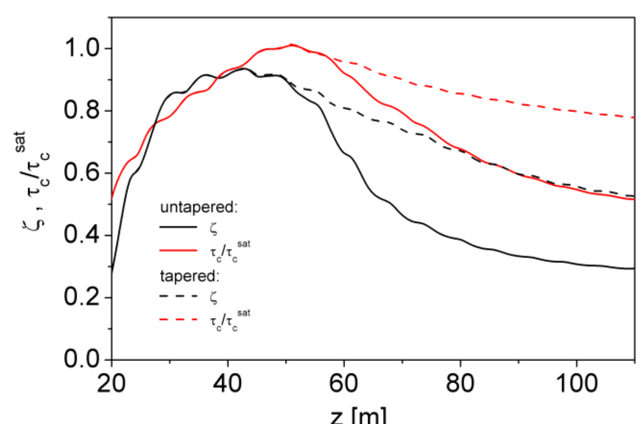
$$\zeta = \frac{\int |\gamma_1(\vec{r}_\perp, \vec{r'}_\perp)|^2 I(\vec{r}_\perp) I(\vec{r'}_\perp)\,\mathrm{d}\,\vec{r}_\perp\,\mathrm{d}\,\vec{r'}_\perp}{[\int I(\vec{r}_\perp)\,\mathrm{d}\,\vec{r}_\perp]^2} ,$$

- Temporal corellations and coherence time:

$$g_1(\vec{r}, t-t') = \frac{\langle \tilde{E}(\vec{r}, t)\tilde{E}^*(\vec{r}, t')\rangle}{\left[\langle|\ \tilde{E}(\vec{r}, t)\ |^2\rangle\langle|\ \tilde{E}(\vec{r}, t')\ |^2\rangle\right]^{1/2}} , \qquad \tau_c = \int\limits_{-\infty}^{\infty} |g_1(\tau)|^2\,\mathrm{d}\,\tau .$$

Degree of transverse coherence

Coherence time



Summary

- We derived the general law for optimum undulator tapering in the presence of diffraction effects. It is simple analytical expression with two fitting coefficients
- Key elements are knowledge of the radiation properties of modulated electron beam and application of similarity techniques in FEL theory.
- Investigation of the case of “cold” electron beam allows one to isolate diffraction effects in the most clear form, and the optimum tapering law is the function of the only diffraction parameter B.
- Extension of this approach with including energy spread and emittance effects is straightforward and will result just in corrections to the fitting coefficients without changing the general law as we demonstrated for the case of SASE FEL.

Thank you very much for your attention!