

## RECENT STUDY IN iSASE

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### Abstract

The Improved Self-Amplified Spontaneous Radiation (iSASE) scheme has the potential to reduce SASE FEL bandwidth. This is achieved by repeatedly delaying the electrons with respect to the radiation pulse using phase shifters in the undulator break sections. It has been shown that the strength, locations and sequences of phase shifters are important to the iSASE performance. The particle swarm optimization algorithm is used to explore the phase shifters configuration space globally.

### INTRODUCTION

Improved Self Amplified Spontaneous Emission (iSASE) [1, 2] is capable of improving spectrum by increasing cooperation length. Electrons are delayed with respect to the optical field by phase shifters in the FEL lattice. And connection is built up between electrons that are separated by several coherent spikes width away. With proper interference between new grown field and optical field, bandwidth can be reduced.

There has been effort to investigate the mechanism of study the proper phase delay configuration. There is study proposes to arrange phase shifter strength in a geometric or reverse geometric sequence. In this kind of configuration, the largest phase delay creates a small period frequency comb modulation in the power spectrum. When the second largest phase delay, which is half of the largest delay, has a good phase match, it eliminates some of the side band peaks and amplifies the central peak. Using this scheme, the central peak can be effectively selected. There are also other schemes uses prime number phase delay [3] and random phase delay [4] to improve the FEL bandwidth.

Some optimization method, such as simulated annealing method [5], has been used to optimize iSASE. The method is able to explore the solution space locally around a reverse geometric sequence configuration, but not yet conclusive. This study focuses on the global optimization of iSASE phase delay configuration.

### iSASE

FEL bandwidth is improved by repeatedly delaying the electron bunch with respect to the optical field. After each phase delay, the interference effect between the shifted light field and the new grown field from energy and density modulated electron beam appears as a modulation to the FEL power spectrum [6],

$$P(\nu; z) = P_0(\nu; z)T(\nu, \phi, a), \quad (1)$$

$$T(\nu, \phi, a) \propto 1 + |a|^2 + 2|a| \cos(\nu\phi + \varphi). \quad (2)$$

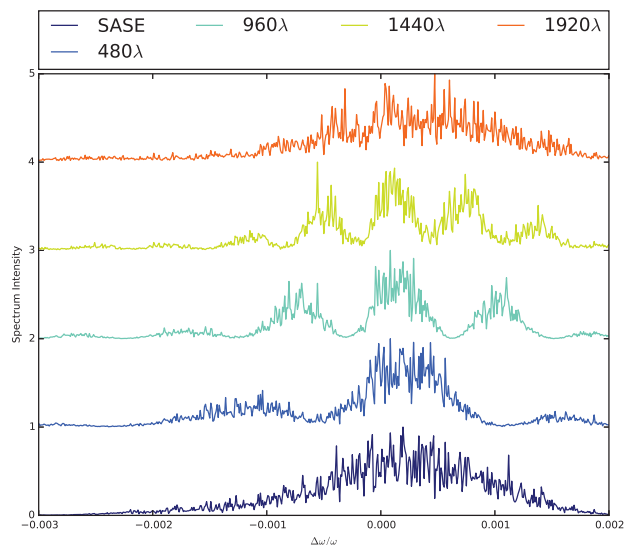


Figure 1: Modulation can be seen in the FEL power spectrum after the first phase shifter. The modulation period decreases with larger phase delay. Yet there is a limit ( $960\lambda$ ) where the modulation pattern no longer exists. This is because the dispersive effect in the phase shifter strong enough to wash out the density modulation in the electron bunch. Only the optical field carries the pure SASE spectrum through the phase shifter.

Here  $\phi$  is the integer phase delay. The modulation period is inversely proportional to  $\phi$ .  $\varphi$ , the fractional phase delay, controls the center of the modulation function.  $a$  is the relative amplitude between the shifted optical field and the new grown field. The dispersive effect in the phase shifter can cause damping to electron bunching and even distort the electron bunch density modulation. The interference effect is degraded by the dispersive effect. Therefore it sets a upper limit to the tolerable phase delay value (Fig. 1). A narrow filtering function can be generated using multiple modulation functions with different modulation periods.

### PARTICLE SWARM OPTIMIZATION

Particle swarm optimization algorithm mimics the behavior of bird flocking. The candidate solutions, which are called particles, have position and velocity. At the beginning, particles are randomly distributed in the solution space with random velocity. As particles sweep through the solution space, particles find solutions with different cost values. During the process, particles are also attracted by the good solutions, with lower cost values, that have been experienced by the particles. And these good solutions may

be replaced by the even lower cost solutions found by the particles. Eventually particles cluster around the global optimal solutions. Therefore the particle swarm optimization algorithm is known for its ability to search for global minimum. The evolution equation for the algorithm can be written as,

$$x^i(n+1) = x^i(n) + v^i(n) \quad (3)$$

$$v^i(n+1) = \alpha v^i(n) + U(0, \beta)(p^i(n) - x^i(n)) + U(0, \beta)(g^i(n) - x^i(n)). \quad (4)$$

Here  $x^i(n)$ ,  $v^i(n)$  are the  $i^{\text{th}}$  particle position and velocity respectively after the  $n^{\text{th}}$  iteration.  $\alpha$  is the particle inertia.  $U(0, \beta)$  is a random number that sampled uniformly between  $(0, \beta)$ .  $p^i(n)$  is the best solution the  $i^{\text{th}}$  particle has experienced after  $n$  iterations.  $g^i(n)$  is the best solution among the neighbors of the  $i^{\text{th}}$  particle. There are two general definition of neighbors. One assumes that the all particles fully connected. In this topology, particles are attracted by the best solution that ever experienced by the whole group. The second setting assumes that particles can only communicate with their adjacent neighbors. The first setting usually converges faster than the latter topology. Yet, the latter one is less likely to be attracted by local minimums than the first one. In our study, we consider the particles are only connected to their adjacent neighbors.

## OPTIMIZATION RESULT

Particle swarm optimization algorithm is applied to optimize iSASE configuration. The algorithm searches for the optimal bandwidth in the phase delay variable space. In this study, we consider inserting five phase shifters to a LCLS-II type machine. In such machine, the gain length is about the length of a undulator section. The phase shifters are placed between the gap sections with the first one locating after the fourth undulator section. Drift space is taken out so that the center of modulations function align with SASE center frequency.

Phase delay upper bound in the optimization is set at  $1800\lambda$  so that the electron beam can maintain its bunching after a dispersive chicane (Fig. 1). 100 particles are uniformly distributed in the parameter space (Fig. 2). The algorithm converges after 200 iterations (Fig. 3). Particles are clustering around the global minimum. There are particles scattering around the cluster. This is coming from the last two terms on the right hand side of Eq. 4. Particles tend to oscillate around the global minimum.

The global best solution (Fig. 4) yields a  $9 \times 10^{-5}$  bandwidth (Fig. 5), which is 4 times smaller than the SASE bandwidth. Figure 6 demonstrates that the solution is a minimum point. The FEL bandwidth tends to increase, as the Euclidian distance to the global best solution increases. By investigating the particles that has Euclidian distance larger than 10, we find that they are solutions that basically switch the values of the first two phase shifters. It is one of the local minimums the algorithm finds before converging to the global minimum. This confirms with the fact that as the

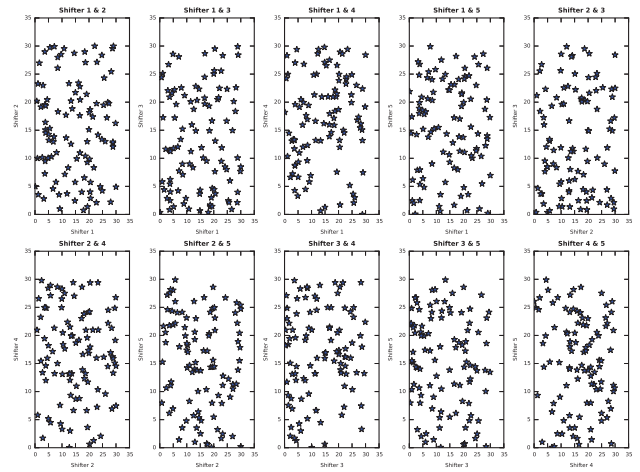


Figure 2: Particles are distributed uniformly in the solution space initially.

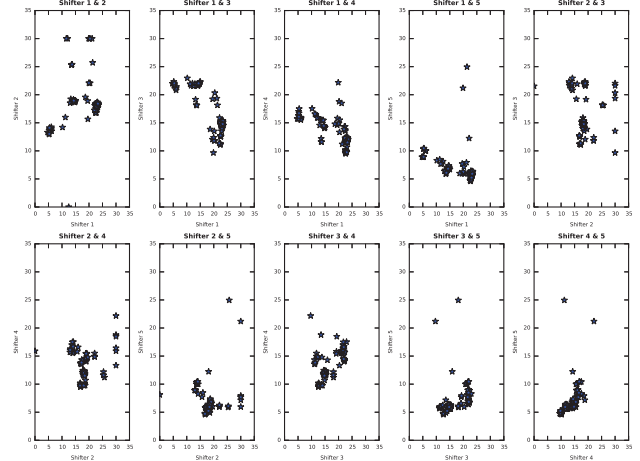


Figure 3: At the end of the optimization, particles cluster around the global best solution.

energy modulation amplitude increases in the linear regime, the tolerable phase delay decreases. Thus the global best solution has large phase delay at beginning and small phase delay at the end.

We also notice that the first phase delay value almost reaches the limit of the phase delay values where interference can be well maintained. The first phase shifter defines the finest modulation period. The configuration can be divided into two parts. The 1<sup>st</sup>, 4<sup>th</sup> and 5<sup>th</sup> phase delays are  $1380\lambda$ ,  $720\lambda$  and  $360\lambda$  respectively. This three phase delays form a reverse geometric sequence. This kind of configuration can effectively eliminate the side bands as it is shown in the middle plot of Fig. 7. However, since the largest one has already reach the limit, the reverse geometric sequence cannot continue. With an isochronous chicane, it can be shown that a 20 times bandwidth reduction can be achieved by extending the existing reverse geometric sequence with five phase shifters (Fig. 5). On the other hand, the algorithm is able to find a compromising solution. The 2<sup>nd</sup> and 3<sup>rd</sup>

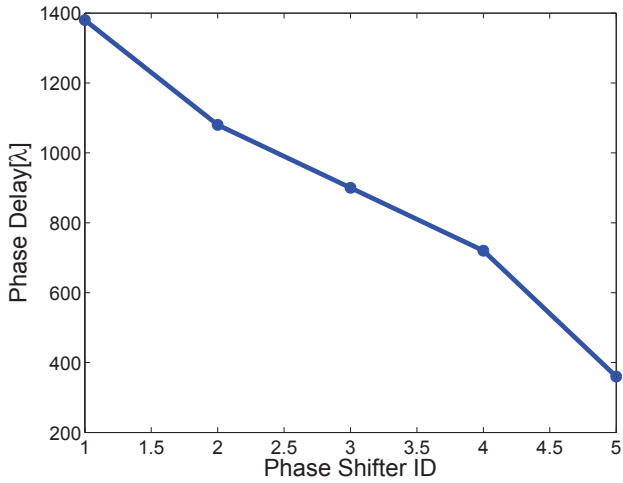


Figure 4: The optimal phase shifter values from the particle swarm optimization.

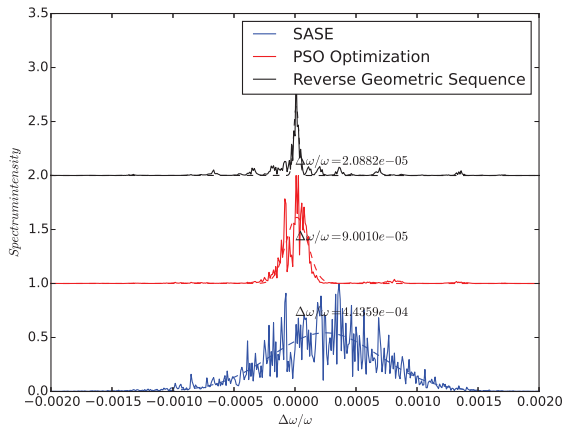


Figure 5: The FEL bandwidth is reduced by a factor of four from  $4.4 \times 10^{-4}$  to  $9.0 \times 10^{-5}$  with dispersive effect. It can be further reduced to  $2 \times 10^{-5}$  with isochronous chicane.

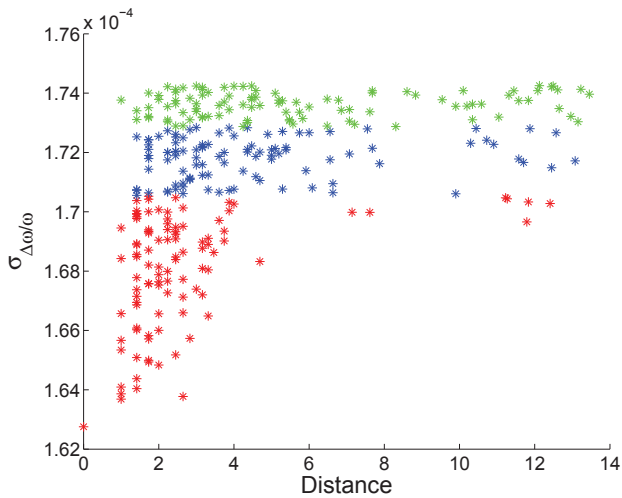


Figure 6: The Euclidian distance to the global best solution for the first 300 best solutions is plotted against their bandwidth.

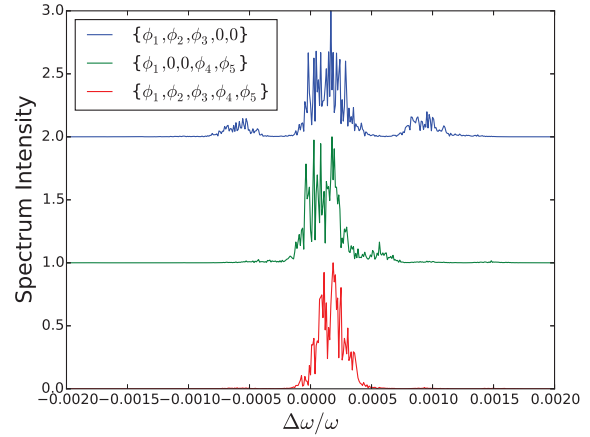


Figure 7: The upper figure plots the final power spectrum with first three phase shifters. The middle figure plots the spectrum with reverse geometric sequence. The last one is the global best solution found by the optimization.

phase shifters are used to further narrow the central peak. With the first three phase shifters, the FEL power spectrum has a narrower distribution and with small sidebands (the upper plot in Fig. 7). Therefore the combination of these two mechanisms finally shape the FEL power spectrum and yield a narrow bandwidth.

## CONCLUSION

Particle swarm optimization is applied to iSASE to search the optimal phase shifter configuration. The algorithm is able to avoid multiple local minimums and find the global minimum. The FEL bandwidth is reduced by a factor of 4 from  $4.4 \times 10^{-4}$  to  $9 \times 10^{-5}$ . And the global best solution also confirms that the reverse geometric sequence configuration is able to remove sidebands effectively. The limitation is caused by the dispersive effect of the phase shifter. By introducing nonlinear chicane, the reduction factor can at least be further enhanced to 20.

## ACKNOWLEDGEMENT

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