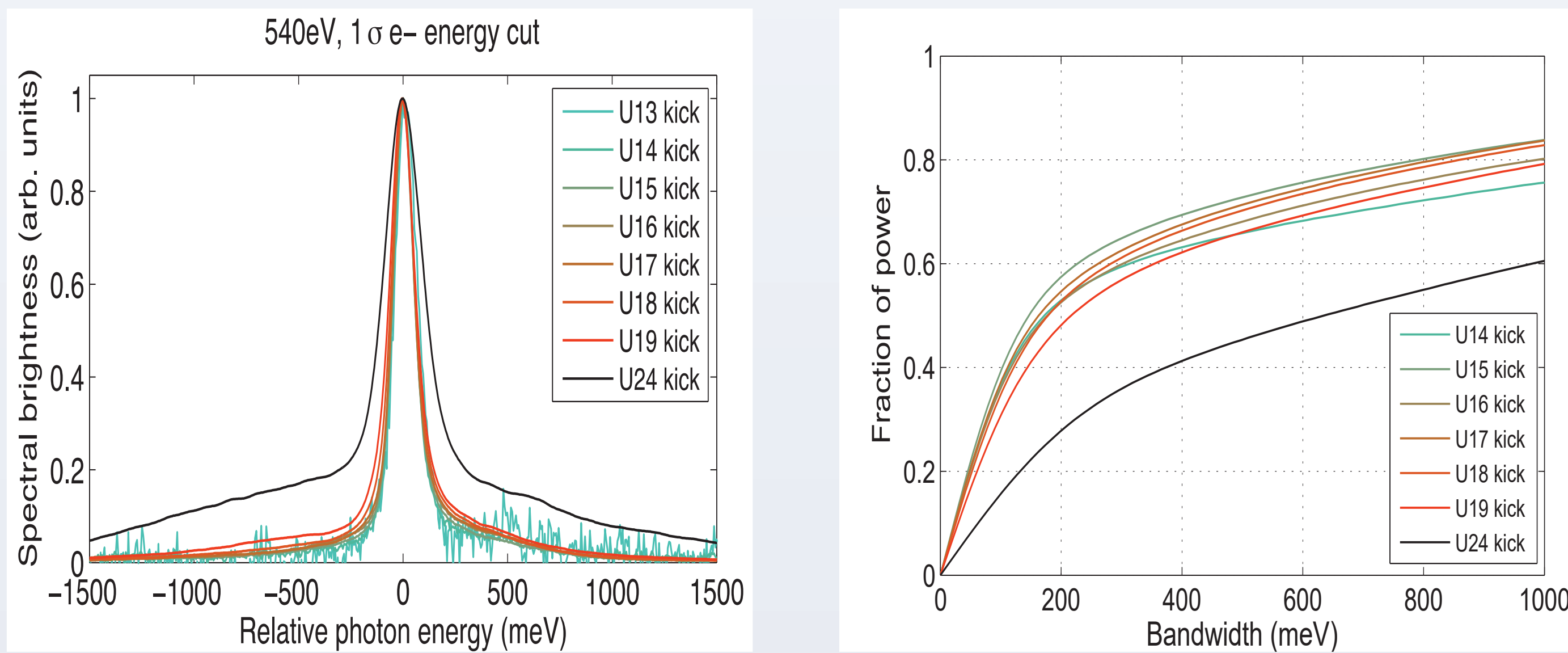


## Introduction

The measured spectrum of the soft X-ray self-seeding at the LCLS has a pedestal-like distribution around the seeded frequency, which limits the spectral purity and seeding applications without a post-undulator monochromator. We study the origins of the pedestals and focus on the contributions of microbunching instability prior to the FEL undulator.



Parameters: 540eV, 180pC cut to 120pC, 1.4kA, 90fs

## Theoretical Analysis

A two-frequency system: the seed  $\omega_1$  and the sideband  $\omega_s$   
Use the dimensionless variables

$$\hat{z} \equiv 2k_u \rho z \quad \hat{\eta} \equiv \frac{\eta}{\rho} \quad \nu \equiv \frac{\omega}{\omega_1}$$

The pendulum equations of the two-frequency system are

$$\begin{aligned} \frac{d\theta}{d\hat{z}} &= \hat{\eta}, \\ \frac{d\hat{\eta}}{d\hat{z}} &= a_1 e^{i\theta} + a_s e^{i\nu\theta} + c.c., \\ \frac{da_1}{d\hat{z}} &= -b_1, \\ \frac{da_s}{d\hat{z}} + i\Delta\nu a_s &= -b_s, \\ a_{1,s} &\equiv \frac{eK[JJ]}{8\gamma_0^2 mc^2 k_u \rho^2} E_{1,s}, \\ b_{1,s} &\equiv \langle e^{-i\nu\theta} \rangle, \\ p_{1,s} &\equiv \langle \hat{\eta} e^{-i\nu\theta} \rangle \end{aligned}$$

### Initial energy modulation

Consider an initial energy modulation

$$A(s) = A_0 \cos(k_s s) = \Delta\gamma/\gamma \cos(k_s s)$$

Initial condition in scaled energy variable

$$\hat{\eta}_0 = \frac{\hat{A}}{2} e^{i\Delta\nu\theta} + c.c. \quad \hat{A} \equiv A_0/\rho$$

With the assumptions that  $|a_s| \ll |a_1|$ ,  $|b_s| \ll |b_1|$  and  $|p_s| \ll |p_1|$

$$\begin{aligned} \frac{d^3 a_1}{d\hat{z}^3} &\approx i a_1, \\ \frac{d^3 a_s}{d\hat{z}^3} + i\Delta\nu \frac{d^2 a_s}{d\hat{z}^2} &\approx i\nu a_s + \nu^2 \hat{A} p_1. \end{aligned}$$

Assume high-gain regime ( $\hat{z} \gg 1$ ) and small detune ( $\Delta\nu < \rho$ )

$$a_s(\hat{z}) = -\frac{i\hat{A}D_3}{3} \hat{z} e^{-i\mu_3 \hat{z}} = -\frac{i\hat{A}}{3} \hat{z} a_1.$$

The power ratio between the sideband and the seed

$$\frac{P_s(\hat{z})}{P_1(\hat{z})} = \frac{\hat{A}^2}{9} \hat{z}^2 = \frac{(2k_u \rho z)^2}{9} \frac{A_0^2}{\rho^2}.$$

\* This work was supported under US Department of Energy contract DE-AC02-76SF00515.

## Theoretical Analysis

Generalized to broadband sidebands

$$\frac{P_s(\hat{z})}{P_1(\hat{z})} = \frac{\hat{A}^2}{9} \hat{z}^2 = \frac{(2k_u \rho z)^2}{9} \int_0^{\Delta s} \frac{A(s)^2}{\rho^2} \frac{ds}{\Delta s}.$$

Initial density modulation

The above analysis can be applied to modulations in density

$$\frac{d^3 a_s}{d\hat{z}^3} \approx i\nu a_s + i\nu a_1 b_0, \quad b_0 = \langle e^{-i\Delta\nu\theta} \rangle$$

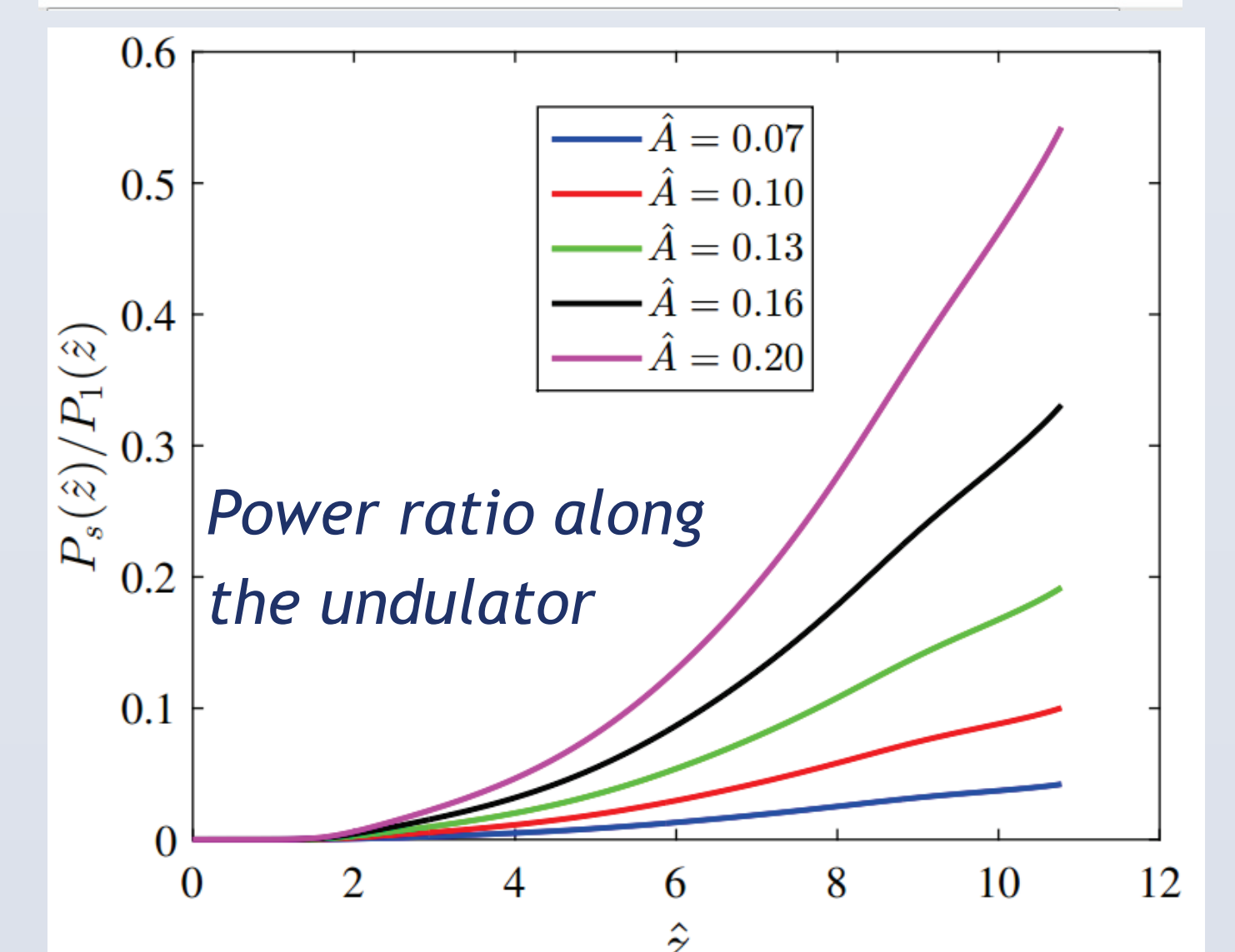
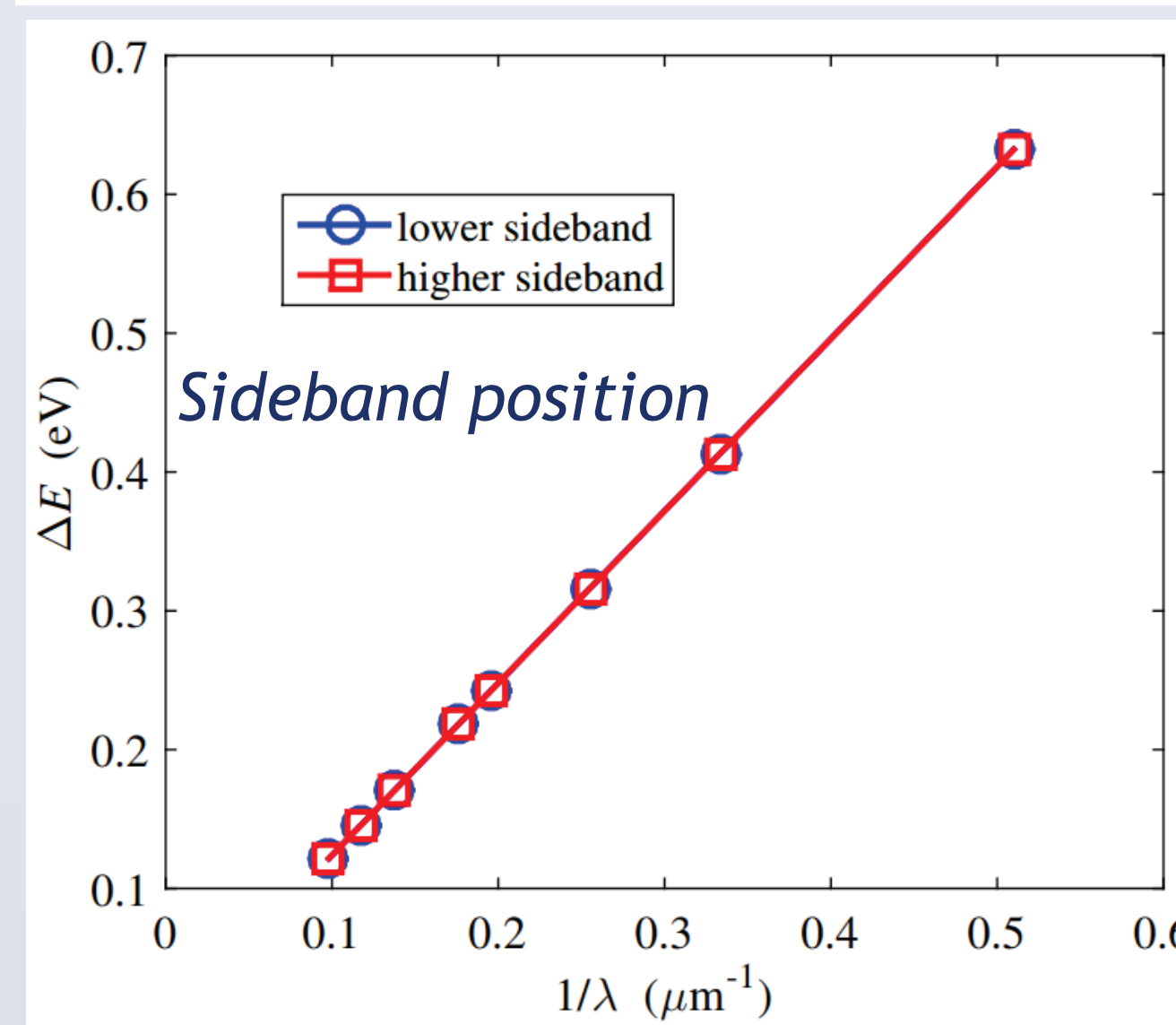
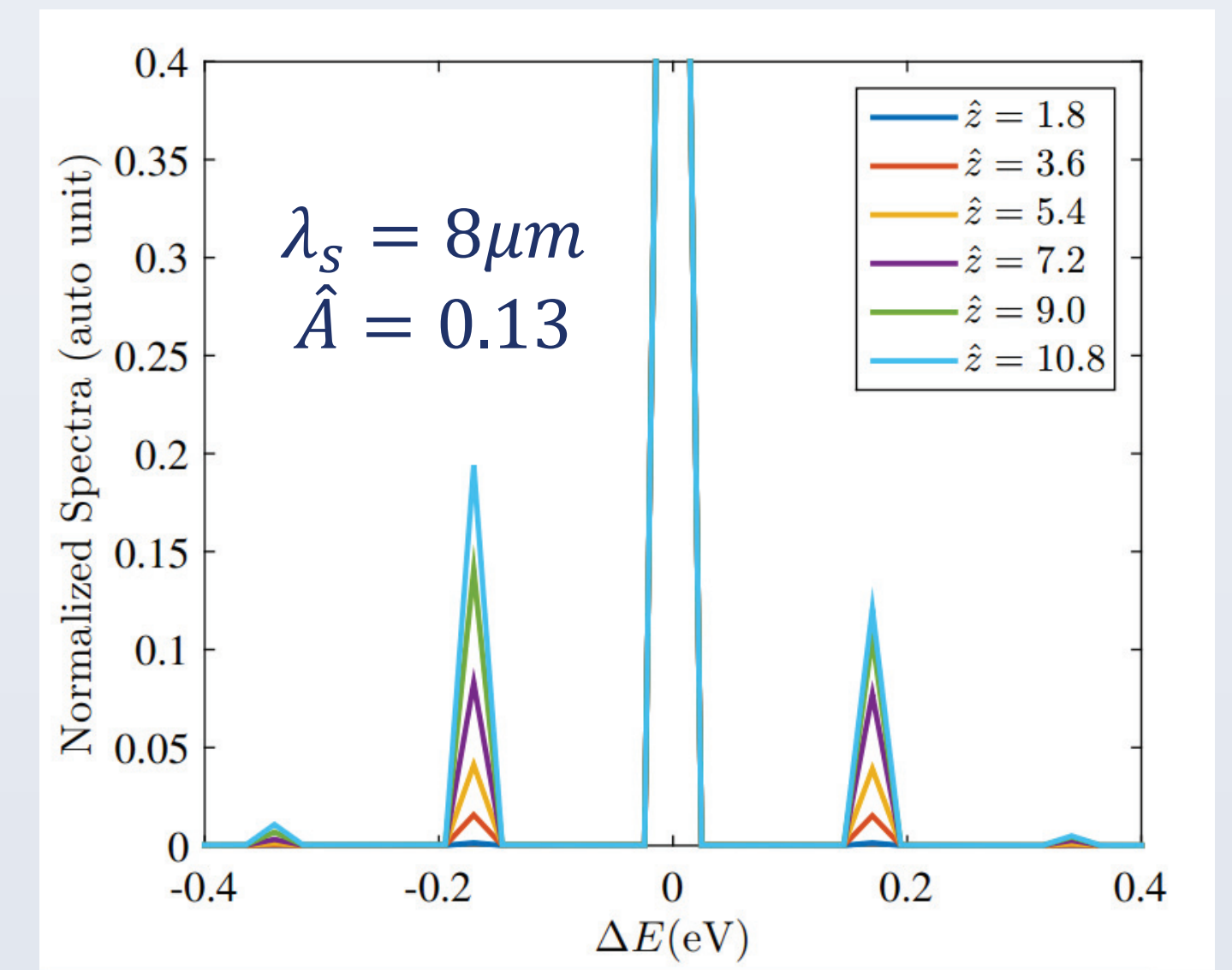
The power ratio between the sideband and the seed

$$\frac{P_s(\hat{z})}{P_1(\hat{z})} = \frac{b_0^2}{9} \hat{z}^2 = \frac{(2k_u \rho z)^2}{9} b_0^2$$

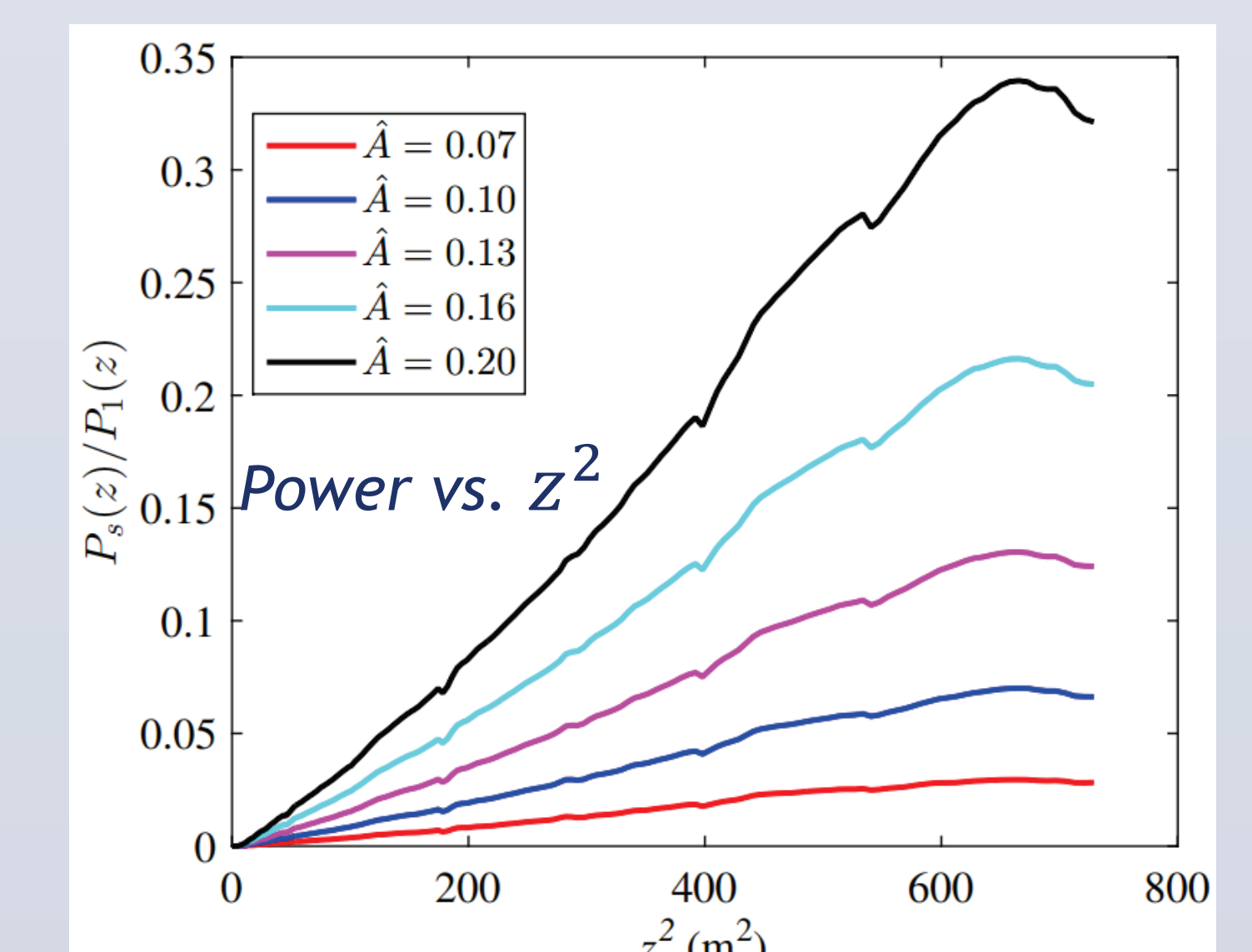
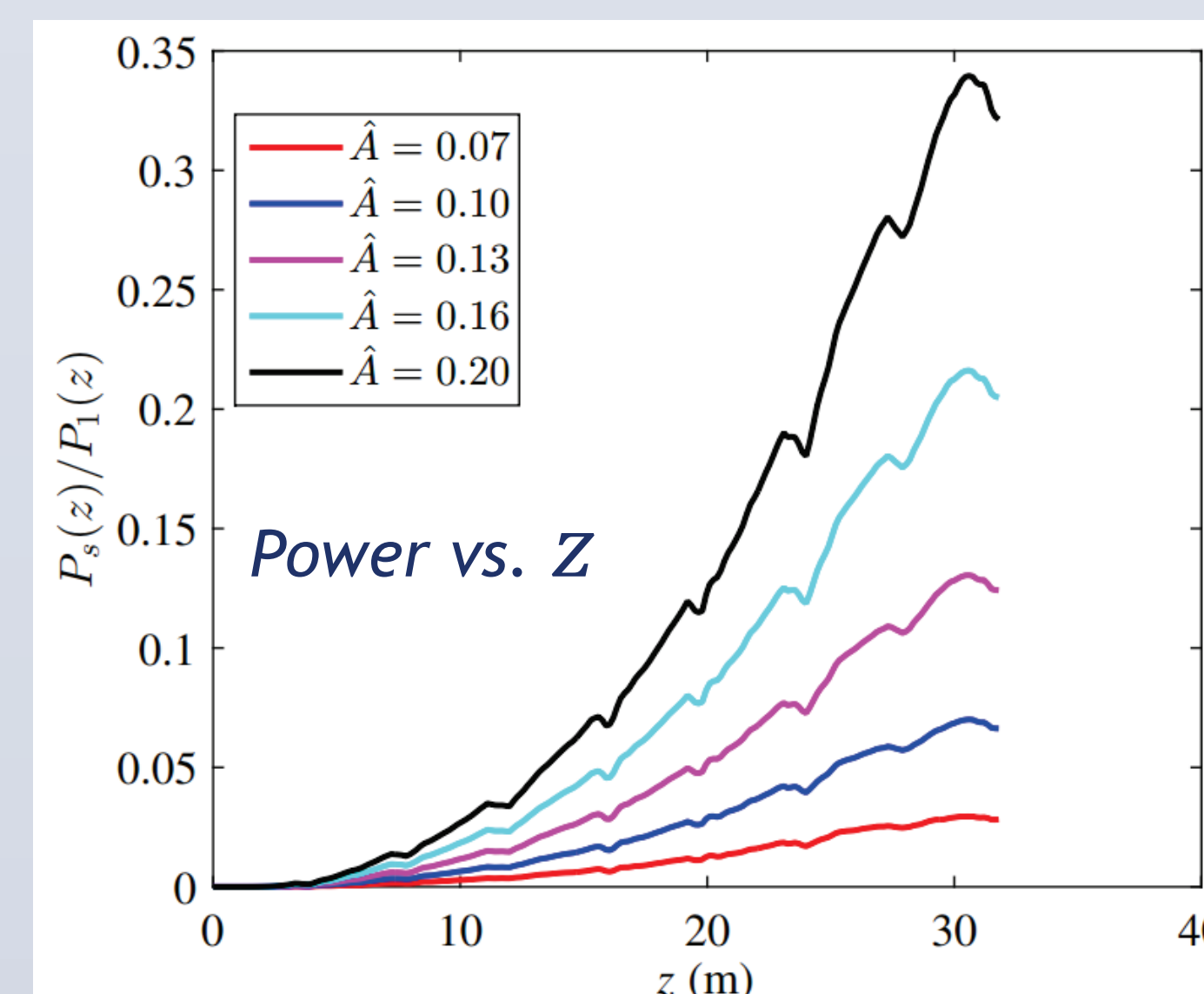
## 1D Simulation

Table 1: Simulation parameters in 1-D FEL code

Parameter	Value	Unit
Beam energy $E$	3.48	GeV
Slice energy spread	0	MeV
Normalized emittance $\epsilon_N$	0.9	$\mu\text{m}$
Current $I$	1.4	kA
Average $\beta$	30	m
Undulator period $\lambda_u$	3	cm
FEL parameter $\rho$	$8.6 \times 10^{-4}$	
Gain length $L_G$	1.6	m
Seeding wavelength $\lambda_r$	2.29	nm
Seeding power	20	kW
Energy modulation wavelength	2-10	$\mu\text{m}$
Energy modulation amplitude	0.1-0.6	MeV



## Genesis Simulation



## Summary

- Both energy and density modulation can induce sidebands in a seeded FEL.
- A simple 1D theory is developed to estimate the sideband content and agrees well with simulations.
- The power ratio of the sidebands to the seeded signal grows quadratically with the modulation amplitude and undulator length before FEL saturation.
- Further work includes detailed comparison with the experimental observations and developing methods to produce a more uniform electron beam in the longitudinal phase space.