TERAHERTZ SOURCE UTILIZING RESONANT COHERENT DIFFRACTION RADIATION AT KEK ERL TEST ACCELERATOR*

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Abstract

A test accelerator of energy recovery linac scheme, cERL, has been under commissioning at KEK. One of the feature of ERL is that it can realize a high repetition rate and continuous operation of a short bunched beam. It is a suitable place to test a light source based on resonant coherent radiation, such as an resonant coherent diffraction radiation (CDR) system. An optical cavity is formed on the beam orbit to build-up CDR. If the fundamental frequency of the cavity coincides with the beam repetition rate, the stored radiation can stimulate the radiation in the following bunches. We show a simple estimation of the radiation power based on a model of coupling between beam and cavity eigen modes. An ideal case example for cERL beam parameter is shown.

INTRODUCTION

One of the feature of ERL type of accelerator is that it can produce a short bunched beam at high repetition rate. It enables us to use it as a THz radiation source based on coherent radiation. CDR (Coherent Diffraction Radiation) is a coherent radiation produced by beam passing near a conductive target. Since it does not destroy a beam, the radiator can be installed in a loop of high averaged current ERL machine. As an advanced layout of CDR, it can be arranged to be a resonator scheme [1]. By coherently adding the radiation in a multi-bunched beam, it can extract radiation power much effectively. A test accelerator, cERL, which has been constructed recently in KEK, should be an ideal place to test the resonant CDR scheme.

Figure 1 shows the schematic of the resonant CDR system. An optical resonator of fundamental frequency that matches with beam repetition is placed on the beam axis. The cavity mirrors has a hole in the center so that beam can pass through. Electromagnetic wave excited in the resonator by a beam can be understood as CDR or higher-order modes of the resonator. Since the transverse profile of the mode is a donuts shape, it can be stored in a resonator formed by mirrors with hole.

Since electromagnetic wave in the resonator positively stimulates the radiation of the following bunches, the radiation power grows in square relation to bunch number. In order to extract the radiation, one of the cavity mirrors is designed to have transmission. Then it can be reflected to a transverse port using a parabolic mirror.

Here, we show the calculation of interaction of beam and resonator, and estimate radiation power assuming cERL beam parameter in an ideal case [2].

* Work supported by KAKENHI

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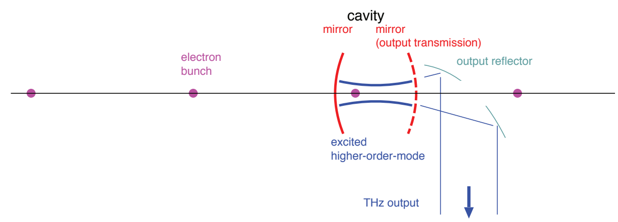

Figure 1: Schematic of resonant CDR.

HIGHER-ORDER GAUSS BEAM

The excited modes are odd order transverse modes of the resonator. Here we calculate the lowest one, TM_{10} mode.

Transverse field of TM_{10} mode is written as follows.

$$E^x_{10} = \frac{A}{w(z)}\frac{x}{w(z)}\exp(-\frac{x^2+y^2}{w^2(z)}) \cdot \exp[i(\omega t - kz + \phi(z))] \quad (1)$$

$$w(z) = w_0\sqrt{1+\frac{z^2}{z_0^2}} \quad (2)$$

$$z_0 = \frac{\pi w_0^2}{\lambda} \quad (3)$$

$$\phi(z) = 2\tan^{-1}(\frac{z}{z_0}) \quad (4)$$

ν is the optical frequency of radiation, c is the speed of light, $k = 2\pi/\lambda$, $\omega = 2\pi\nu$, $\omega/k = c$. A is a scale factor for normalization. $w(z)$ is the size at location z, w_0 is the size at the waist. $\phi(z)$ is known as Gouy phase which depends on the order of transverse mode, the factor 2 means the first order mode. z_0 is Rayleigh length.

Electromagnetic wave is a transverse wave in the case of an uniform plane wave. But, in cases of waves with spatial structure such as higher-order transverse modes, there exists a longitudinal field. The following relation can be shown from Helmholtz equation.

$$ikE^z = \frac{\partial E^x}{\partial x} \quad (5)$$

From Eq. 1, longitudinal field of TM_{10} mode is obtained as follows.

$$E^z_{10} = -\frac{i}{k}\frac{A}{w^2(z)}(1-\frac{2x^2}{w^2(z)})\exp(-\frac{x^2+y^2}{w^2(z)}) \cdot \exp[i(\omega t - kz + \phi(z))] \quad (6)$$

When beam of speed c passes on the center axis, it feels the longitudinal field of

$$E^z_{10}(x=y=0) = -\frac{A}{kw^2(z)}\exp[i2\phi(z)] \quad . \quad (7)$$

ISBN 978-3-95450-134-2

It accumulates the field while experiencing phase shift due to Gouy phase.

INTERACTION WITH THE RESONATOR

Here, we compare two types of two-mirror resonator shown in Fig. 2. (A) is formed with two concave mirrors of same radius of curvature. (B) is a half-cavity, formed with one flat mirror and one concave mirror. One of the mirror has power transmission ratio of T, it is the extraction port.

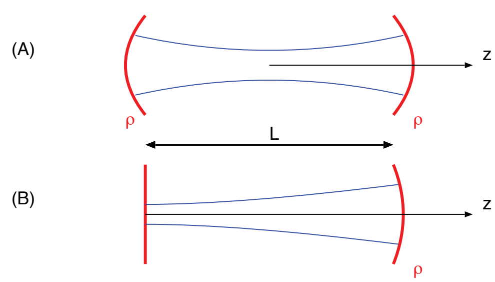

Figure 2: Configuration of resonator.

Electromagnetic energy of TM_{10} mode U can be written as

$$U = 2 \times \frac{\epsilon_0}{2} \int |E^x_{10}|^2 dV \quad , \tag{8}$$

here, we ignore contribution of longitudinal field. Using Eq. 1, it is calculated to be

$$U = \frac{\epsilon_0 \pi A^2 L}{8} \quad . \tag{9}$$

We calculate excitation of one longitudinal mode by a charged particle. (R/Q) of the mode is defined as follows.

$$(R/Q) = \frac{|\int E^z dz|^2}{\omega U} \tag{10}$$

Using Eq. 7, excited energy by charge q is

$$U_1 = \frac{\omega}{4}(R/Q)q^2 \tag{11}$$

$$= \frac{1}{2\epsilon_0 \pi L}\left|\int \frac{1}{1+p^2}\exp[i2\tan^{-1}p]dp\right|^2 \tag{12}$$

here $p = z/z_0$.

Integration of p depends on design of the resonator. For example, in the case of Fig. 2(A), z should be integrated in $-\alpha z_0 \sim \alpha z_0$, and in the case of Fig. 2(B), z should be integrated in $0 \sim \alpha z_0$.

The integration part of Eq. 12 is calculated as a function of α, which is the half-cavity length in the unit of z_0. Figure 3 and Fig. 4 show the case (A) and (B), respectively. In the case (A), longer cavity than optimum can cancel the field due to the phase shift. The optimal case is $\alpha = 1$, it means $L = 2z_0$. As for in the case (B), phase shift is half of case (A), it can not be cancelled by phase shift. In both cases, the maximum of the integral part is 1. So, in the ideal case,

$$U_1 \sim \frac{q^2}{2\epsilon_0 \pi L} \quad . \tag{13}$$

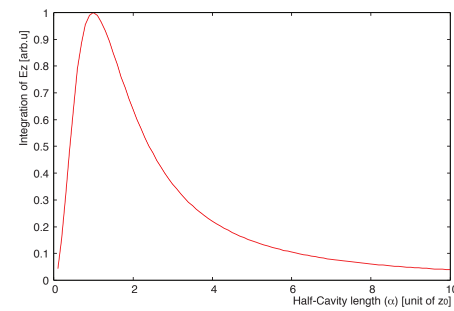

Figure 3: Integration in case (A).

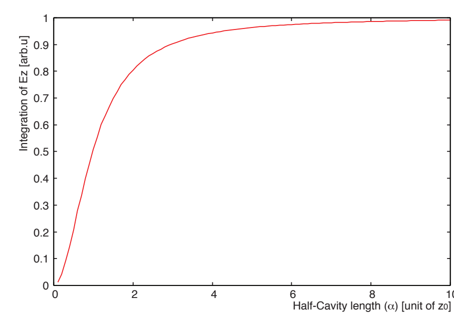

Figure 4: Integration in case (B).

EXTRACTED POWER

Power in the resonator P_{in} can be written as

$$P_{in}\frac{2L}{c} = U_1 \quad , \tag{14}$$

The power extracted from the resonator becomes

$$P_{out} = P_{in}T = U_1 \frac{c}{2L} T \tag{15}$$

here T is the mirror transmittance.

We consider the case of multi-bunch excitation. Here, we assume an ideal case that the resonator loss is negligible to the excitation mirror transmittance. And bunch repetition perfectly matches with resonator fundamental frequency, so that the radiation adds up coherently.

Field in the resonator decays due to the power extraction at the mirror. The amplitude becomes factor $\sqrt{1-T}$ in one

ISBN 978-3-95450-134-2

round-trip. Field extracted in multi-bunch excitation V_∞ can be written using the single bunch excitation V_1 as follows.

$$V_\infty = V_1 + V_1\sqrt{1-T} + V_1(\sqrt{1-T})^2 + \cdots = \frac{V_1}{1-\sqrt{1-T}} \sim \frac{2V_1}{T} \quad (16)$$

Power is the square of amplitude.

$$P_\infty = V_\infty^2 = \frac{4}{T^2}V_1^2 = \frac{4}{T^2}P_{out} \sim \frac{cq^2}{\epsilon_0 \pi L^2 T} \quad (17)$$

So far, we discussed about single longitudinal mode. There should be many longitudinal mode spaced in every FSR (free-spectral-range) of the resonator. These are excited by the beam at the same time. It forms a mode-locked pulse in the resonator. Since Eq. 17 does not depend on frequency or w_0, all the longitudinal modes can be excited, except for diffraction loss or frequency dependence of mirrors.

For example, we assume the center frequency to be 1.6 THz and FSR to be 160 MHz, there are 100 longitudinal modes in 1 % bandwidth. The total power is sum of all the modes.

CALCULATION IN THE CASE OF CERL

We assume bunch compression mode of operation. So, we assume form-factor to be 1 around a few THz region. The parameter is as follows. Bunch charge is q = 10 pC/bunch, bunch repetition rate is FSR=162 MHz, number of longitudinal mode in 1% band width is $N_{mode} = 10^2$, the resonator length L = 0.925 m (162 MHz), transmission of extraction mirror $T = 10^{-3}$.

Photon flux F is calculated to be

$$F = \frac{cq^2}{\epsilon_0 \pi L^2 T}\frac{N_{mode}}{h\nu} \sim 2\times 10^{23} \quad \text{photons/s/1\%BW} \quad (18)$$

In averaged power, it is $P_\infty = F \times h\nu \sim$ 30W. By counting another transverse mode of perpendicular direction, TM_{01}, there should be another factor 2.

Finally, we compare with a single-path radiation without resonator. We use the formula of transition ratiation (TR), called Ginzburg-Frank equation in the follwing.

$$\frac{dW_{TR}}{d\omega d\Omega} = \frac{e^2}{4\pi^3\epsilon_0 c}\frac{\beta^2 \sin^2\theta}{(1-\beta^2\cos^2\theta)^2} \quad (19)$$

θ is radiation angle.

Integrating all solid angle and coherently add number of particle (N^2), and also integrating in 1% bandwidth around 1.6 THz, the radiation flux is calculated to be $\sim 3\times 10^{19}$ photons/s/1%BW at same beam parameter. Comparing with Eq. 18, it shows that there is four orders of magnitude enhancement because of the stimulation by resonator configuration.

SUMMARY

Here, we discussed CDR in a resonator configuration. It can be understood as excitation of a higher-order transverse mode of the resonator through its longitudinal electric field. So, it can be calculated from beam to cavity mode interaction. Calculating coherent addition of excitation in a multi-bunch beam, in an ideal case of loss is dominated by power extraction, the photon flux in cERL beam parameter results in $\sim 10^{23}$ photons/s/1%BW at THz region. This can be a very strong radiation source in this frequency.

This work is partially supported by Photon and Quantum Basic Research Coordinated Development Program from the Ministry of Education, Culture, Sports, Science and Technology, Japan, and JSPS KAKENHI 25600146.

REFERENCES

[1] A. Aryshev *et al.* "Observation of the stimulated coherent diffraction radiation in an open resonator at LUCX facility", Nucl. Instr. Meth. A 763, pp 424-432 (2014)

[2] Y. Honda "THz source utilizing an resonant CDR system at cERL", Proceedings of 11th Japanese Particle Society Meeting (2015)

ISBN 978-3-95450-134-2