SMITH-PURCELL RADIATION FROM MICROBUNCHED BEAMS MODULATED AFTER PASSING THE UNDULATORS IN FELS*

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Abstract

We suggest using the Smith-Purcell effect from microbunched beams modulated after passing the undulators in FELs as an extra source of monochromatic radiation. We investigate theoretically characteristics of Smith-Purcell radiation in THz and X-ray frequency regions for two types of distribution of the particles in the beam. The expression for spectral-angular distribution of such radiation is obtained and analyzed, both for fully and partially modulated beams. The intensity of Smith-Purcell radiation is shown to be able to increase both due to the periodicity of the beam and the periodicity of the target. The numerical results prove that such radiation source can be an effective instrument for different FEL users, supplementary for the main FEL source.

INTRODUCTION

Smith-Purcell radiation (SPR) is a promising scheme for creating the intense source of radiation. SPR is convenient for beam diagnostics because of large emission angles. The intensity of SPR is proportional to the squared number of strips in the periodic target (grating). Besides the intensity of radiation can be increased if it is generated by the beam having periodic inner structure. Such microbunched beams can be obtained in FEL, in the process of the beam modulation in undulator. Therefore, the beam after passing the undulator in FEL can generate intensive radiation from the grating before passing to a dump (see scheme in Fig. 1). Changing an emission angle it is possible to produce quasimonochromatic radiation with different wavelengths in a broad range in comparison with the modulation period.

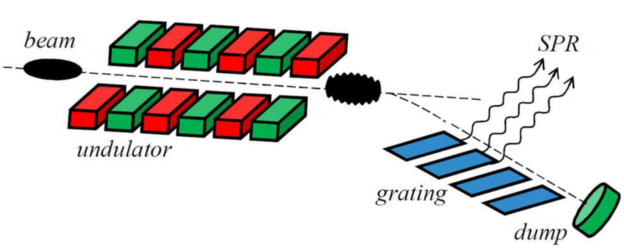

Figure 1: Scheme of using the microbunched beam modulated after passing the undulator for generating intensive Smith-Purcell radiation.

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We theoretically investigate SPR generated by the microbunched beam of relativistic electrons. The beam is assumed to have periodic internal structure with the period λ_0. The number of the particles with the charge e is N, the number of microbunches is N_b. The beam moves at a constant distance h above the grating surface with the constant velocity $\mathbf{v} = (v, 0, 0)$. The period of the grating is d, the single strip width is a, N_{st} is the number of the strips in the grating. The qualitative scheme is shown in Fig. 2.

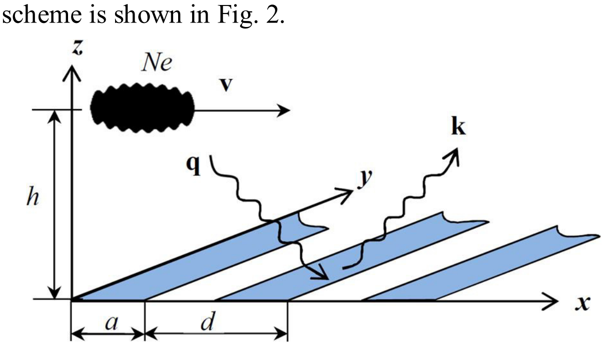

Figure 2: Qualitative scheme of generating the Smith-Purcell radiation.

MODULATED BEAM

The distribution of the particles in the modulated beam can be described by two ways. We will mark the values obtained for these distributions by f and g as the superscripts.

The first one is convenient to describe the beam which has a lot of microbunches with rather short delay between them. In this case the inner structure of each microbunch is negligible. Such kind of beams is produced, for example, in FELs like FLASH, Germany. The longitudinal profile of beam modulated in the undulator in this case can be described by the function

$$f_{long}(x) = \frac{2}{\sqrt{\pi}\sigma_x} \frac{\exp\left[-x^2/\sigma_x^2\right]\left(\mu + \sin^2(\pi x/\lambda_0)\right)}{1 + 2\mu - \exp\left[-\pi^2\sigma_x^2/\lambda_0^2\right]}, \quad (1)$$

with σ_x being the character size of the bunch in x direction; μ defining the "depth" of the modulation: if $\mu = 0$ then the beam is fully modulated, if $\mu \to \infty$ then the beam has the Gaussian form; λ_0 being the period of the modulation. The function in Eq. (1) is shown in Fig.3.

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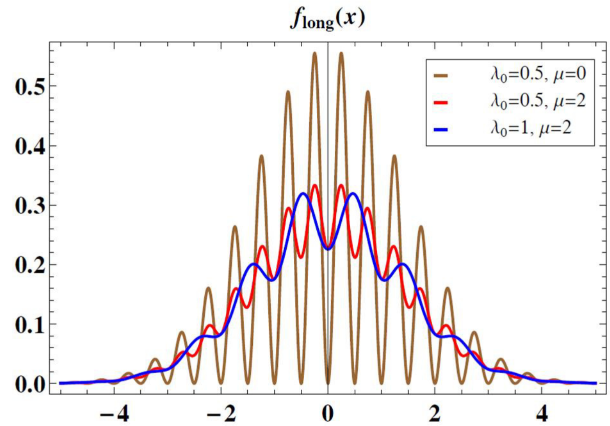

Figure 3: The longitudinal profile of the beam described by Eq. (1) for different parameters λ_0 and μ. For all curves $\sigma_x = 2$. The values x, λ_0, σ_x have the dimension of length and are measured in identical units.

The second distribution is convenient to describe the beam with not small delay between microbunches, or large size of microbunch. In this case the inner structure of each microbunch is taken into account. Such kind of the beam is produced, for example, in LUCX, Japan. There are about ten microbunches in the "train"[1]. The longitudinal distribution can be written as:

$$g_{long}(x) = \frac{1}{N_b\sqrt{\pi}\sigma'_x}\sum_{s=0}^{N_b-1}\exp\left[-(x-s\lambda_0)^2/\sigma'^2_x\right]. \qquad (2)$$

Here σ'_x is the dispersion of a single microbunch, unlike σ_x in Eq. (1).

Function in Eq. (2) also can describe the partially modulated beam at $\lambda_0 < 4\sigma'_x$. This function is plotted in Fig. 4.

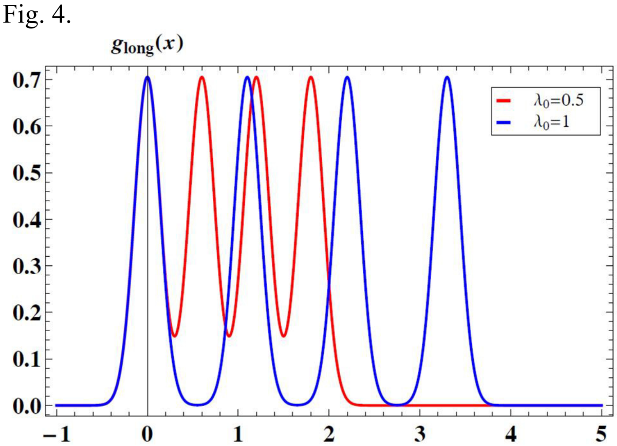

Figure 4: The longitudinal profile of the beam described by Eq. (2) for different parameters λ_0. For all curves $\sigma'_x = 0.2$, $N_b = 4$. The values x, λ_0, σ_x have the dimension of length and are measured in identical units.

The transversal distribution is assumed to be the same both for the $f_{tr}(x)$ and $g_{tr}(x)$:

$$f_{tr}(y,z) = g_{tr}(y,z) = \begin{cases} \dfrac{e^{-y^2/\sigma_y^2}}{\sqrt{\pi}\sigma_y\sigma_z}, & h-\dfrac{\sigma_z}{2} \le z \le h+\dfrac{\sigma_z}{2}, \\ 0, & z < h-\sigma_z/2,\ z > h+\sigma_z/2. \end{cases} \qquad (3)$$

FORM-FACTOR FOR POLARIZATION RADIATION

In general the expression for the spectral-angular distribution of SPR has the following form [2]:

$$\frac{d^2W(\mathbf{n},\omega)}{d\omega d\Omega} = \frac{d^2W_1(\mathbf{n},\omega)}{d\omega d\Omega}GF, \qquad (4)$$

where $d^2W_1(\mathbf{n},\omega)/d\omega d\Omega$ is the spectral-angular distribution of the radiation from a single electron (the center of the bunch) moving above a single strip, G is the factor defining the radiation from the grating, F is the so-called "form-factor" of the bunch, which can be written in the form [2-7]:

$$F = NF_{inc} + N(N-1)F_{coh}. \qquad (5)$$

In Eq. (5) F_{inc} and F_{coh} are incoherent and coherent parts of the form-factor, correspondingly. We have to notice that $F_{inc} \ne 1$ in general case.

The expression for F can be found from qualitative approach in form similar to the form-factor for synchrotron radiation:

$$F_{inc} = \int_V d^3r \left|e^{-i\mathbf{r}_m\mathbf{q}}\right|^2 f(\mathbf{r}), \qquad F_{coh} = \left|\int_V d^3r e^{-i\mathbf{r}_m\mathbf{q}} f(\mathbf{r})\right|^2, \qquad (6)$$

where $f(\mathbf{r})$ is the function of distribution of the particles in the bunch, V is the volume of the bunch, $\mathbf{r}_m = \mathbf{r} - \mathbf{r}_0$, $\mathbf{r}_0$ is the radius-vector of the bunch center. It is important that for considered type of radiation the phase $\mathbf{r}_m\mathbf{q}$ is a complex value. All the values in Eq. (6) have to be written in a laboratory system of coordinates. That is why as opposed to the synchrotron radiation in the problems of diffraction or Smith-Purcell radiation there is a side of the target. The phase $\mathbf{r}_m\mathbf{q}$ for the case of polarization radiation was found in detail in the paper [8] from the conversation laws. For the coordinate system and geometry showing in the Fig. 2 the laws have the form

$$q_y = k_y, \qquad q = k, \qquad (7)$$

and the dispersion relation for the virtual photons:

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$$\omega = \mathbf{q}\mathbf{v}, \qquad (8)$$

where $\mathbf{k} = (k_x, k_y, k_z) = \mathbf{n}\,\omega/c$ is the wave-vector of the radiation in vacuum, $\mathbf{v}$ is the speed of the electron.

Solving the system of Eqs. (7) and (8) with help of equation

$$q = \sqrt{q_x^2 + q_y^2 + q_z^2}, \qquad (9)$$

one can find the value $\mathbf{q}$ in Eq. (6) for the case of polarization radiation from the particle moving parallel to the target surface in form:

$$\mathbf{q} = \left(\frac{\omega}{v}, k_y, -i\sqrt{\frac{\omega^2}{v^2} + k_y^2 - \frac{\omega^2}{c^2}}\right). \qquad (10)$$

Due to the qualitative difference between the natures of polarization types of radiation (diffraction radiation, transition radiation from a target of limited size, Smith-Purcell radiation) and synchrotron radiation, the differences in $\mathbf{q}$ and in the form-factor of the bunch arise.

RADIATION FROM MODULATED BEAM

Let us give the explicit form of expression for the spectral-angular distribution of the radiation for considered geometry.

Integrating in Eq. (6) with use of Eq. (10) for the beam with the function of distribution described in Eqs. (1)-(3), one can easily find the expression of the incoherent form-factor:

$$F_{inc} = F_{inc}^f = F_{inc}^g = \frac{sh(\rho\sigma_z)}{\rho\sigma_z}, \qquad (11)$$

and coherent form-factors for different distributions $f_{long}(x) f_{tr}(y,z)$ and $g_{long}(x) g_{tr}(y,z)$:

$$F_{coh}^f = \frac{sh^2(\rho\sigma_z/2)}{(\rho\sigma_z/2)^2}\exp\left[-\frac{\sigma_y^2 k_y^2}{2} - \frac{\sigma_x^2\xi^2}{2}\right]\times \left(\frac{1 + 2\mu - \exp\left[-\pi^2\sigma_x^2/\lambda_0^2\right] ch\left(\pi\xi\sigma_x^2/\lambda_0\right)}{1 + 2\mu - \exp\left[-\pi^2\sigma_x^2/\lambda_0^2\right]}\right)^2, \qquad (12)$$

and

$$F_{coh}^g = \frac{sh^2(\rho\sigma_z/2)}{(\rho\sigma_z/2)^2}\exp\left[-\frac{\sigma_y^2 k_y^2}{2} - \frac{\sigma_x'^2\xi^2}{2}\right]\times \frac{1}{N_b^2}\frac{\sin^2(N_b\lambda_0\xi/2)}{\sin^2(\lambda_0\xi/2)}, \qquad (13)$$

where we denoted

$$\xi = \omega/v, \quad \rho = \sqrt{\xi^2 + k_y^2 - \omega^2/c^2}. \qquad (14)$$

As a spectral-angular distribution of the radiation from a single electron at the optical and lower frequencies we chose the expression for the radiation from ideal conducting infinitely thin target. The expression was derived in the paper [9] and based on the theory of A.P. Kazantsev and G.I. Surdutovich [10] and adapted to the concerned coordinate system and geometry in[8]:

$$\frac{d^2W_1(\mathbf{n},\omega)}{d\hbar\omega d\Omega} = \frac{1}{137}\frac{\exp[-2\rho h]}{\pi^2\beta^3\rho^2\varphi^2}\left(\frac{\omega}{c}\right)^4\sin^2\left(\frac{a\varphi}{2}\right)\times \left[\frac{\gamma^{-2}(1-n_y^2) + 2n_y^2}{\sqrt{1-n_y^2}}(1-\beta n_x) + \gamma^{-2}\left(\beta(1-n_y^2) - n_x\right)\right]. \qquad (15)$$

For X-ray frequency range we use the theory developed in [8, 11]:

$$\frac{d^2W_1(\mathbf{n},\omega)}{d\hbar\omega d\Omega} = \frac{1}{137}e^{-2\rho h}\left(\frac{\varepsilon(\omega)-1}{2\pi\beta\varphi\rho}\right)^2\frac{\omega^4}{c^4}\sin^2\left(\frac{a\varphi}{2}\right)\times \frac{\left|\left[\mathbf{n}'\times\mathbf{n}'\times\left(\frac{\omega}{\beta c\gamma^2}\mathbf{e}_x + k_y\mathbf{e}_y - i\rho\mathbf{e}_z\right)\right]\right|^2}{\left|\rho - i(\omega/c)\sqrt{\varepsilon(\omega) - 1 + n_z^2}\right|^2}, \qquad (16)$$

where $\varepsilon(\omega)$ is the dielectric permittivity of the target material described by the plasma frequency ω_p as

$$\varepsilon(\omega) = 1 - \omega_p^2/\omega^2, \ \omega >> \omega_p. \qquad (17)$$

The factor G was also derived in the paper [8]:

$$G = \frac{\sin^2(N_{st}d\varphi/2)}{\sin^2(d\varphi/2)}, \qquad (18)$$

where $\varphi = (\beta^{-1} - n_x)\omega/c$, d is a period of the grating, N_{st} is the number of the strips in the grating. This factor gives the well-known dispersion relation of SPR for $N_{st} >> 1$:

$$\lambda m = d(\beta^{-1} - n_x), \ m = 1,2... \qquad (19)$$

As a result, the spectral-angular distribution of the radiation at optical and lower frequencies has the form of Eqs. (4)-(5) with Eqs. (12)-(18).

RADIATION CHARACTERISTICS

For beam parameters of LUCX and FLASH the incoherent radiation is suppressed in comparison with the coherent one. That is why below we shall concentrate only on coherent radiation. As an example, the

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comparison between F_{inc} and F_{coh}^{g} for terahertz frequencies is shown in Fig. 5. It can be seen that $F_{inc} \approx 1$ for wide range of wavelength. If F_{inc} and F_{coh}^{g} are multiplied by $N \sim 10^{10}$ and N^2 correspondingly, then coherent part will be more intensive (in more detail see [4]). In Fig. 5 we denote:

$$\mathbf{n} = \left(\cos\theta\cos\phi, \cos\theta\sin\phi, \sin\theta\right). \quad (20)$$

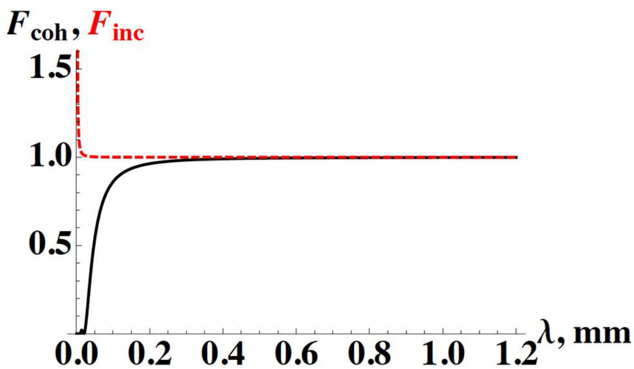

Figure 5: The comparison between F_{inc} (red dashed curve) and F_{coh}^{g} (black solid curve) for the beam of the size $\sigma_x = 3\mu m$, $\sigma_{y,z} = 14\mu m$, $\lambda_0 = 2\mu m$, $\phi = 0$, $\theta = 30^0$, $N_b = 10$, $\gamma = 16$ (energy of LUCX $E_e = 8MeV$).

From Eq. (13) the condition of strong enhancement of radiation intensity follows:

$$\lambda_0 = \beta\lambda s, \; s = 1, 2..., \quad (21)$$

that for ultrarelativistic particles, i.e. for $\gamma >> 1$, $\beta = \sqrt{1-\gamma^{-2}} \approx 1$ is $\lambda_0 \approx \lambda s, s = 1, 2...$. The similar condition can be found from Eq. (12):

$$\lambda_0 = \beta\lambda. \quad (22)$$

The spectral angular distribution of SPR at THz frequencies is shown in Fig. 6, at X-ray ones in Fig. 7.

For observation the most strong enhancement the wavelength of radiation λ should be very close to λ_0. For example, in Fig. 6 black curve is plotted for $\lambda_0 = \beta\lambda \approx 299\mu m$ and the radiation is most intensive; red curve is plotted for $\lambda_0 = 300\mu m$ and the radiation is less intensive. In Fig. 7 for $\lambda_0 = 10.001 nm$ the distributions of the radiation from the modulated beam and from Gaussian distributed beam in Fig. 7 are indistinguishable. If $\lambda_0 = 10.006 nm$, then all curves in Fig. 7 coincide with the blue curve. If $\lambda_0 = \lambda\beta$, then the radiation from modulated beam will be even much more intensive than in Fig. 7.

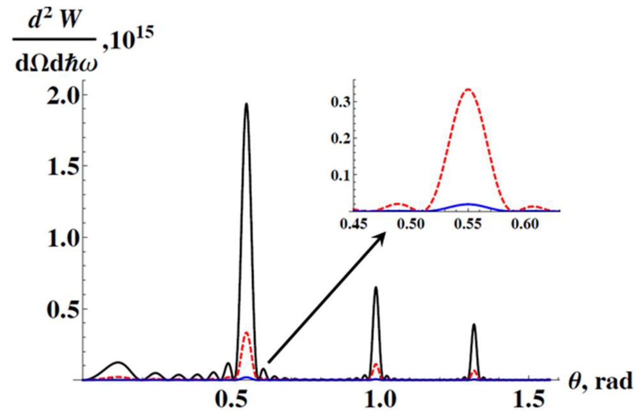

Figure 6: Spectral-angular distribution of SPR at THz frequencies plotted using Eqs. (4), (5), (13), (15), (18). Black solid curve: radiation from fully modulated beam with $\lambda_0 = \beta\lambda \approx 299\mu m$ (see Eq. (21) for $s = 1$), $N = 10^{10}$; red dashed curve: fully modulated beam with $\lambda_0 = 280\mu m$, $N = 10^{10}$; blue curve for the single microbunch ($N = 10^9$). For all curves $\sigma_x = 150\mu m$, $\sigma_{y,z} = 14\mu m$, $\lambda = 300\mu m$, $\phi = 0$, $N_b = 10$, $N_{st} = 7$, $\gamma = 16$ (energy of LUCX $E_e = 8MeV$), $h = 2mm$, $d = 2mm$, $a = 1mm$.

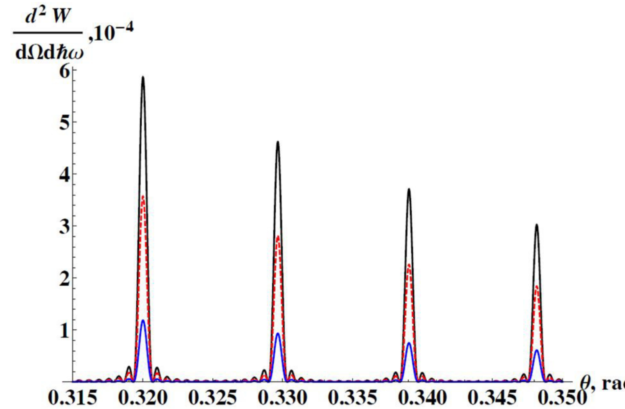

Figure 7: Spectral-angular distribution of SPR at X-ray frequencies plotted using Eqs. (4), (5), (12),(16), (18). Black solid curve: radiation from fully modulated beam $m = 0$; red dashed curve: partially modulated beam $m = 0.2$; blue dotted curve for the single not modulated bunch. For all curves $\sigma_x = 20\mu m$, $\sigma_{y,z} = 10\mu m$, $\lambda = 10nm$, $\phi = 0$, $N_b \approx 2000$, $N_{st} = 7$, $\gamma = 2000$ (energy of FLASH $E_e = 1GeV$), $h = 0.05mm$, $d = 6.5\mu m$, $a = d/2$, $\lambda_0 = 10.005nm$ (see Eq. (22)), $N = 10^{10}$, $\hbar\omega_p = 26.1eV$ (beryllium).

SUMMARY

In this paper we considered Smith-Purcell radiation generated by a microbunched beam modulated after passing the undulator in FELs, both for a conventional FEL, like FLASH, and also for the pre-bunched FEL, like LUCX (KEK).

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We calculated the general analytical expression for the form-factor of the beam for polarization types of radiation that are defined by the edge of the target (diffraction radiation, Smith-Purcell radiation), and this expression differs from the form-factor for synchrotron radiation.

We show that the intensity of such radiation can be increased due to both the periodicity of the target and the periodicity of the beam, and strongly depends on the depth of modulation. So, the beam after undulator in FEL can be used as an effective supplementary source of radiation, in a wide range from THz to X-rays.

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