

CONCEPTUAL THEORY OF SPONTANEOUS AND TAPER-ENHANCED SUPERRADIANCE AND STIMULATED SUPERRADIANCE*

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Abstract

In the current work we outline the fundamental physical concepts of Spontaneous Superradiance (SR), Stimulated Superradiance (ST-SR), Taper-Enhanced Superradiance (TES) and Taper-Enhanced Stimulated Superradiance Amplification (TESSA), and compare their Fourier and Phasor formulations in a model of radiation mode expansion. Detailed further analysis can provide better design concepts of high power FELs and improved tapering strategy for enhancing the power of seeded short wavelength FELs. We further discuss the extensions of the model required for full description of these radiation processes, including diffraction and spectral widening effects.

INTRODUCTION

In the context of radiation emission from an electron beam Dicke's superradiance (SR) [1] is the enhanced radiation emission from a pre-bunched beam. Stimulated Superradiance (ST-SR) is the further enhanced emission of the bunched beam in the presence of a phase-matched radiation wave. These processes were analyzed for Undulator radiation in the framework of radiation field mode-excitation theory [2]. In the nonlinear saturation regime the synchronism of the bunched beam and an injected radiation wave may be sustained by wiggler tapering [3]. Same processes are instrumental also in enhancing the radiative emission in the tapered wiggler section of seeded FEL [4]. Here we outline the fundamental physical concepts of Spontaneous Superradiance (SR), Stimulated Superradiance (ST-SR), Taper-Enhanced Superradiance (TES) and Taper-Enhanced Stimulated Superradiance Amplification (TESSA), and compare their Fourier and Phasor formulations in a model of radiation mode expansion. Detailed further analysis can provide better design concepts of high power FELs and improved tapering strategy for enhancing the power of seeded short wavelength FELs. We further discuss the extensions of the model required for full description of these radiation processes, including diffraction [5] and spectral widening effects.

SUPERRADIANT AND STIMULATED SUPERRADIANCE OF SPONTANEOUS EMISSION

As a starting point we review the theory of superradiant (SR) and stimulated superradiant (ST-SR) emission from

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free electrons in a general radiative emission process. In this section we use a spectral formulation, namely, all fields are given in the frequency domain as Fourier transforms of the real time dependent fields:

$$\check{A}(\underline{r}, \omega) = \int_{-\infty}^{\infty} A(\underline{r}, t) e^{i\omega t} dt \quad (1)$$

We use the radiation modes expansion formulation of [2], where the radiation field is expanded in terms of an orthogonal set of eigenmodes in a waveguide structure or in free space (eg. Hermite-Gaussian modes):

$$\{\check{E}_q(\underline{r}), \check{H}_q(\underline{r})\} = \{\check{E}_q(\underline{r}_\perp), \check{H}_q(\underline{r}_\perp)\} e^{ik_q z} \quad (2)$$

$$\check{E}(\underline{r}, \omega) = \sum_{\pm q} C_q(z, \omega) \check{E}_q(\underline{r}) \quad (3)$$

$$\check{H}(\underline{r}, \omega) = \sum_{\pm q} C_q(z, \omega) \check{H}_q(\underline{r}) \quad (4)$$

The excitation equations of the mode amplitudes is:

$$\frac{d\check{C}_q(z, \omega)}{dz} = \frac{-1}{4P_q} \int d^2 \underline{r}_\perp \check{J}_\perp(\underline{r}, \omega) \cdot \check{E}_q^*(\underline{r}), \quad (5)$$

which is formally integrated and given in terms of the initial mode excitation amplitude and the currents

$$\check{C}_q(z, \omega) - \check{C}_q(0, \omega) = -\frac{1}{4P_q} \int dV \check{J}_\perp(\underline{r}, \omega) \cdot \check{E}_q^*(\underline{r}), \quad (6)$$

where

$$P_q = \frac{1}{2} Re \iint \check{E}_q \times \check{H}_q d^2 r_\perp = \frac{|\check{E}_q(\underline{r}_\perp = 0)|^2}{2Z_q} A_{em}, \quad (7)$$

That defines the mode effective area A_{em} in terms of the field of the mode on axis $\check{E}_q(\underline{r}_\perp) = 0$.

For a particulate current (an electron beam):

$$J(\underline{r}, t) = \sum_{j=1}^N -ev_j(t) \delta(\underline{r} - \underline{r}_j(t)) \quad (8)$$

The field amplitude increment appears as a coherent sum of contributions (energy wavepackets) from all the electrons in the beam: The contributions can be split into a spontaneous part (independent of the presence of radiation field) and stimulated (field dependent) parts:

$$\check{C}_q^{out}(\omega) - \check{C}_q^{in}(\omega) = -\frac{1}{4P_q} \sum_{j=1}^N \Delta \check{W}_{qj} \quad (9)$$

$$\Delta\check{W}_{qj} = -e \int_{-\infty}^{\infty} v_j(t) \cdot \tilde{E}_q^*(r_j(t)) e^{i\omega t} dt \quad (10)$$

$$\Delta\check{W}_{qj} = \Delta\check{W}_{qj}^0 + \Delta\check{W}_{qj}^{st} \quad (11)$$

Assuming a narrow cold beam where all particles follow the same trajectories, the spontaneous emission wavepacket contributions are identical except for a phase factor corresponding to their injection time t_{0j} .

$$\Delta\check{W}_{qj}^0 = \Delta\check{W}_{qe}^0 e^{i\omega t_{0j}} \quad (12)$$

where

$$\Delta\check{W}_{qe}^0 = -e \int_{-\infty}^{\infty} v_e^0(t) \cdot \tilde{E}_q^*(r_e^0(t)) e^{i\omega t} dt. \quad (13)$$

The radiation mode amplitude at the output is composed of a sum of wavepacket contributions including the input field contribution (if any):

$$\begin{aligned} \check{C}_q^{out}(\omega) &= \check{C}_q^{in}(\omega) + \Delta\check{C}_{qe}^0(\omega) \sum_{j=1}^N e^{i\omega t_{0j}} + \sum_{j=1}^N \Delta\check{C}_{qj}^{st} = \\ \check{C}_q^{in}(\omega) - \frac{1}{4P_q} \Delta\check{W}_{qe}^0 \sum_{j=1}^N e^{i\omega t_{0j}} - \frac{1}{4P_q} \Delta\check{W}_{qe}^0 \sum_{j=1}^N \Delta\check{W}_{qj}^{st} \end{aligned} \quad (14)$$

so that the total spectral radiative energy from the electron pulse is

$$\begin{aligned} \frac{d\mathcal{W}_q}{d\omega} &= \frac{2}{\pi} P_q |C_q^{in}(\omega)|^2 = \\ \frac{2}{\pi} P_q &\left\{ |C_q^{in}(\omega)|^2 + \left| \Delta\check{C}_{qe}^{(0)}(\omega) \sum_{j=1}^N e^{i\omega t_{0j}} \right|^2 + \right. \\ &\left[C_q^{in*}(\omega) \Delta\check{C}_{qe}^{(0)}(\omega) \sum_{j=1}^N e^{i\omega t_{0j}} + c.c. \right] + \\ &\left[C_q^{in*}(\omega) \sum_{j=1}^N \Delta\check{C}_{qj}^{st}(\omega) + c.c. \right] \Big\} \equiv \\ \left(\frac{d\mathcal{W}_q}{d\omega} \right)_{in} &+ \left(\frac{d\mathcal{W}_q}{d\omega} \right)_{sp/SR} + \left(\frac{d\mathcal{W}_q}{d\omega} \right)_{ST-SR} + \left(\frac{d\mathcal{W}_q}{d\omega} \right)_{st} \end{aligned} \quad (15)$$

Figures 1(a) and (b) represent the conventional spontaneous emission and superradiance emission that correspond to the second term in Eq. (15) where in 1(a) the wavepackets interfere randomly and in 1(b), in phase. Figure 1(d) represents the third term in Eq. (15) where the coherent constructive interference of a prebunched beam interferes with the input field with some phase offset. Figure 1(c) represents regular stimulated emission from a randomly injected electron beam (regular FEL). When the electrons in the beam are injected at random in a long pulse the second term in Eq. (15) contributes only to conventional shot-noise driven spontaneous emission [2, 6].

$$\left(\frac{d\mathcal{W}_q}{d\omega} \right)_{sp} = \frac{1}{16} |\Delta\check{W}_{qe}^{(0)}|^2 N \quad (16)$$

Only when the electrons are bunched into a pulse shorter than an optical period $\omega(t_{0i} - t_{0j}) \ll \pi$ or are periodically bunched, one gets enhanced superradiant spontaneous emission. Here we focus on periodic bunching, and following the formulation of [2] we write

$$\sum_{j=1}^N e^{i\omega t_{0j}} = \sum_{k=1}^{N_M} \sum_{j=1}^{N_b} e^{i\omega t_{0j}} = N M_b(\omega) M_M(\omega) e^{i\omega t_0}, \quad (17)$$

where

$$M_b(\omega) = \frac{1}{N_b} \left\langle \sum_{j=1}^{N_b} e^{i\omega t_{0j}} \right\rangle, \quad (18)$$

$$M_M(\omega) = \frac{1}{N_M} \left\langle \sum_{k=1}^{N_M} e^{i\omega t_{0k}} \right\rangle, \quad (19)$$

and

$$t_{0k} = t_0 + [k - (N_M/2)] 2\pi/\omega_b \quad (20)$$

where now, neglecting beam noise, we assume identical microbunches of equal number of particles N_b and a uniform train of N_M microbunches (macropulse), such that the total number of particles is $N = N_b N_M$. For normalized Gaussian shaped microbunches

$$f(t) = \frac{1}{\sqrt{\pi} t_b} e^{-t^2/t_b^2} \quad (21)$$

and

$$M_b(\omega) = e^{-\omega^2 t_b^2 / 2}. \quad (22)$$

$M_M(\omega)$ comes out

$$M_M(\omega) = \frac{\sin(N_M \pi \omega / \omega_b)}{N_M \sin(\pi \omega / \omega_b)} \quad (23)$$

and consequently the superradiant spectral energy of the pulse is

$$\left(\frac{d\mathcal{W}_q}{d\omega} \right)_{SR} = \frac{N^2}{8\pi P_q} |\Delta\check{W}_{qe}^{(0)}|^2 |M_b(\omega)|^2 |M_M(\omega)|^2, \quad (24)$$

and the stimulated superradiant at zero order approximation (the third term in Eq. (15) is

$$\left(\frac{d\mathcal{W}_q}{d\omega} \right)_{ST-SR} = \frac{N}{2\pi} |\check{C}_q^{in}(\omega)| |\Delta\check{W}_{qe}^{(0)}|^2 |M_b(\omega)| |M_M(\omega)| \cos\varphi, \quad (25)$$

where φ is the phase between the radiation field and the periodically bunched beam. For Undulator Radiation [2]:

$$\Delta\check{W}_{qj}^0 = -e \frac{v_{\perp 0} \cdot \tilde{E}_q^*}{2v_z} L \text{sinc}(\theta L/2) e^{i\theta L/2} e^{i\omega t_{0j}}, \quad (26)$$

and the detuning parameter $\theta(\omega)$ is

$$\theta(\omega) = \frac{\omega}{v_z} - k_{zq}(\omega) - k_w. \quad (27)$$

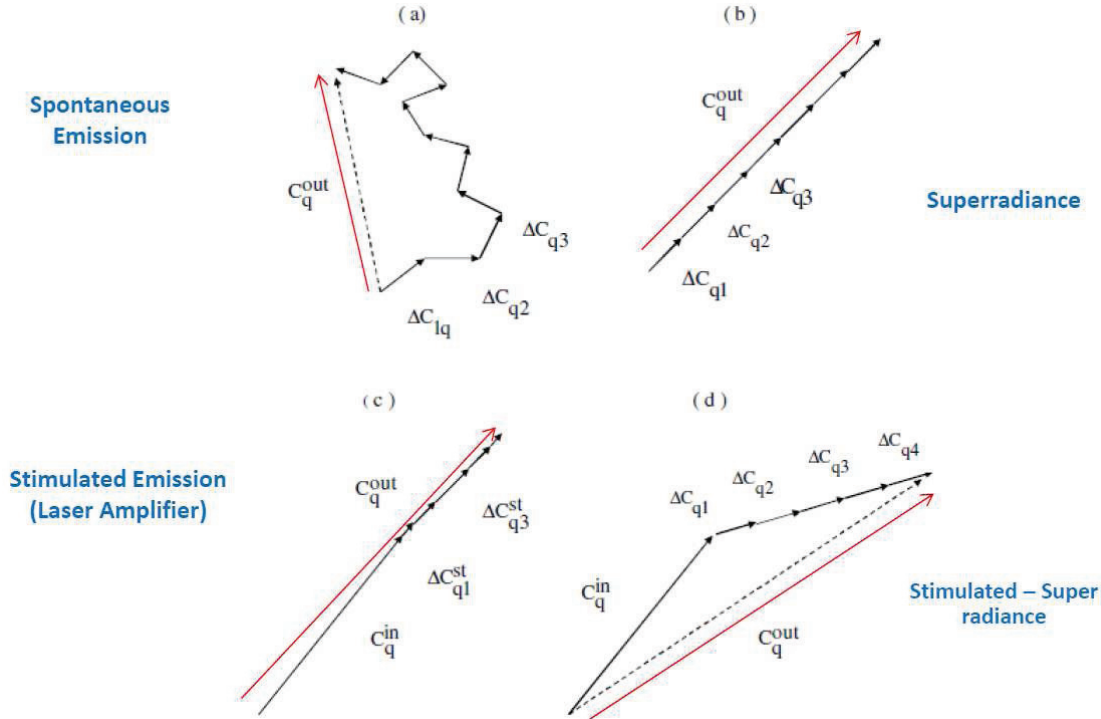


Figure 1: Different cases of radiation: (a) spontaneous emission, (b) superradiance, (c) stimulated emission and (d) stimulated superradiance.

The superradiant term is

$$\left(\frac{d\mathcal{W}_q}{d\omega}\right)_{SR} = \frac{N_b^2 e^2 Z_q}{16\pi} \left(\frac{a_w}{\beta_z \gamma}\right)^2 \frac{L^2}{A_m} |M_b(\omega)|^2 \text{sinc}^2(\theta L/2) \quad (28)$$

and the stimulated superradiant term is

$$\left(\frac{d\mathcal{W}_q}{d\omega}\right)_{ST-SR} = |\check{C}_q^{\text{in}}(\omega)| \frac{N_b^2 e^2 Z_q P_q}{4\pi N} \left(\frac{a_w}{\beta_z \gamma}\right) \frac{L}{A_m} |M_b(\omega)| \text{sinc}(\theta L/2) \cos(\varphi - \theta L/2) \quad (29)$$

SINGLE FREQUENCY (PHASOR) FORMULATION

In the limit of a continuous train of microbunches or a long macropulse $N_M \gg 1$, the grid function $M_M(\omega)$ behaves like a comb of delta functions and narrows the spectrum of the prebunched beam SR and ST-SR Undulator Radiation to harmonics of the bunching frequencies $\omega = n\omega_b$. Instead of spectral energy one can then evaluate the average radiation power output by setting $M_M(\omega) = 1$ and dividing the spectral energy by the pulse duration: $T_M = N_M 2\pi/\omega_b$. Alternatively, one may have analyzed the continuous bunched beam problem from the start in a single frequency model using phasor formulation:

$$A(\underline{r}, t) = \text{Re}[\tilde{A}(\underline{r}, \omega) e^{-i\omega t}] \quad (30)$$

As in [5, 7, 8], we take a model of a periodically modulated e-beam current of a single frequency ω :

$$I(z, t) = I_0 \{1 + \text{Re}[\tilde{M}_b e^{-i\omega(t-z/v_z)}]\} \quad (31)$$

Assuming the beam has a normalized transverse profile distribution $f(\underline{r}_\perp)$. The transverse current density in the wiggler is:

$$\underline{J}_\perp(\underline{r}, \omega) = \frac{I_{m\perp} \hat{\underline{e}}_\perp}{2} f(\underline{r}_\perp) e^{i(\omega/v_z - k_w)z} \quad (32)$$

Where:

$$I_{m\perp} \hat{\underline{e}}_\perp = I_0 \tilde{M}_b \frac{\tilde{\beta}_w}{\beta_z} \quad (33)$$

Writing now the excitation equation in phasor formulation:

$$\tilde{C}_q(z) = \tilde{C}_q(0) - \frac{1}{4P_q} \int dV \tilde{\underline{J}}_\perp(\underline{r}, \omega) \cdot \tilde{\underline{E}}_q^*(\underline{r}_\perp) e^{-ik_z z}, \quad (34)$$

One obtains:

$$\tilde{C}_q(z) = \tilde{C}_q(0) - \frac{I_m}{8P_q} |\tilde{E}_q(0)| F z e^{i\theta z/2} \text{sinc}(\theta z/2) \quad (35)$$

where

$$F = \frac{1}{|\tilde{E}_q(0)|} \int_{-\infty}^{\infty} \hat{\underline{e}}_\perp \cdot \tilde{\underline{E}}_q^*(\underline{r}_\perp) d^2 r_\perp. \quad (36)$$

We remark that $F \simeq 1$ for a narrow beam. The time averaged radiation power will then be given by:

$$P_q(z) = P_q |C_q(z)|^2 = P_q(0) + P_{SR}(z) + P_{ST-SR}(z) \quad (37)$$

Where the superradiant and stimulated superradiant powers are:

$$P_{SR}(z) = \frac{1}{32} Z_q |I_m|^2 F^2 \frac{z^2}{A_{em}} \text{sinc}^2(\theta z/2) \quad (38)$$

and

$$P_{ST-SR}(z) = \frac{1}{4} |I_m| |E_{\perp}(0)| Fz \cos(\varphi_0^r - \varphi_0^b - \theta z/2) \operatorname{sinc}(\theta z/2) \quad (39)$$

TAPER ENHANCED SUPERRADIANCE (TES) AND TAPER ENHANCED STIMULATED SUPERRADIANCE AMPLIFICATION (TESSA)

The underlying assumption in the calculation of spontaneous emission, superradiant spontaneous emission and (zero order) stimulated superradiant emission is that the beam energy loss as a result of radiation emission is negligible. When this is not the case the problem is a much harder nonlinear evolution problem. We now extend our model to the case of a continuously bunched electron beam interacting with a strong radiation field in an undulator, so that the electron beam loses an appreciable portion of its energy in favor of the radiation field. However in the case the injected radiation field power $P(0)$ is high enough to trap the bunched electrons in the buckets of the ponderomotive potential, one can conceive a concept of tapering the period or amplitude of the wiggler in such a way, that the phase velocity of the ponderomotive potential buckets will slow down in accordance with the energy loss of the bunched beam and the synchronism condition ($\theta \approx 0$) continues to be kept [9]. In such a configuration Taper Enhanced Superradiance (TES) can continue to be produced [4, 5], but also as suggested in [3] there may be significant emission of Taper Enhanced Stimulated Superradiant Amplification (TESSA). This observation is particularly relevant to the case of seed injected tapered wiggler FEL. In this case (see Fig. 2) the tapered wiggler section would emit both TES and TESSA radiation, but the input field for the TESSA process is not arbitrary, but determined by the saturation power emitted by the constant wiggler parameters FEL section preceding the tapered wiggler section.

Start to end analysis and simulation of the tapered wiggler FEL were presented by numerous authors in attempt to maximize the radiation extraction efficiency and output power of the FEL [4, 5, 10–18]. A single frequency phasor analytical model has been recently presented by Schneidmiller and Yurkov [5] drawing attention to the radiation diffraction effect that must be taken into account in the tapered wiggler section. Emma et al have shown that a spectral analysis approach is necessary for including non negligible effects of shot noise and synchrotron oscillation side band radiation [18]. In this section we consider the tapering enhancement effect in the framework of the simple analytical model of radiation mode expansion and particularly drawing attention to the role of TESSA radiation emission process. Clearly a full 3D spectral numerical analysis is necessary for considering all the above mentioned effects and getting to reliable quantitative estimates of the radiation emission. But following [5] we suggest here an extended analytical

analysis that can be used as a guideline for wiggler tapering strategy including TESSA contribution.

Applying for now our simplistic phasor analysis we assume that the bunched beam loses its average energy uniformly as a function of z in a way that depends on the radiation field evolution on axis $E(z)$:

$$\gamma(z) = \gamma(0) + \delta\gamma(z, E(z)) \quad (40)$$

The average axial velocity of the beam changes as a function of z both because because of wiggler amplitude tapering β_z^0 and because of the energy loss $\delta\beta_z(z, E(z))$

$$\beta_z(z) = \beta_z^0 + \delta\beta_z(z, E(z)) \quad (41)$$

The phase of the bunched beam relative to the ponderomotive wave is then:

$$\varphi(z) = \int_0^z \left[\theta^0(z') - k \frac{\delta\beta_z(z')}{[\beta_z^0(z')]^2} \right] dz' = \int_0^z \theta^E(z') dz', \quad (42)$$

where $\theta^E(z')$ is the resultant field dependent detuning parameter, and

$$\theta^0(z') = \frac{\omega}{v_z^0(z')} - k_w(z') - k_{zq}(z') \quad (43)$$

is the would-be detuning parameter in the tapered wiggler in the absence of energy loss, consequently, from Eq. (34)

$$\begin{aligned} \check{C}_q(z) = & \check{C}_q(0) - \frac{1}{8} I_0 F \frac{|\check{E}_q(0)|}{P_q} e^{i\varphi_b} \\ & \int_0^z \frac{a_w(z')}{\gamma(z')\beta_z(z')} |M_b(z')| e^{i \int_0^{z'} \theta^E(z'') dz''} dz' \end{aligned} \quad (44)$$

Of course, in order to know $\delta\gamma(z, E(z))$ and consequently $\theta^E(z)$ one must solve the force equation for the bunched electron beam dynamics in the buckets of the slowing down ponderomotive potential. This part of the analysis is not attempted in the present work. We only assume the tapering strategy is optimal, such that we may assume with the consideration of the beam energy loss that the net taper and field depending detuning is constant: $\theta^E(z) = \theta^E(0)$. If we also assume as in [5] that the bunching amplitude and amplitude coefficient in the integrand are approximately constant, then the equation is integrable, resulting in:

$$\check{C}_q(z) = \check{C}_q(0) - \frac{I_m}{8P_q} |\check{E}_q(0)| Fz e^{i\theta_0^E z/2} \operatorname{sinc}(\theta_0^E z/2) \quad (45)$$

Similarly to Eqs. (37), (38) and (39) for SR/ST-SR we get then for a tapered wiggler with tapering matched t_D the beam energy loss

$$P(z) = P(0) + P_{TES}(z) + P_{TESSA}(z) \quad (46)$$

where

$$P_{TES}(z) = \frac{1}{32} Z_q |I_m|^2 F^2 \frac{z^2}{A_{em}} \operatorname{sinc}^2(\theta_0^E z/2) \quad (47)$$

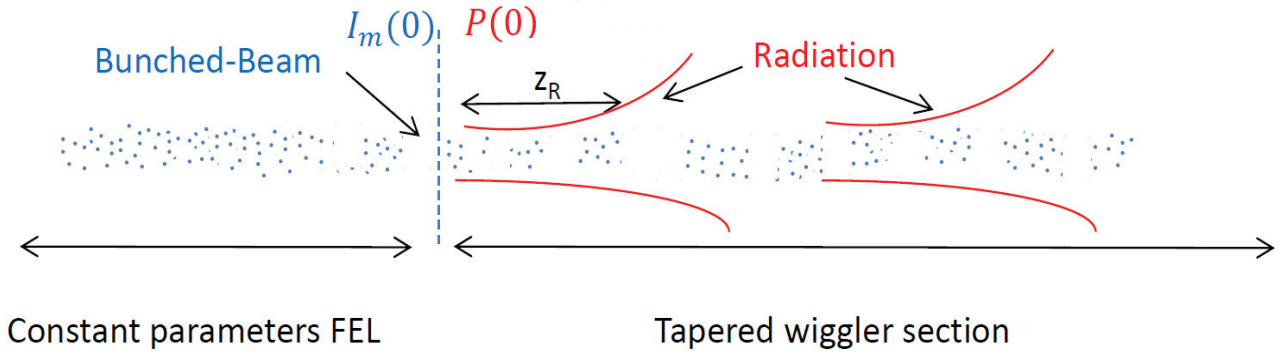


Figure 2: Constant wiggler vs taped wiggler.

and

$$P_{TESSA}(z) = \frac{1}{4} |I_m| |E_{\perp}(0)| F z \cos(\varphi_0^r - \varphi_0^b - \theta_0^E z/2) \text{sinc}^2(\theta_0^E z/2). \quad (48)$$

For $\theta_0^E = 0$ and phase matched bunched current and radiation field $\varphi_0^r = \varphi_0^b$

$$P_{TES}(z) = \frac{1}{32} Z_q |I_m|^2 F^2 \frac{z^2}{A_{em}} \quad (49)$$

and

$$P_{TESSA}(z) = \frac{1}{4} |I_m| \sqrt{\frac{2Z_q}{A_{em}}} \sqrt{P_{in}} F z \quad (50)$$

The ratio between the two contributions to the radiation power is

$$\frac{P_{TESSA}}{P_{TES}} = 8 \frac{A_{em}}{Z_q I_m F z} \sqrt{\frac{2Z_q}{A_{em}}} \sqrt{P_{in}} = 8 \frac{A_{em}}{Z_q I_m F z} E_{in}(0) \quad (51)$$

In Fig. 3 we show the Ratio of 0-order TESSA to TES for different initial power at $z = z_0$. Initially the TESSA

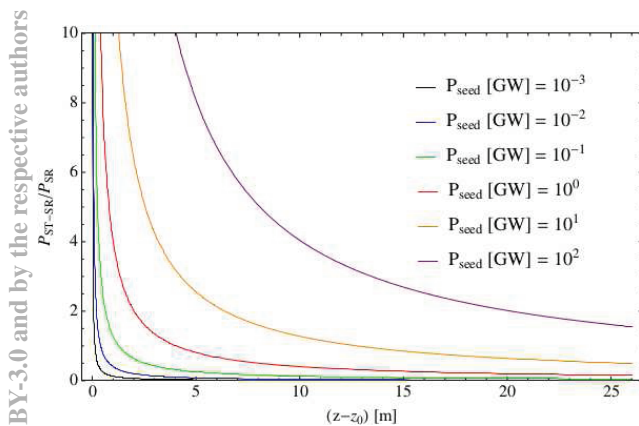


Figure 3: Ratio of 0-order TESSA to TES for different initial power at $z = z_0$.

power dominates the TES power, but evidently, for long interaction length the TES power that grows like z^2 exceeds

the TESSA power that grows like z . At the beginning stages of interaction in the tapered wiggler the TESSA power may be significantly higher than the TES power if the initial radiation power P_{in} injected into the tapered section is large enough. This balance is demonstrated in Fig. 3 for the parameters of LCLS [15].

ELABORATION OF THE CONCEPTUAL MODEL

The classification of the spontaneous and stimulated emission processes of electron beams (shot noise spontaneous emission, SR and ST-SR emission) and the counterpart processes of TES and TESSA in a tapered wiggler is helpful as a guideline and general framework for more detailed and accurate analysis of realizable radiative emission devices. Some progress has been made recently by various authors, but substantial analytical and simulation analyses are still required in order to optimize the radiation emission from practical sources.

Diffraction Effects

The radiation mode expansion formulation would be valid for a free diffraction case only for a wiggler length smaller than one Rayleigh length $L_w < z_R = \pi W_0^2/\lambda$. For longer lengths one should use a full multimode expansion analysis of the radiation field or rather a Fresnel diffraction analysis as was done by Schneidmiller et al for TES only [5].

Single vs Multi Frequency Analysis

The single frequency beam current steady state modulation model is too crude for obtaining reliable quantitative results in the case of the taper enhanced SR and ST-SR. A multifrequency spectral analysis (as in Eqs. (1,15)) is required for taking full account of the trapped particle beam dynamics in the ponderomotive bucket in the tapered wiggler. This includes simulation of the synchrotron oscillations trajectories of the electron of the bunch trapped in the buckets, that differs from each other due to incomplete bunching, finite energy spread and emittance. This means that the beam modulation current evolves and diminishes (due to detraping) along the interaction length. Further spectral

broadening effects that require more detailed spectral (time dependent) numerical simulations are evolution of synchrotron oscillation side-band frequencies and shot noise effect [18].

TESSA

The TESSA contribution given in the present model calculation should be regarded only as "zero-order TESSA", because Eq. (48) does not take into consideration the enhanced stimulated SR emission that takes place when the radiation power grows along the interaction length, providing deepening of the trapping buckets and allowing more aggressive tapering strategy. This kind of non-linear dynamics "high gain TESSA" was analyzed numerically in [3].

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