# MINMIZATION OF THE EMITTANCE GROWTH INDUCED BY COHERENT SYNCHROTRON RADIATION IN ARC COMPRESSOR*

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*Abstract*

Coherent synchrotron radiation (CSR) is a critical issue when electron bunches with short bunch length and high peak current transporting through a bending system in high-brightness light sources and linear colliders. For example, a high peak current of electron beam can be achieved by using magnetic bunch compressor, however, CSR induced transverse emittance growth will limit the performance of bunch compressor. In this paper, based on our 'two-dimensional point-kick analysis', an arc compressor with high compression factor is studied. Through analytical and numerical research, an easy optics design technique is introduced that could minimize the emittance dilution within this compressor. It is demonstrated that the strong compression of bunch length and the transverse emittance preservation can be achieved at the same time.

## INTRODUCTION

In ERL designs, recirculation arcs are often used to compress the bunch length. In order to achieve compression, an ultra-relativistic electron beam with energy chirp passes through the bending system. Since the trajectory is curved, electrons emit coherent synchrotron radiation (CSR) and may induce energy modulation along the bunch and dilutes transverse emittance, leading to degradation of the beam quality [1-3]. To suppress the undesirable emittance growth, several design strategies have been proposed [4-8]. However, most of these designs reach a high compression factor by adopting a low bunch charge. In this paper, the point-kick analysis [8] is reviewed and be applied to an arc compressor consists of double bend achromats (DBAs) to achieve emittance preservation with high bunch charge.

## POINT-KICK ANALYSIS OF CSR EFFECT

In the "steady-state" approximation for a Gaussian line-charge distribution beam, the CSR-induced rms relative energy spread depends linearly on both $L_b$ and $\rho^{2/3}$ [9-11].

$$\Delta E_{rms} = 0.2459 \frac{eQ\mu_0 c_0^2 L_b}{4\pi\rho^{2/3}\sigma_z^{4/3}}, \tag{1}$$

where $e$, $Q$, $\rho$, $\sigma_z$, $L_b$, $\mu_0$, $c_0$ represent the charge of a single particle, the bunch charge, the bending radius of the orbit, the rms bunch length, the bending path, the permeability of vacuum, and the speed of light, respectively.

Therefore the CSR effect in a dipole was linearized by assuming $\delta(csr) = kL_b/\rho^{2/3}$, where $k$ depends only on the bunch charge $Q$ and the bunch length $\sigma_z$, and is in unit of $m^{1/3}$. In addition, it was shown that the CSR-induced coordinate deviations after a passage through a dipole can be equivalently formulated with a point-kick at the centre of the dipole (see Fig. 1), which is of the form [8]

$$X_k = \begin{pmatrix} \rho^{4/3}k[\theta\cos(\theta/2) - 2\sin(\theta/2)] \\ \sin(\theta/2)(2\delta + \rho^{1/3}k\theta) \end{pmatrix}, \tag{2}$$

where $\delta = \delta_0 + \delta_{csr}$, is the particle energy deviation at the entrance of the dipole, with $\delta_0$ being the initial particle energy deviation and $\delta_{csr}$ being that caused by CSR in the upstream path.

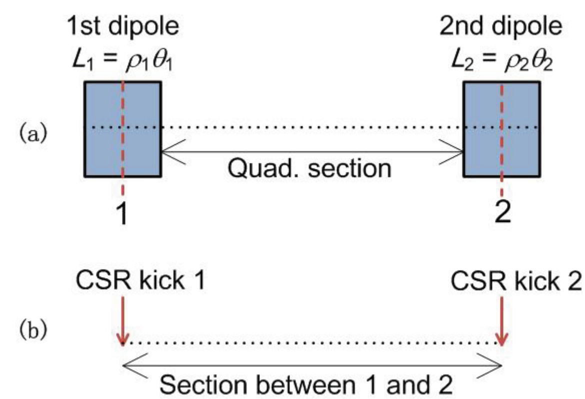


Figure 1: Schematic layout of a two-dipole achromat and physical model for the analysis of the CSR effect with two point kicks. The point 1 and 2 indicate the centres of the first and the second dipole, respectively.

## EMITTANCE PRESERVATION OF A CHIRPED BEAM AFTER A DBA

In this section, we will present the derivation of the CSR-minimization condition for a DBA with symmetric layout. As sketched in Fig. 1, the bending angles of the first and the second dipole are denoted by $\theta$, the bending radii of these dipoles are the same, denoted by $\rho$. According to the point-kick analysis, CSR kicks occur at the centres of the two dipoles (denoted by 1, 2, in Fig. 1), and between the adjacent kicks only one 2-by-2 transfer matrix of the horizontal betatron motion is considered. For simplicity, it is assumed that the initial particle coordinates relative to the reference trajectory are $X_0$ =

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$(x_0, x_0')^\dagger = (0, 0)^\dagger$ and the energy deviation is $\delta = 0$. According to Eq. 2, the coordinates remain zero until the particle experiences the CSR-kick at point 1, where the particle coordinates are given by

$$X_1 = \begin{pmatrix} -\rho^{4/3} k_1 r \\ S\rho^{1/3} k_1 \theta \end{pmatrix}. \quad (3)$$

where $r = 2\sin(\theta/2) - \theta\cos(\theta/2)$, and $S = \sin(\theta/2)$. Since the symmetric DBA satisfied the achromat condition, the transport matrix can be expressed as (we assume that the Courant-Snyder (C-S) parameters are symmetric)

$$M_{12} = \begin{pmatrix} -1 & 0 \\ -\frac{2\alpha_1}{\beta_1} & -1 \end{pmatrix}. \quad (4)$$

After passing through the section between point 1 and 2, the particle experiences the second kick. Note that due to energy chirp, the bunch length of the beam is compressed and the coefficient $k_2 = m^{4/3} k_1$, where m is the compression factor of one DBA. Thus the CSR-induced particle coordinate deviations at point 2 are (the energy spread $\delta$ grows to $k\rho^{1/3}\theta$ at point 2)

$$X_2 = \begin{pmatrix} -rk_1\rho^{4/3}(-1+m^{4/3}) \\ k_1\rho^{1/3}[\frac{2r\alpha_1\rho}{\beta_1} + (1+m^{4/3})\theta S] \end{pmatrix}. \quad (5)$$

The final geometric emittance (the geometric emittance at the end of the DBA) in presence of the CSR effect can be estimated by

$$\varepsilon^2 = \varepsilon_0^{\ 2} + \varepsilon_0 \cdot d\varepsilon,$$
$$d\varepsilon = \frac{\theta^2}{144}(m^{4/3}+1)\left(\frac{q^2\rho^2 r^2}{\beta_1 S^2} + \left(\frac{-\rho\alpha_1 r}{\sqrt{\beta_1} S} + \sqrt{\beta_1}\theta\right)^2\right). \quad (6)$$

where $\varepsilon_0$ is the unperturbed geometric emittance and $\alpha_2, \beta_2, \gamma_2$ are the C-S parameters at the centre of the second dipole of the DBA. In most cases $d\varepsilon \ll \varepsilon_0$, therefore the growth in unnormalized and normalized emittance due to CSR can be estimated by

$$\Delta\varepsilon = \varepsilon - \varepsilon_0 \approx \frac{1}{2} d\varepsilon,$$
$$\Delta\varepsilon_n = \varepsilon_n - \varepsilon_{n0} = \gamma\beta(\varepsilon - \varepsilon_0) \approx \frac{1}{2}\gamma\beta d\varepsilon, \quad (7)$$

where $\beta$ is the particle velocity relative to the speed of light, $\gamma$ is the relativistic Lorentz factor，and the subscript $n$ represents the normalized emittance. To achieve the emittance preservation, it is required to minimize Eq. 6, from which the linear CSR-suppression condition can be obtained,

$$\beta_1 / \alpha_1 = \rho(\theta\cot\frac{\theta}{2} - 2)/\theta. \quad (8)$$

In the particular case with $\theta \ll 1$, Eq. 8 can be simplify as

$$\beta_1 / \alpha_1 = -L_b / 6. \quad (9)$$

By expressing $\beta_1$ and $\alpha_1$ in terms of $\beta_0$ and $\alpha_0$, the above equation can be written as

$$\beta_0 = \frac{5\alpha_0 L_b}{12} \pm \frac{L_b}{12}\sqrt{\alpha_0^2 - 24}. \quad (10)$$

where in Eqs. 9 and 10 only the first significant terms with respect to $\theta$ are kept. It should also mentioning that in Eq. 10 the negative sign results better CSR-suppression effect, which is illustrated in Fig. 2.

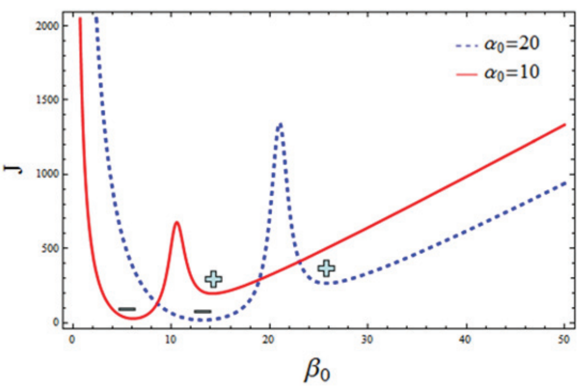


Figure 2: (color online) The lower $\beta_0$ solution has a lower particle invariant.

## ARC COMPRESSOR

The compression arc can be consists of several identical DBAs, and the total compression factor is defined as $C = 1/(1+hR_{56})$ in first order, where $h$ and $R_{56}$ is the bunch energy chirp and the z-δ correlation term of transport matrix, respectively. To minimize the CSR-induced emittance growth after the arc, one should choose the proper initial C-S parameters, which satisfying Eq. 10. Since the energy chirp $h$ increases along the arc, we concern about the emittance growth in the last DBA which has the largest compression factor. To contrast, the dependency of the emittance growth $\Delta\varepsilon_n$ on the initial C-S parameters is investigated with ELEGANT simulations, the initial beam distribution in phase space is generated accordingly to match the optics. The bunch charge in the simulations are 500 pC and the compression factor of the DBA is 4.5. The dipole bending radius and bending angle is 10m and 10 degrees, respectively. It can be seen from Fig. 3 that the emittance growth reaches a minimum as C-S parameters are close to the optimal value, which agrees reasonably well with the analytical prediction. Note that,



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the minimum $\Delta\varepsilon_n$ is not exactly on the optimal value, this is because the bunch length is not invariant in the dipole.

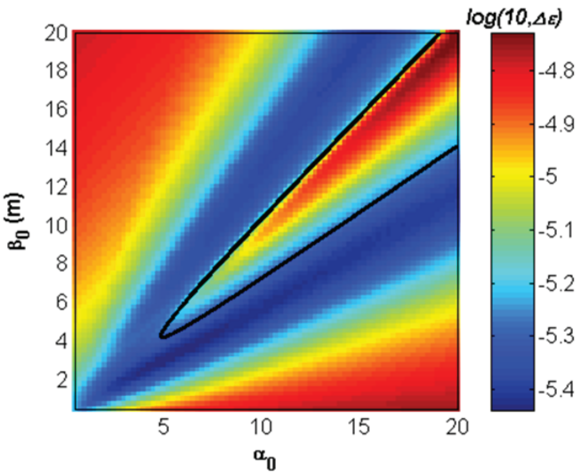


Figure 3: (color online) Numerical simulations of final emittance growth by scanning initial C-S parameters in a symmetric DBA with large compression factor 4.5. The initial transverse emittance is 1μm.rad. Black Line satisfying the CSR-suppression condition Eq. 10.

Now consider a periodical 192° arc compressor made of 8 DBA which satisfying the condition deduced above, some parameters of the lattice and the beam are listed in Table. 1 and $R_{56}$ of the arc is 0.128m. To achieve a total compression factor $C$ = 45, one need the energy chirp h = -7.6m$^{-1}$ at the entrance of the arc. The initial C-S parameters can be chosen as $\beta_0$ = 3m, $\alpha_0$ = 9, which close to the suppression condition. It can be seen from Fig. 4 that at the end of the arc, the normalized emittance growth is at 0.1μm.rad level, which has a good agreement with Eq. 6 with scaling of the CSR effect with the beam parameters.

Table 1: Summary of the Simulation Parameters

| Parameter | Value | Units |
|---|---|---|
| Bunch charge | 500 | pC |
| Normalized emittance | 0.5 | μm.rad |
| Beam energy | 1 | GeV |
| Energy spread | 0.05 | % |
| Initial bunch length | 0.9 | mm |
| Dipole bending radius | 5.24 | m |
| Dipole bending angle | 12 | degree |
| Final bunch length | 2 | μm |
| Final Normalized emittance | 0.74 | μm.rad |

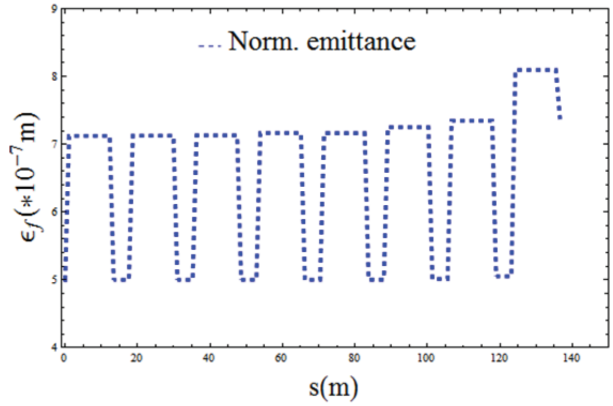


Figure 4: The normalized emittance along the arc. The emittance growth at the end is 0.24μm.rad.

## DISCUSSIONS

By adopting a few modifications on the point-kick analysis, we have derived the generic condition for minimizing the CSR kicks in a linear regime. This condition provide a new way to suppress the CSR-induced growth emittance in an arc compressor made of identical symmetric DBAs with the beam of high bunch charge. It is worth mentioning that this analysis is based on the assumption that the bunch length does not change in the dipole, thus the compression factor in the last DBA should not be too large. On the other hand, from Eq. 6 the final emittance growth depends on $\beta_1$, the beta function at the centre of dipole should be cautiously tuned for both emittance preservation and optics symmetry.

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