Using Pipe With Corrugated Walls for a Sub-Terahertz FEL

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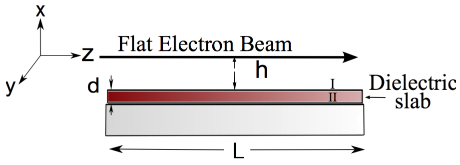
Outline of the talk

- Introduction and motivation
- Resonant mode propagating in pipe with corrugated walls
- Connection between FEL theory and the formalism of longitudinal beam instabilities
- Resonant mode beam instability—analogy with 1D FEL
- Parameter choice for the THz FEL in corrugated pipe
- Discussion

Introduction

Interaction of electron beams with metallic corrugated walls and the radiation has a long history and numerous publications. It goes under the names "Smith-Purcell FEL", "Smith-Purcell Traveling Wave Tube", "Čerenkov FEL"[1].



[1] Y. Kalkal and V. Kumar. PRST-AB **18**, 030707 (2015); H. P. Freund and T. M. Abu-Elfadl, IEEE TRANSACTIONS ON PLASMA SCIENCE, **32**, NO. 3, (2004); V. Kumar and Kwang-Je Kim. PRST-AB **12**, 070703 (2009)

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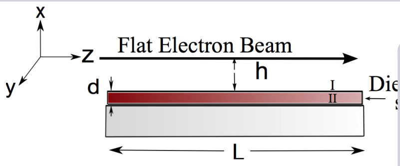

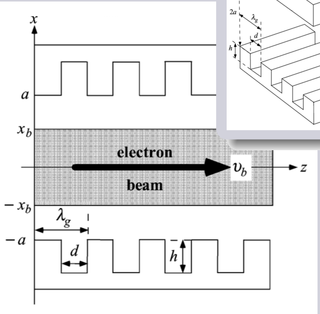

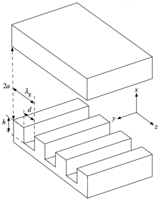

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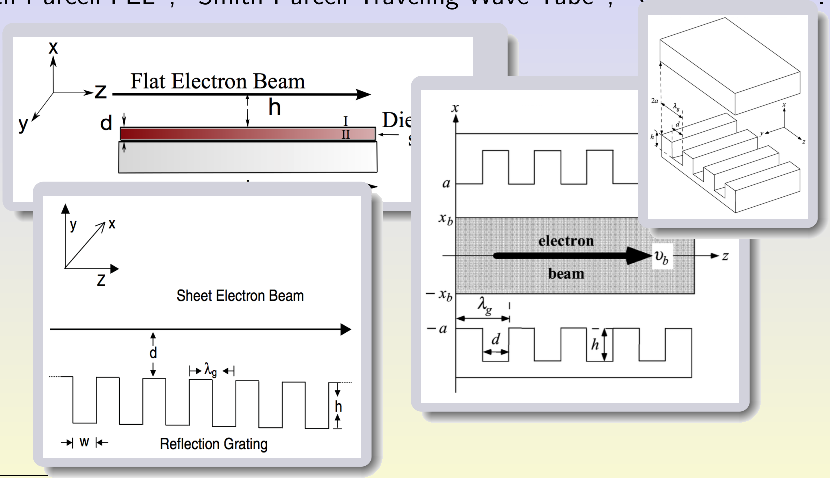

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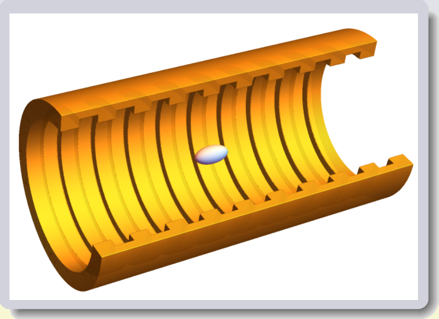
Motivation for the round geometry

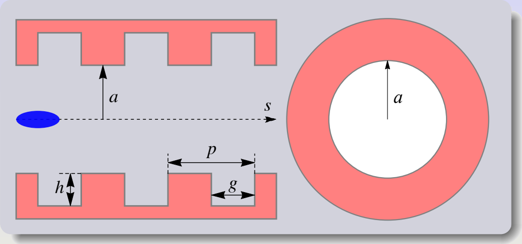
- Coupling to the beam is more efficient because the corrugations are closer.
- Calculations are much simpler.
- This is a continuation of our studies with K. Bane of various applications of wakefield generated in a pipe with corrugated walls (THz radiation by a short bunch, dechirper[2]).



[2] K. Bane and G. Stupakov, NIMA **677**, 67 (2012); K. Bane and G. Stupakov, NIMA **690**, 106 (2012).

Resonant mode in a pipe with corrugated walls

Geometry: a long round metallic pipe with inner radius a and small rectangular wall corrugations that have depth h, period p and gap g.



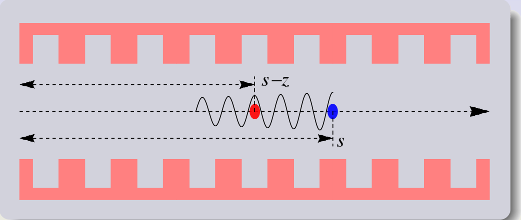
When $h, p \ll a$ and $h \gtrsim p$, the fundamental resonant mode[3] with the phase velocity equal to the speed of light has the frequency $\omega_0 = ck_0$ and the group velocity v_g

$$k_0 = \left(\frac{2p}{agh}\right)^{1/2} \gg \frac{1}{a}, \qquad 1 - \frac{v_{g0}}{c} = \frac{4gh}{ap} \ll 1$$

[3] A. Mosnier and A. Novokhatskii. PAC 1997, p. 1661; K. L. F. Bane and A. Novokhatskii, SLAC-AP-117, 1999.

Resonant mode: dynamic wakefield

A relativistic point charge entering the pipe at $s = 0$ and moving with $v \approx c$ excites the resonant mode and generates a longitudinal wakefield. This wake is localized behind the charge, $w(s, z)$. The s dependence makes this wakefield *dynamic*.



The variable z is equal to the distance between the test and the source charges, measured in the direction of motion.

When the source charge reaches position s, the longitudinal wake at distance z is given by

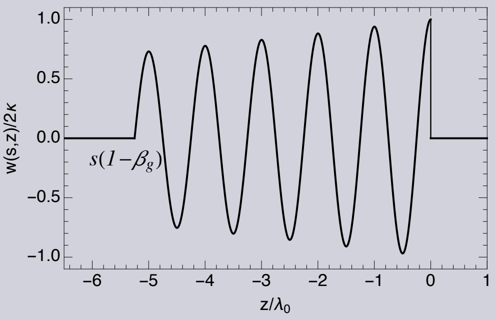
$$w(s,z) = \begin{cases} 2\varkappa_0 e^{-\mu z}\cos(k_0 z), & \text{for } -s(c-v_g)/c < z < 0 \\ 0, & \text{otherwize} \end{cases},$$

where $\varkappa$ is the loss factor

$$\varkappa_0 = 2/a^2$$

and μ describes the the damping of the mode due to the wall resistivity.

Resonant mode wakefield



We assume that the damping is weak, $\mu \ll k_0$. The sign of the wake is such that the positive wake corresponds to the energy loss, and negative wake means the energy gain. The wake extension (along z) grows with distance s (cooperation length).

Numerical example: $a = 2$ mm, $h = 50$ μm, $p = 40$ μm, $g = 20$ μm gives $\omega_0/2\pi \approx 0.3$ THz, $\varkappa = 4.5$ kV/(m·pC).

Digression: connection between formalism in FEL theory and longitudinal beam instabilities

SASE FEL is an instability that develops due to the particles' interaction via electromagnetic field in the undulator. Beam instabilities caused by the longitudinal wakefield $w(z)$ has a similar nature. Their description however uses different formalisms.

1-D FEL equations[4]:

$$\frac{\partial \tilde{F}_1}{\partial z} + i\,2k_u\eta\tilde{F}_1 - \frac{e}{m_e c^2\gamma_r}\frac{dF_0}{d\eta}\left[\frac{\widehat{K}}{2\gamma_r}\tilde{E}_x + \tilde{E}_z\right] = 0 \qquad \frac{d\tilde{E}_x}{dz} = -\frac{\mu_0 c\widehat{K}}{4\gamma_r}\tilde{j}_1 = \frac{\mu_0 c^2\widehat{K} n_e e}{4\gamma_r}\int_{-\delta}^{\delta}\tilde{F}_1(\eta, z)d\eta$$

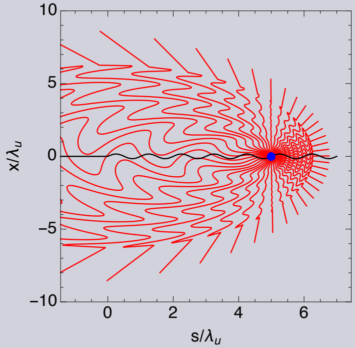
Equations for longitudinal instability of a coasting beam:

$$\frac{\partial F_1}{\partial z} - \alpha\eta\frac{\partial F_1}{\partial s} + K(z,s)\frac{dF_0}{d\eta} = 0 \qquad K(s,z) = -\frac{r_e}{\gamma}\int_{-\infty}^{\infty} ds' d\eta F_1(z,s',\eta)w(s'-s)$$

[4] P. Schmüser et al. "Free-Electron Lasers in the Ultraviolet and X-Ray Regime", Springer, 2013.

Digression: 1D FEL equations are derivable from *dynamic* wake potential

One can derive 1D FEL equations using the dynamic longitudinal wake[5].



Field lines of the electric field of radiation in undulator. The field propagates ahead of the particle, and generates dynamic wakefield $w(s, z)$. For 1D (helical) undulator:

$$w(s, z) = 4\pi \frac{K^2}{1 + K^2} \cos\left(\frac{\omega_0 z}{v_z}\right)$$

for $0 \le z < (1 - \beta_z)s$; here $\omega_0 = \omega_r$ is the undulator radiation frequency.

One has to take into account retardation effects associated with the emission of the wake field.

[5] G. Stupakov and S. Krinsky, in Proceedings of the 2003 Particle Accelerator Conference (Portland, Oregon USA, 2003) p. 3225.

1D Vlasov equations with retardation

The distribution function $f(\eta, z, s)$ is a function of the relative energy deviation, $\eta = \Delta\gamma/\gamma$, longitudinal position inside the bunch z, and the distance s from the entrance to the pipe.
The Vlasov equation for f is

$$\frac{\partial f}{\partial s} - \alpha\eta\frac{\partial f}{\partial z} - \frac{r_0}{\gamma}\frac{\partial f}{\partial \eta}\int_{-\infty}^{\infty} dz' \\ \times \int_{-\infty}^{\infty} d\eta' w(s, z - z') f\left(\eta', z', s - c\frac{z' - z}{c - v_g}\right) = 0$$

where $\alpha = -\gamma_z^{-2}$ is the slip factor per unit length and $r_0 = e^2/mc^2$. The third argument of f in the integrand takes into account the retardation.

Retardation effects are typically ignored in classical instabilities, and the wake is treated as a function of z only, $w(z)$. The dynamic nature of the wake is sometimes refered to as a catch-up distance.

Effective λ_w in pipe with corrugations

To establish a closer analogy with the standard FEL theory, it is convenient to introduce a parameter k_w (an analog of the FEL undulator wave number $2\pi/\lambda_w$). In undulator-based FELs we have

$$\frac{k_0}{k_w} = \frac{c}{c - \langle v_z \rangle}$$

In an undulator the particles are moving slower than the wave, $\langle v_z \rangle < c$, and the wave is ahead of the particle.

In the corrugated pipe we define k_w through

$$\frac{k_0}{k_w} = \frac{c}{c - v_g}.$$

Equivalently, $k_w = k_0 \Delta\beta_g$ with $\Delta\beta_g = 1 - v_g/c$. For the parameters of the corrugations $a = 2$ mm, $h = 50$ μm, $p = 40$ μm, $g = 20$ μm we have $\lambda_w \approx 2$ cm.

Dispersion relation for the resonant mode instability

Consider a long bunch (coasting beam) with the equilibrium energy distribution $n_0 F(\eta)$, where n_0 is the number of particles per unit length of the beam. The dispersion relation for the dimensionless propagation constant q (defined so that $f_1 \propto e^{qk_w s}$) as a function of the frequency detuning $\nu = (\omega - \omega_0)/\omega_0$, is

$$\frac{1}{2}\frac{(2\rho)^3}{q + \mu/k_0 - i\nu}\int_{-\infty}^{\infty} d\eta \frac{F'(\eta)}{q - i\alpha\eta(k_0/k_w)} = 1$$

where the parameter ρ (an analog of the Pierce parameter) is

$$(2\rho)^3 = \frac{2n_0\varkappa_0 c r_0}{k_w\gamma\omega_0}$$

Gain length

Consider a cold beam, $F(\eta) = \delta(\eta)$ and zero detuning, $\nu = 0$, and neglect the damping μ. Use $\alpha = -1/\gamma^2$,

$$q^3 = i\frac{n_0 \varkappa_0 r_0}{k_w^2 \gamma^3}$$

Introducing the power gain length $L_g = (2\mathrm{Re}\, q k_w)^{-1}$, and using the Alfven current $I_A = 17.5$ kA

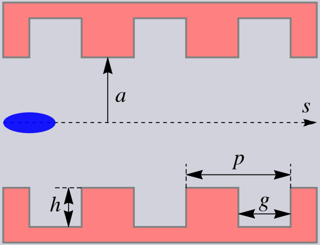
$$L_{g0} = \frac{a}{2\sqrt{3}} \gamma \left(\frac{I_A}{I}\right)^{1/3} \left(\frac{ap}{2gh}\right)^{1/6}.$$

The gain length decreases with growth of γ. The most interesting case is a mildly relativistic beam.

Numerical example

Consider the pipe [6] and beam parameters:

Pipe radius, mm	2
Depth h, μm	50
Period p, μm	40
Gap g, μm	20
Bunch charge, nC	0.5
Bunch length , ps	25
Beam energy, MeV	5



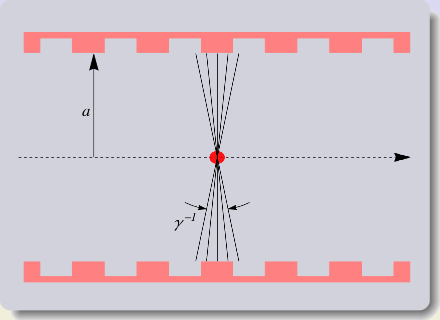
The beam current is $I_b = 20$ A. The frequency $\omega_0/2\pi$ corresponding to these parameter is 0.3 THz, the gain length is $L_g = 9.9$ cm. The THz FEL would saturate in ≈ 2 m.

[6] K. Bane and G. Stupakov, NIMA **677**, 67 (2012).

Effect of moderately large γ

For a beam of energy ~ 5 MeV the approximation $v = c$ is not good and one has to take into account that the phase velocity of the resonant wave is not equal to c.

The spot size of the point charge field on the wall of the pipe has the size on the order of a/γ, and for not very large values of γ can be comparable with the inverse wave number of the wake ω_0/c. The parameter



$$u = \frac{ak_0}{\gamma} = \frac{2\pi a}{\gamma\lambda_0}$$

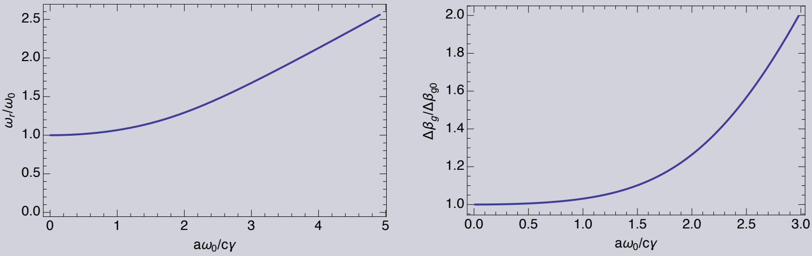
Our previous results are valid in the limit $u \ll 1$ or

$$\gamma \gg \frac{2\pi a}{\lambda_0} \gg 1$$

For the parameters from the table $u = 1.3$.

Frequency of the resonant mode when $u \sim 1$

Calculations for $u \sim 1$ are carried out in[7]. The frequency of the resonant mode increases with $u \propto 1/\gamma$ and the group velocity becomes smaller.



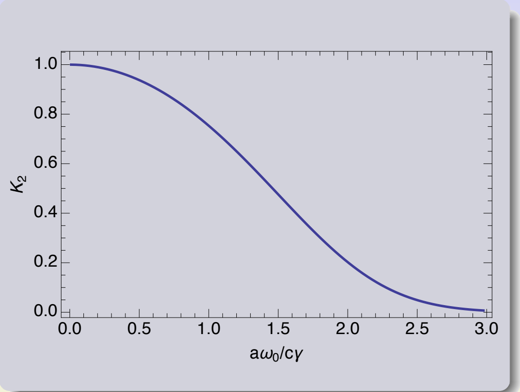
$$\omega_0 = c\left(\frac{2p}{agh}\right)^{1/2}, \qquad K_1 = \frac{\Delta\beta_g}{\Delta\beta_{g0}}$$

$(k_w \propto \Delta\beta_g)$.

[7] G. Stupakov. PRST-AB **18**, 030709 (2015).

The loss factor decreases for less relativistic beams

The loss factor $\varkappa$ decreases with increase of $u \propto 1/\gamma$.



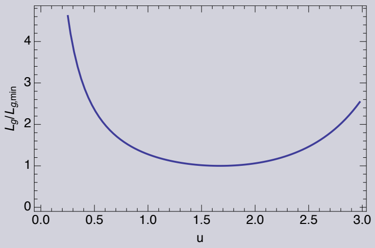
$$K_2 = \frac{\varkappa}{\varkappa_0}$$

Modification of the gain length for moderate γ

With account of the factors K_1 and K_2 the gain length is

$$L_g = (K_1 K_2)^{-1/3} L_{g0}$$

Minimization of the gain length as a function of u (beam energy)



The minimal value is reached at $u = 1.9$ and is equal to 0.74. For the parameter considered above this gives the optimal value of the beam energy: $\gamma = 6.6$ and $L_g = 9.4$ cm. For $\gamma = 10$ the new formula gives $\ell = 11.1$ cm.

Saturated radiation power

Standard FEL formulas can be used for estimation of the saturation power

$$P_{sat} \approx \rho\, \gamma mc^2 \frac{I_b}{e}$$

For our parameters $\rho \approx 0.02$

$$P_{sat} \approx 2.2 \text{ MW}$$

This makes $U \approx 50\ \mu J$ in the pulse.
Note that P_{sat} scales like

$$P_{sat} \propto \gamma^{2/3} I_b^{4/3}$$

Choosing $\gamma = 20$ and $I_b = 100$ A makes $P_{sat} \approx 27$ MW, $U = 0.13$ mJ ($L_g = 11.8$ cm).

Discussion

- Resistive wall dissipation of the resonant mode: the dissipation length is ~ 66 cm, much longer than the gain length.
- The bunch length should be longer than the cooperation length:

$$l_c = 2L_g \Delta\beta_g = 8.7 \text{ mm}$$

  The bunch length 25 ps = 7.5 mm.
- Corrugations can be replaced by a dielectric layer.
- Space charge may play a role, can be suppressed by increasing γ.
- Effect of the resonant transverse wake on the beam.

Can one reach 1 THz? Yes, with even smaller corrugations: $a = 0.5$ mm, $p = 60\ \mu\text{m}$, $h = 20\ \mu\text{m}$, $g = 30\ \mu\text{m} \rightarrow f = 0.95$ THz.

Conclusions

Sub-THz FEL using a small-radius pipe with corrugated wall offers an option of a compact device with multi-MW peak power. It may not require bunch compression but favors a large beam charge and long bunches. It is possible that it can use a thermionic electron gun. More studies are needed with parameters optimization and evaluation of the transverse wakefield effects.