

# ON THE IMPORTANCE OF ELECTRON BEAM BRIGHTNESS IN HIGH GAIN FREE ELECTRON LASERS\*

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## Abstract

Linear accelerators delivering high brightness electron beams are essential for driving short wavelength, high gain free-electron lasers (FELs). The FEL radiation output efficiency is often parametrized through the power gain length that relates FEL performance to the electron beam quality at the undulator. Experimental data and simulation results of existing and planned FEL facilities, collected in [1], are used to explicit the relationship between the FEL output wavelength and the electron beam six-dimensional brightness. Following [2], practical formulas are provided that show the dependence of the exponential gain length on the beam brightness.

## 6-D ELECTRON BEAM BRIGHTNESS

In the context of an electron bunch, the 4-dimensional (4-D) brightness,  $B_{4D}$ , can be defined as the peak current divided by its 4-D transverse phase space volume that is the product of the transverse emittances [3]. Owing to the fact that linac-driven free electron lasers (FELs) are sensitive to the beam relative energy spread and local charge density, it is convenient to parameterize the linac performance in terms of the 6-D brightness,  $B_{6D}$ , which is the total bunch charge divided by its 6-D phase space volume. The 6-D volume includes, in addition to the four transverse positions and slopes, the normalized longitudinal emittance, which scales as the product of bunch length and absolute energy spread. In the following, we assume the particle beam in the ultra-relativistic approximation, so that the longitudinal charge distribution is assumed to be constant during acceleration.

In general, we may define the brightness either locally, *i.e.*, for each bunch slice (in this case, the brightness depends on the z-coordinate inside the bunch), or for the whole bunch, thus involving the bunch total charge and *projected* emittances. The transverse rms normalized emittances are invariant under acceleration and linear transport, presuming collective effects, such as space charge, may be neglected. The same is true for the longitudinal rms normalized emittance if the energy spread is intended as uncorrelated, *i.e.*, without any energy chirp.

The presence of nonlinear motion and collective effects along the beam delivery system may dilute the normalized emittances from their values at the injection point. Following [2], we introduce an effective degradation factor  $\zeta \geq 1$  in each plane of the particle motion so that  $\epsilon_{nx,f} = \zeta_x \gamma_0 \epsilon_{x,0}$ ,  $\epsilon_{ny,f} = \zeta_y \gamma_0 \epsilon_{y,0}$ , and

$\epsilon_{nz,f} = \sigma_{z,f} \sigma_{E,f} = \zeta_z \sigma_{z,0} \sigma_{E,0}$ , with obvious notation. We are

now able to relate the 6-D normalized brightness at the undulator,  $B_{n,f}$ , to that at the linac injection,  $B_{n,0}$ :

$$B_{n,f} \equiv \frac{Q}{\epsilon_{nx,f} \epsilon_{ny,f} \epsilon_{nz,f}} = \frac{Q}{\zeta_x \zeta_y \zeta_z \gamma_0^2 \epsilon_{x,0} \epsilon_{y,0} \epsilon_{z,0}} = \frac{B_{n,0}}{\zeta_x \zeta_y \zeta_z} \quad (1)$$

In the ideal case of vanishing nonlinear and collective effects,  $\zeta_x, \zeta_y, \zeta_z \rightarrow 0$  in Eq. 1 and thereby the 6-D normalized brightness is preserved at the injector level under acceleration and linear bunch length compression.

## IMPORTANCE OF PROJECTED BEAM PARAMETERS

In contrast to linear colliders, where particle collisions effectively integrate over the entire bunch length, the FEL process takes place over short fractions of the electron bunch length. In fact, slice emittance and slice energy spread may vary significantly along the bunch and thus give local regions where lasing may or may not occur [4]. One could therefore argue that only *slice* electron beam quality is of interest, each slice being at maximum as long as the slippage length of the photon beam over the electrons, cumulated along the undulator length. In the following, we make the case that other considerations related to the electron beam control and optimization of the FEL performance justify an optimization of  $B_{6D}$  defined in terms of the projected beam emittances. We will limit the discussion to the transverse emittances; correlations in the longitudinal plane are discussed in [1,2].

The need to control beam size and angular divergence along the undulator calls for measurements and manipulation of the electron beam optical (Twiss or envelope) parameters, which have to be matched to the design ones [5–8]. As a practical matter, optics matching is routinely performed by measuring the *projected* electron bunch transverse size [9]. From an operational point of view, it is therefore important to ensure that the projected transverse emittances and Twiss parameters be as close as possible to the slice ones, because this guarantees that most of the bunch slices are matched to the design optics and that they overlap in the transverse phase space. During beam transport and acceleration, at least two collective effects threaten locally to offset bunch slices in the transverse (and longitudinal) phase space, namely coherent synchrotron radiation (CSR) and geometric transverse wakefield (GTW). Specific optics designs can be adopted to minimize those collective effects (for a review of these topics, see for example [1]).

The projected emittance can be considered a good marker also for externally-seeded FEL performance. In

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such FELs, output FEL properties reflect the high longitudinal coherence of the seeding laser, which can be tens to hundreds of femtoseconds long. In order to maximize the FEL efficiency and the peak current, the final electron bunch duration is commonly specified to be as long as the seed laser duration plus some room (typically, from tens to hundreds of femtoseconds) for accommodating the shot-to-shot arrival time jitter of the electron bunch with respect to the seed laser. The electron bunch duration has to be even longer when the so-called “fresh bunch” scheme is implemented for lasing at high harmonic jumps [10–12]. Consequently, high performance from a seeded FEL requires uniformity of the slice beam parameters over most of the bunch duration in order to ensure the same strength of lasing from different slices. That is, seeded FELs also require a large value of electron beam brightness, defined in terms of projected transverse and longitudinal emittance.

Another point in favor of carefully considering projected beam parameters is illustrated in [13], where it is shown that the output power gain length of a self-amplified spontaneous emission (SASE) FEL [14,15] depends on the mismatch of bunch slices in the transverse phase space, thus on the projected emittance, even if the slice emittance is unperturbed. A similar result is expected to be valid for externally seeded FELs as well. The projected emittance growth due to mismatch of bunch slices in the transverse phase space is taken into account through the mechanism described by Tanaka *et al.* [16]. In that work, the authors identify two distinct processes that increase the FEL gain length. The first effect is referred to as the (lack of) electron-photon transverse spatial overlap along the undulator. The second one describes the accumulation of longitudinal phase error between electrons and radiation by virtue of the slowing down of individual electrons due to their local angular divergence. We recognize that the electrons’ angular divergence has two contributions (similar considerations can be found in [17,18]): one is incoherent and due to the non-zero beam emittance as depicted in Xie’s [19] and Saldin’s [20] models; the other is coherent, originating from the possible tilt of the slices’ centroid with respect to the reference trajectory. The coherent divergence adds to (and in some cases, surpasses) the incoherent one and may amplify the effect of bunching smearing. One source of coherent divergence occurs when each slice is transversely kicked by collective effects in the linac and moves along the undulator on a trajectory different from that of other slices. In this case, Tanaka’s formula for the gain length is revised via the following *ansatz*, to estimate the 3-D gain length in the presence of collective effects [13]:

$$L_{G, coll} \approx \frac{L_{G, 3D}}{1 - \pi \langle \theta_{coll}^2 \rangle / \theta_{th}^2} \quad (2)$$

$L_{G, 3D}$  is the 3-D power gain length as calculated by Xie [19] and  $\theta_{th} = \sqrt{\lambda / L_{G, 3D}}$ . The electron beam slice

transverse emittance and the slice energy spread at the undulator are taken into account in  $L_{G, 3D}$ ; the information on the projected emittance growth  $\Delta\epsilon$ , which is uniquely determined by the initial beam parameters and its dynamics in the linac, is brought about by  $\theta_{coll} \propto \Delta\epsilon / \beta_u$  [13], with  $\beta_u$  the average betatron function along the undulator.

## FEL REQUIREMENTS FOR THE ELECTRON BEAM

It is well-known that in the so-called 1-D, cold limit, where electron beam energy spread, transverse emittance and radiation diffraction effects are all neglected, the radiation peak power at the resonant wavelength grows exponentially along the undulator with a gain length  $L_G = \lambda_u / (4\pi\sqrt{3}\rho)$ . Here  $\lambda_u$  is the undulator period length, and  $\rho$  is the “FEL parameter” [4]:

$$\rho = \left( \frac{\Omega_p \lambda_u a_w [JJ]}{8\pi c \gamma} \right)^{2/3} = \frac{1}{2\gamma} \left( \frac{I}{I_A} \right)^{1/3} \left( \frac{\lambda_u a_w [JJ]}{2\pi \sigma_x} \right)^{2/3}, \quad (3)$$

with  $\Omega_p$  being the plasma frequency,  $I$  the electron bunch peak current,  $I_A = 17045$  A the Alfvén current,  $\gamma$  the relativistic Lorentz factor for the beam mean energy,  $\sigma_x$  the standard deviation of the (assumed round) electron beam transverse size;  $a_w = K$  for helically- and  $a_w = K/\sqrt{2}$  for planar-polarized undulator, where  $K = 0.934 B_0 [T] \lambda_u [cm]$  in practical units, is the so-called undulator parameter,  $B_0$  the undulator peak magnetic field, and  $c$  the speed of light in vacuum.  $[JJ]$  is the undulator-radiation coupling factor [21], equal to 1 for a helical undulator, and to  $J_0(\xi) - J_1(\xi)$  for a planar undulator, where  $J_0$  and  $J_1$  are Bessel’s functions of the first kind with argument  $\xi = K^2 / (4 + 2K^2)$ . The FEL fundamental wavelength of emission satisfies:

$$\lambda = \frac{\lambda_u}{2\gamma^2} (1 + a_w^2) \quad (4)$$

Typically  $\rho \approx 10^{-3}$  in the UV wavelength regime but may drop to  $\sim 10^{-4}$  in the X-ray regime for kA-current beams. If the undulator length is equal or longer than  $\sim 18L_G$ , the conversion of electrons’ kinetic energy to photon energy considerably enlarges the electron beam energy spread. Once the spread in the longitudinal momentum of the electrons becomes sufficiently large to cause significant de-bunching over one power gain length, the FEL gain process strongly diminishes. Consequently, the FEL gain grows exponentially as far as the following numerical condition applies to the beam fractional energy spread [22]:

$$\sigma_\delta \leq 0.5\rho, \quad (5)$$

with an eventual reduction in the FEL gain; the associated FEL power saturates at a level  $P_{\text{sat}} \approx 1.6\rho EI/e$ . An electron beam at multi-GeV energies and kA-scale peak currents is able to produce GW-scale radiation peak powers. For SASE devices, the value of  $\rho$  also defines the approximate number of undulator periods  $N_{\text{sat}} \sim 1/\rho$  and the length  $L_{\text{sat}} \approx \lambda_u/\rho$  necessary to reach saturation.

The spread in longitudinal momentum in Eq. 5 has two major sources: (1) the incoherent energy spread that is “uncorrelated” with the particle longitudinal position inside the bunch, and (2) the non-zero transverse emittance. In other words, Eq. 5 refers both to the spread of longitudinal momentum and to the energy spread associated with the square of the beam transverse angular divergence [23]. Beam divergence scales as  $(\varepsilon/\beta_u)^{1/2}$ , with  $\beta_u$  the average betatron function along the undulator and  $\varepsilon$  the geometrical electron beam transverse emittance (the two parameters are measured in the same plane; the divergences in the two planes add in quadrature). At the same time, in order to minimize emittance effects and to ensure optimal transverse overlap of the co-propagating radiation and electron beam, the electron beam trajectory, transverse size and angular divergence must be controlled with steering and quadrupole magnets that are interleaved between the undulator segments. The most efficient electron-photon beam interaction occurs when the transverse beam phase space area and distribution matches that of the radiation, whereas the transverse electron beam size scales as  $(\varepsilon\beta_u)^{1/2}$ . Considerations on both the maximum allowable effective energy spread and the transverse overlap lead to an rms value of  $\varepsilon$  that must be smaller than, or of the same order as, that of the diffraction-limited photon beam [24]:

$$\varepsilon_{x,y} \leq \frac{\lambda}{4\pi} \quad (6)$$

Equation 6 implies an “optimum” average betatron function along the undulator of the order of the FEL power gain length. More details on this relationship are discussed in the following Section. In general, Eq. 6 allows to maximize the FEL gain and it also optimizes the FEL transverse coherence.

### OPTIMUM BETATRON FUNCTION

It should be noticed that the term “optimum” used above refers to the condition for minimum SASE FEL power gain length. Some deviations are usually found when the SASE FEL output power at saturation is maximized. The existence of an “optimum” average betatron function in the undulator, in the assumption of periodic smooth focusing, can be inferred already by considering the effective size and divergence of a photon beam propagating over one FEL gain length, in the presence of a non-zero emittance electron beam. Those are, respectively,  $\sigma_{\gamma,\text{eff}}^2 = \varepsilon\beta_u + \lambda L_G/(4\pi)^2$  and  $\sigma_{\gamma,\text{eff}}^{\prime 2} = \varepsilon/\beta_u + \lambda/L_G$ , whereas the expressions apply to

each transverse plane separately, or we may intend the (square of) electron beam size and divergence as the sum in quadrature of the size and divergence in the two planes. The photon beam *brilliance* is maximized when the effective photon beam emittance  $\sigma_{\gamma,\text{eff}}\sigma_{\gamma,\text{eff}}^{\prime}$  is minimized, which implies simultaneously  $\varepsilon \leq \lambda/(4\pi)$ ,  $\beta_u \leq L_G/(4\pi)$  and  $\beta_u \geq L_G/(4\pi)$ . That would be true when Eq. 6 holds and at the same time  $\beta_u \approx L_G/(4\pi)$ .

The found expression, however, suggests an optimum value of  $\beta_u$  which is commonly at the sub-meter level, and therefore not practical. As a matter of fact, it is not correct since that was derived by considerations solely related to the transverse overlap of the electron beam and the FEL radiation. When the electrons’ longitudinal motion w.r.t. the FEL radiation is also considered, one finds that the betatron motion affects the synchronism between electron and emitted photons, so that the transverse emittance causes Landau damping of the electrons bunching. Considerations on the bunching smearing leads to the lower limit  $\beta_u \geq (0.25-0.50)L_G$  for the optimum betatron function [25,26]. When the constraint on the total energy spread is considered, Eq. 6, together with the target of maximum FEL gain, Eq. 3, an equation for the optimum  $\beta_u$  is found [27]. In this case, when the cold beam limit (no energy spread) and the emittance diffraction limit are considered at the same time, we find an optimum value  $\beta_u \approx 2L_G$  (actually, a betatron phase advance of 0.5 rad over one gain length) [27]. This analytical evaluation was somehow supported from simulation results [28], which provided the optimum condition  $\beta_u \approx 3L_G$  for non-zero energy spread. One should notice that those results are consistent with a photon beam weakly affected by radiation diffraction. Indeed, if we assume a photon beam size at waist that matches the electron beam size, then the ratio of the Rayleigh length over the FEL gain length must be  $4\pi\varepsilon\beta_u/\lambda L_G \geq 1$ . At the diffraction limit, we have again  $\beta_u \geq L_G$ . In conclusion, in most practical cases the approximate equation  $\beta_u \approx L_G$  is taken as a reference.

### SCALING LAWS

Since a smaller transverse emittance is usually associated with shorter FEL wavelengths, and since we can observe a proportionality between transverse emittance and  $B_{n,f}$ , we wonder if we could establish any relationship between  $B_{n,f}$  and  $\lambda$ . This is done below, neglecting for the moment any emittance dilution, by substituting Eq. 4 into Eq. 1, and assuming the electron beam transverse emittance (equal in the two planes) at the diffraction limit (see Eq. 6):

$$B_{n,f} = \frac{Q}{\varepsilon_{nx,f}\varepsilon_{ny,f}\varepsilon_{nz,f}} = \frac{I}{c\sigma_E\gamma_0^2\alpha_0^2} \approx \frac{32\pi^2}{c} \frac{I}{\sigma_E} \frac{1}{\lambda_u(1+a_w^2)} \frac{1}{\lambda} \quad (7)$$

It is worth noticing that the ratio  $I/\sigma_E$  is invariant under acceleration and compression (whereas the peak current

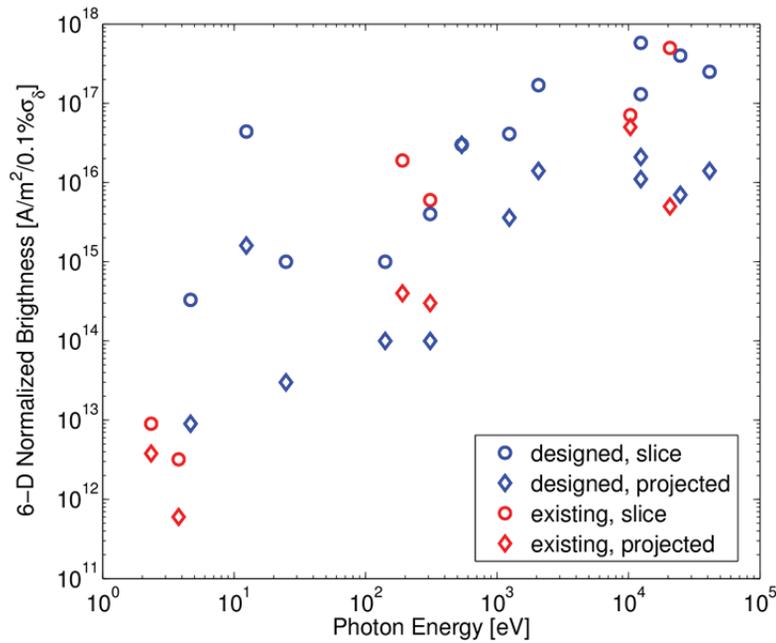


Figure 1: Six-dimensional normalized electron beam brightness vs. maximum photon energy at fundamental FEL emission, for facilities in the ultra-violet (UV) to X-rays, designed (blue) or running (red). Data taken from [1] and updated to 2013. From lower to higher energies, now-running facilities are: SPARC (Italy), SDUV-FEL (China), FLASH-I (Germany), FERMI (Italy), LCLS (USA), SACLA (Japan). The brightness refers to the projected (circle) or slice value in the bunch (diamond). Copyright of Photonics MDPI [2].

and the energy spread must be evaluated at the same location along the accelerator), when collective effects are ignored. We find that, for any given undulator, a shorter  $\lambda$  requires a higher  $B_{n,f}$ . This is confirmed in Fig. 1, where  $B_{n,f}$  of designed and existing single-pass linac-driven FEL facilities, is shown as a function of the maximum photon energy (*i.e.*, minimum fundamental wavelength) from UV to X-rays (inferred or measured data are taken from [1] and updated to 2013). Moreover, Fig. 1 shows  $B_{n,f}$  evaluated for projected and slice emittances (where the slice length is approximately one tenth of the total bunch duration, and located in the bunch core). A gap of one or two orders of magnitude occurs typically between the two brightness values. The closer the projected and the slice brightness, the more efficient the FEL process is, since most of the electrons are distributed in identical manner in 5-D ( $x, x', y, y', \gamma$ ) phase space along the bunch. Usually, a smaller gap between the projected and the slice brightness is gained at the expense of the flexibility of the FEL facility in wavelength, intensity, polarization, *etc.*

Since  $\rho(\lambda)$  determines the efficiency of the electron-to-photon energy transfer in the undulator at a given wavelength, a large  $\rho$  is typically desired because that implies a shorter gain length, or a higher FEL power at saturation. Some restrictions to the upper value of  $\rho$  may be considered in a SASE FEL that targets a relatively narrow spectral bandwidth because in this kind of FEL the output bandwidth is also proportional to  $\rho$ . We can explicit the dependence of  $\rho$  on  $B_{n,f}$  by substituting Eq. 7 into Eq. 3, similarly to what was done in [29] for the longitudinal brightness. We impose  $4\pi\epsilon = \lambda$ , re-define the energy spread like the rms value of  $\gamma$ , and consider a

specific, typical value  $K = 1$  in a helical undulator. Finally we get:

$$\rho \approx 0.016 \frac{E[\text{GeV}]^{4/3} \lambda[\text{nm}]}{\beta_u[\text{m}]^{1/3}} \sigma_\delta^{1/3} B_{n,f} \left[ \frac{A}{\mu\text{m}} \right]^{-1/3}, \quad (8)$$

from which we see that the strongest dependence of  $\rho$  is on the electron beam energy. The latter can be increased with a longer linac or higher accelerating gradient RF structures, but it is also quite expensive. It is worth noting that since the FEL resonance condition in Eq. 4 imposes  $\lambda \sim 1/E^2$ ,  $\rho$  is not expected to vary much when  $\lambda$  is made short, and in fact we typically have  $\rho \approx 10^{-3} - 10^{-4}$  in the entire XUV range (*i.e.*,  $\lambda \approx 0.1 - 100$  nm).

Equation 8 can be further manipulated and  $\rho$  written as a function of the electron beam transverse and longitudinal parameters at the undulator, whereas still we retain  $4\pi\epsilon = \lambda$  and  $K = 1$ :

$$\rho \approx 3.1 \times 10^{-4} \left( \frac{I[A] \epsilon_{n,x}[\mu\text{m}]}{\beta_u[\text{m}]} \right)^{1/3} \quad (9)$$

Equation 9 tells us that, in order to have  $\rho$  large at any given  $\lambda$  and for any fixed optics in the undulator, it is always convenient to increase the peak current, while there might be no practical convenience in reducing the *normalized* emittance below the diffraction limit, because this could reduce  $\rho$  with much improvement neither in the FEL output power, nor in the FEL transverse coherence. A closer look to Eq. 9 tells us that, when Eq. 6 is forced to equality, if  $\epsilon_{n,x} = \gamma\epsilon_x$  is lowered because  $\gamma$  is lowered

(while  $\varepsilon_x$  is kept fixed), then  $\lambda$  is fixed that implies  $\lambda_u$  is lowered (see Eq. 4);  $\rho \sim \gamma^{1/3}$  is also lowered. If  $\varepsilon_{n,x} = \gamma\varepsilon_x$  is lowered because  $\varepsilon_x$  is lowered (while  $\gamma$  is kept fixed), then  $\lambda$  is lowered that implies  $\lambda_u$  is lowered;  $\rho \sim \lambda_u^{1/3}$  is lowered as well.

Alternatively, if the equality in Eq. 6 is broken and the geometric emittance is left “free” to span lower values than  $\lambda$ , Eq. 9 is not valid anymore and, for same peak current and undulator optics, a higher FEL gain is expected by virtue of both a reduced beam size (see Eq. 3), and of an overall smaller effective energy spread (see Eq. 5). A notable improvement of SASE FEL output power by virtue of a transverse emittance well below the diffraction limit was in fact observed in simulation runs and recently reported in [30].

We conclude this Section by noticing that by replacing that “optimum” value  $\beta_u \approx L_G$  in Eq. 9, we find that  $\rho$  scales like  $\sim\sqrt{I}$ , instead of the cubic power of  $I$  as in Eq. 3. In fact, Eq. 3 assumes a scaling with  $I$  which is independent from the transverse beam size. The condition  $\beta_u \approx L_G$ , instead, implies that the transverse charge density changes as the current changes and, in particular, the beam size squeezes as the current increases, so leading to a more favourable dependence of  $\rho$  on  $I$ . Moreover, by virtue of Eq. 2, a larger  $\beta_u$  than usually considered on the basis of the previous discussion could be considered, when the projected emittance *growth* becomes comparable to the unperturbed value of the slice emittance.

## CONCLUSIONS

The importance of projected electron beam parameters for FEL performance were highlighted, in regard of operational aspects of an FEL facility and of a new definition for the SASE FEL 3-D gain length. Scaling laws for the FEL parameter in the 1-D approximation with the electron beam 6-D brightness were discussed, as well as the relationship between the brightness and the FEL wavelength. Considerations on the beam optics in the undulator were refreshed, which suggest an optimum range for the average betatron function also in consideration of the importance of the projected emittance for the FEL performance.

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