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# Multistage CSR Microbunching Gain Development in Transport or Recirculation Arcs

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## ABSTRACT

Coherent synchrotron radiation (CSR) induced microbunching instability has been one of the most challenging issues in the design of modern accelerators. A linear Vlasov solver has been developed [1] and applied to investigate the physical processes of microbunching gain amplification for several example lattices [2]. In this paper, by further extending the concept of stage gain as proposed by Huang and Kim [3], we develop a method to characterize the microbunching development in terms of stage orders that allow the quantitative comparison of optics impacts on microbunching gain for different lattices. We find that the microbunching instability in our demonstrated arcs has a distinguishing feature of multistage amplification (e.g, up to 6th stage amplification for our example transport arcs, in contrast to two-stage amplification for a typical 4-dipole bunch compressor chicane). We also try to connect lattice optics pattern with the obtained stage gain functions by a physical interpretation. This Vlasov analysis is validated by ELEGANT [4] tracking results with excellent agreement.

## OVERVIEW OF MBI IN A SINGLE-PASS SYSTEM

➢ **Linear Vlasov solver** [3, 5]

$$g_k(s) = g_k^{(0)}(s) + \int_0^s K(s,s')g_k(s')ds' \quad \text{Eq. (1)}$$

$$K(s,s') = \frac{ik}{\gamma}\frac{I(s)}{I_A}C(s')R_{56}(s'\to s)Z_{CSR}^{s.s.}\left(kC(s'),s'\right)\times[\text{Landau damping}]$$

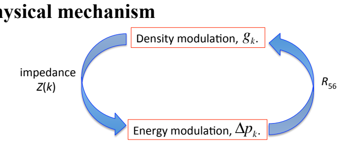
$$Z_{CSR}^{ss}\left(k(s);s\right) = \frac{-ik(s)^{1/3}A}{|\rho(s)|^{2/3}},\ A \approx -0.94+1.63i$$

$$[\text{Landau damping}] = \exp\left\{\frac{-k^2}{2}\left[\varepsilon_{x0}\left(\beta_{x0}R_{51}^2(s,s') + \frac{R_{52}^2(s,s')}{\beta_{x0}}\right) + \sigma_\delta^2 R_{56}^2(s,s')\right]\right\}$$

$$R_{56}(s'\to s) = R_{56}(s) - R_{56}(s') + R_{51}(s')R_{52}(s) - R_{51}(s)R_{52}(s')$$

$$R_{5i}(s,s') = C(s)R_{5i}(s) - C(s')R_{5i}(s')$$

➢ **Physical mechanism**




## DIRECT SOLUTION

The above Volterra integral equation Eq. (1) can be expressed in a vector-matrix form as

$$\mathbf{b}_k = (\mathbf{I}-\mathbf{K})^{-1}\mathbf{b}_k^{(0)}$$

provided the inverse matrix exists.

Define the microbunching gain as the ratio of bunching factor at a certain location to that at the initial location,

$$G(s,k=2\pi/\lambda) \equiv \left|\frac{b_k(s)}{b_k^{(0)}(0)}\right| \quad \text{(gain function)}$$

$$G_f(\lambda) \equiv G(s=s_f) \quad \text{(gain spectrum)}$$

## ITERATIVE SOLUTION

Another approach to solve the integral equation is resorted to iterative method. The 0th order solution is simply the bunching factor in the absence of collective effect, $\mathbf{b}_k^{(0)}$. The first few-order solutions can be obtained

$$\mathbf{b}_k^{(1)} = (\mathbf{I}+\mathbf{K})\mathbf{b}_k^{(0)}$$

$$\mathbf{b}_k^{(2)} = (\mathbf{I}+\mathbf{K}+\mathbf{K}^2)\mathbf{b}_k^{(0)}$$

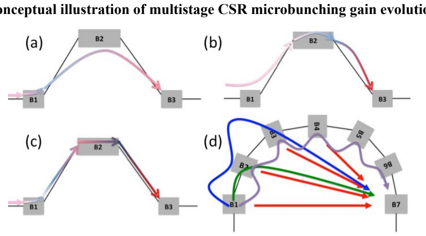
In general, we have n-th order solution

$$\mathbf{b}_k^{(n)} = \left(\sum_{m=0}^{n}\mathbf{K}^m\right)\mathbf{b}_k^{(0)} \quad \text{and} \quad \lim_{n\to\infty}\mathbf{b}_k^{(n)} = \mathbf{b}_k \text{ provided the sum converges.}$$

We define the stage gain function as

$$\tilde{G}^{(n)}(s,k=2\pi/\lambda) = \frac{\mathbf{b}_k^{(n)}(s)}{\mathbf{b}_k^{(0)}(0)}, \quad \text{and} \quad G^{(n)}(s,k) = \left|\tilde{G}^{(n)}(s,k)\right| \quad \text{Eq. (2)}$$

**Conceptual illustration of multistage CSR microbunching gain evolution.**




## STAGE GAIN ANALYSIS

**Goal: (i) extract individual stage gains from the overall CSR gain;**
**(ii) quantify the CSR gains by separating the contribution of beam parameters from the lattice properties.**

Expand Eq. (2) in a series of polynomials of the beam current, $\tilde{G}_f^{(M)} = \tilde{G}^{(M)}(s=s_f) = \tilde{G}_0 + \tilde{G}_1 I_b + ... + \tilde{G}_M I_b^M = \sum_{m=0}^{M}\tilde{G}_m I_b^m$
By inspecting the kernel function, we can further express the stage gain to be

$$\tilde{G}_f^{(M)} = \sum_{m=0}^{M} A^m d_m^{(\lambda)}\left(\frac{I_b}{\gamma I_A}\right)^m \quad \text{Eq. (3)} \quad \text{and} \quad \mathcal{G}_f^{(m)} = \left|A^m d_m^{(\lambda)}\left(\frac{I_b}{\gamma I_A}\right)^m\right| \quad \text{(individual stage gain)} \quad \text{Eq. (4)}$$

Note: (i) once $d_m^{(\lambda)}$ is obtained, it can be used to quantify stage gain amplification (see **Figure 3** and **Figure 4**).
(ii) $d_m^{(\lambda)}$ can also be used to quickly estimate current-dependence of CSR gain (**Figure 5**).

## RESULTS OF SEMI-ANALYTICAL VLASOV SOLVER

➢ **Example: 1.3 GeV High-energy Recirculation Arcs** [8]

| Name | Example 1 (large $R_{56}$) | Example 2 (small $R_{56}$) | Unit |
|---|---|---|---|
| Beam energy | 1.3 | 1.3 | GeV |
| Bunch current | 65.5 | 65.5 | A |
| Norm. emittance | 0.3 | 0.3 | μm |
| $\beta_{x0}$ | 35.81 | 65.0 | m |
| $\alpha_{x0}$ | 0 | 0 | |
| Slice energy spread | $1.23 \times 10^{-5}$ | $1.23 \times 10^{-5}$ | |

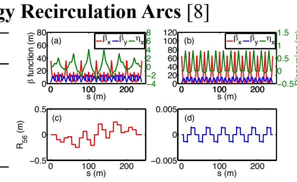



**Figure 3**: stage gain coefficient for Example 1

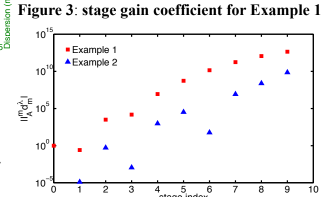
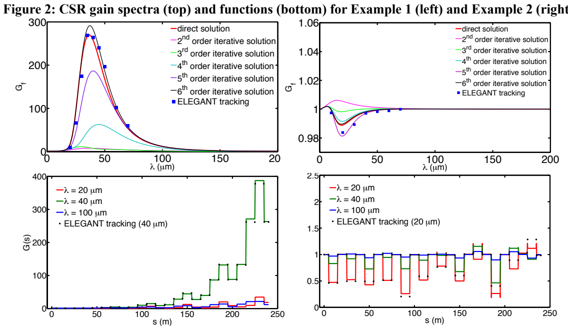



**Figure 2: CSR gain spectra (top) and functions (bottom) for Example 1 (left) and Example 2 (right).**



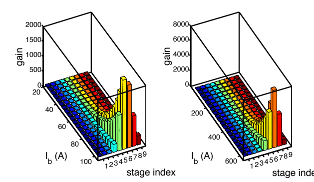




**Figure 4**: bar chart representation for Example 1 (left) and Example 2 (right) lattices, by Eq. (4).

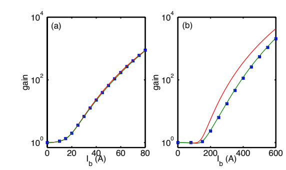



**Figure 5**: maximal CSR gain for Example 1 (left) and Example 2 (right) lattices, by using Eq. (3) and by direct solution [blue dot, Eq. (1)]. Red curve: $M = 6$, green curve: $M = 9$.

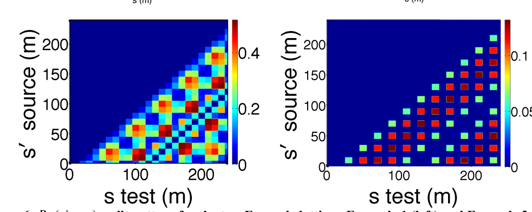



**Figure 6:** $R_{56}(s'\to s)$ **quilt pattern for the two Example lattices. Example 1 (left) and Example 2 (right).**

## SUMMARY

- ➢ Two types of linear Vlasov solutions: direct & iterative solutions
- ➢ Direct solutions ➔ more accurate results
- ➢ Iterative solutions ➔ more insightful pictures:
  - ✓ explore multistage CSR amplification
  - ✓ ≤ 2-stage for a typical bunch compressor chicane [3]
  - ✓ potentially higher stages for a transport or recirculation arc
  - ✓ clarified in both mathematical and physical aspects
- ➢ Derived stage gain coefficients $d_m^{(\lambda)}$ can be applied to
  - ✓ make (quick) estimation of maximal CSR microbunching gain
  - ✓ compare the lattice optics impact on CSR microbunching gain

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