TRANSFORMING THE FEL: COHERENCE, COMPLEX STRUCTURES, AND EXOTIC BEAMS

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Abstract

Modern high brightness electron beams used in FELs are extremely versatile and highly malleable. This flexibility can be used to precisely tailor the properties of the FEL light for improved temporal coherence (as in external or self-seeding), but can also be exploited in new ways to generate exotic FEL modes of twisted light that carry orbital angular momentum (OAM) for new science. In this paper I briefly review the history of the work on OAM light production in FELs, and describe how lasers and undulator harmonics can be combined to produce both simple and complex e-beam distributions that emit intense, coherent, and highly tunable OAM light in future FELs.

INTRODUCTION: FEL TAILORING

Free-electron lasers (FELs) are composite systems of accelerators, electron beam (e-beam) optics, and undulators that produce widely tunable light with exceptional brightness at wavelengths down to hard x-rays for a broad range of studies. The versatility of FELs is derived from the fact that the e-beams that form the lasing medium can be precisely manipulated to tailor the properties of the radiated light. These 'beam shaping' manipulations, which range from coarse shaping of the e-beam current profile to precision shaping of the distribution at optical or shorter wavelengths, are used primarily to tailor the temporal shape of the FEL pulse. Because the typical SASE FEL pulse is composed of many temporal spikes, the aim of these schemes is to improve the longitudinal coherence and produce Fourier Transform-limited pulses. To this end, such techniques can also be combined with 'radiation shaping' techniques that exploit characteristic features of the undulator radiation to further broaden the landscape of designer FEL photon beams.

The past decade has shown tremendous progress in the development of such 'beam by design' concepts [1], in some cases turning proposed techniques into experimental realities over the course of just a few years. This is due in part to the confluence of rapidly advancing technologies that yield higher brightness e-beams, highly stable sub-ps lasers, and tunable undulator systems. The diagram in Figure 1 shows a sample of a number of different schemes designed to tailor the FEL output through either direct shaping of the electron beam (beam shaping) or through shaping using intrinsic features of the undulator radiation (radiation shaping). The slotted foil technique [2], for example, is a method of selecting only a short portion (or portions) of the electron beam to lase by spoiling the emittance of or removing the rest of the beam. Laser-based e-beam shaping techniques (shown

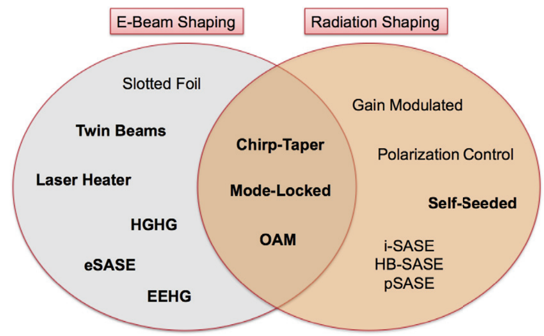

Figure 1: Diagram of example FEL pulse shaping schemes. In bold are those that are based on lasers.

in bold) such as HGHG and EEHG rely on external lasers to precisely rearrange the e-beam phase space to produce coherent density bunching at high harmonics. Such microbunched beams then radiate coherent pulses with bandwidths much narrower than the intrinsic FEL bandwidth. Radiation shaping with the i-SASE, HB-SASE, or pSASE techniques, on the other hand, seeks to take control over the natural slippage between the e-beam and the co-propagating radiation to communicate phase information over different portions of the beam to improve the temporal coherence. In another example, the polarization of the FEL pulse can be controlled using different combinations of linear or circularly polarized undulators, delays, and undulator tapering. Several schemes rely on specific combinations of both types of shaping. In the laser-based 'chirp-taper' technique designed to produce ultrashort pulses, for example, the resonant frequency of the undulators is tapered along the length to exactly match the energy chirp of a short portion of an e-beam that has been modulated by a few-cycle laser pulse.

The concentration on tailoring the temporal profile stems from the fact that the high-gain FEL is nearly diffraction limited and thus already has a high degree of transverse coherence, even for SASE. The lowest order transverse mode also has the highest gain, so the radiation at the fundamental frequency is gaussian-like and is peaked on axis. While it is fortunate that such ubiquitous modes are generated as a matter of course, there are numerous emerging applications and research opportunities where higher order transverse modes, specifically modes that carry orbital angular momentum (OAM), provide additional degrees of freedom that may be specifically exploited to probe the deep structure and behavior of matter.

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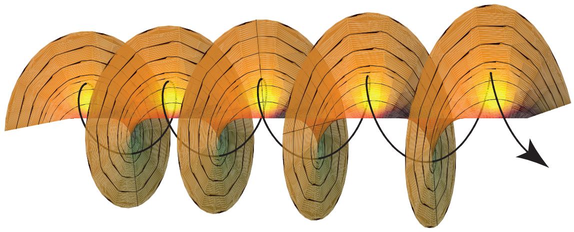

Figure 2: Helical phase and spiraling Poynting vector of OAM light.

A BRIEF HISTORY OF OAM LIGHT

The idea that light can carry spin angular momentum (SAM) dates back to the work of J.H. Poynting in 1909 [3]. He argued that circularly polarized light waves must carry angular momentum by virtue of the energy flux carried by the rotating electromagnetic fields, and that the associated torque could thus be measured in its transfer with matter. This was first confirmed by Beth in 1936 [4] in an experiment that measured the twist on a series of quarter wave plates suspended by a thin torsion fiber and normal to an incident circularly polarized beam. The conversion of the polarization state gave a measured angular momentum that was in agreement with expectations from quantum theory that predicted each photon carried $\pm\hbar$ of SAM.

Analysis of the total angular momentum of the electromagnetic fields reveals, however, that the spin does not necessarily account for all of the angular momentum carried by the light. It wasn't until 1992 that Allen et al [5] showed that Laguerre-Gaussian modes, which are eigenmodes of the paraxial wave equation, also carry discrete values of *orbital* angular momentum per photon[1]. Since then, there has been considerable work devoted to revealing the novel behaviors and uses of these exotic beams, because like the SAM, the OAM can also be transferred to matter. This leads to unique exploits in particle micro-manipulation [9–11], microscopy [12, 13], imaging [14], optical pump schemes [15], quantum entanglement [16, 17], and communications [18].

The most salient feature of OAM light is a characteristic helical phase front and associated spiraling Poynting vector that describes the flow of the linear momentum about the propagation axis. The depicted single twist helix shown in Figure 2 illustrates the phase of the lowest order non-zero OAM mode. Modes described by such phases have associated $l\hbar$ of OAM per photon, where l is an integer. This is in addition to the angular momentum defined by the polarization state where each photon has at most $\pm\hbar$ of SAM. The undefined phase along that axis also gives these modes a characteristic annular intensity profile, which has important implications for their generation in FELs.

OAM AMPLIFICATION IN FELS

Because of their exotic properties and multitude of applications at visible and longer wavelengths, it seems only natural to wonder how OAM light might be folded into the flexible FEL repertoire, particularly since these new insights overlapped with the development of the first x-ray FEL [19]. Among the promising x-ray OAM applications is an expanded x-ray magnetic circular dichroism technique [20] where it is proposed that angle-resolved energy loss spectrometry can distinguish spin-polarized atomic transitions subject to different photon OAM and polarization states [21, 22].

In reality, the first examinations into FEL OAM were more modestly motivated. They began at UCLA initially as an attempt to explain the strange hollow and swirling intensity profiles that were observed at the visible-infrared SASE amplifier (VISA) FEL at the ATF at Brookhaven [23, 24]. While in the end these profiles were likely not OAM in nature[2], it was soon realized that OAM light could indeed be generated in an FEL given the proper conditions.

This came as a result of a theoretical formalism that had been developed by myself, Avi Gover and James Rosenzweig to understand how OAM modes couple to the e-beam and are amplified [25, 26]. The basis of this formalism was the gain-guiding nature of the high-gain FEL process, in which the lasing e-beam tends to guide the radiation rather than let it diffract completely away [27]. This feature compels a mathematical description of the guided FEL radiation in terms of self-similar eigenmodes with fixed profiles rather than the diffracting modes of free space. In our formalism, the guiding properties of the lasing e-beam were modeled through a description of the e-beam as a 'virtual dielectric'. This approach is ultimately mathematically identical to previous analyses that described the radiation field through a mode expansion, but it had a particularly useful advantage. It turned out that, if the virtual dielectric is chosen to have a refractive index with a parabolic radial dependence (i.e., a so-called quadratic index medium), then the basis self-similar eigenmodes have precisely the same form as the OAM modes of free space paraxial propagation evaluated at the beam waist. In this way of describing the FEL, we had a useful connection between the guided FEL modes and the naturally occurring free-space modes such that the coupling characteristics of specific OAM modes in the FEL interaction could be calculated directly [25]. Subsequent extensions to the model were then developed that included energy spread [28] (where the equivalence between ours and other formalisms was also shown) and later on emittance and betatron motion [29]. From the full theoretical description, a fitting formula[3] for the gain length $L_{3D}/L_{1D} = 1 + \Lambda_{0,\pm1}$ of the $l = \pm1$ OAM modes was obtained in at the optimal

[1] To the astute reader troubled by the assignment of OAM to the massless photon which should have only SAM by Lorentz invariance, I defer to the relevant discussions in [6], [7], and [8] regarding the subtleties of intrinsic and extrinsic angular momentum as they apply to the OAM modes.

[2] The donut shapes were most likely the result of off-axis coupling to the radiation due to betatron motion and energy spread.

[3] in the spirit of the example set by Ming Xie for quickly calculating the FEL gain length [30]

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detuning [29],

$$\begin{aligned}\Lambda_{0,\pm1} =&1.1\eta_d^{0.57} + 3.0\eta_\gamma^2 + 0.60\eta_\epsilon^{1.56} + 950\eta_d^{1.5}\eta_\gamma^{3.7}\\ &+ 5.5\eta_d^{1.10}\eta_\epsilon^{0.5} + 11\eta_d^{0.7}\eta_\epsilon^{1.2}\\ &+ 1.14\eta_\gamma^{5.1}\eta_\epsilon^{1.6} + 20300\eta_d^{2.3}\eta_\gamma^{1.75}\eta_\epsilon^{2.1},\end{aligned} \tag{1}$$

where $\eta_d = L_{1D}/2k\sigma_x^2$ is the diffraction parameter, $\eta_\gamma = 2\sigma_\gamma k_u L_{1D}$ is the energy spread parameter for a gaussian distribution, and $\eta_\epsilon = 2k\epsilon_x k_\beta L_{1D}$ is the emittance parameter. An analogous fitting formula for the OAM gain length in a space charge dominated beam is given in Ref [29].

With the description of the FEL as an expansion of OAM modes it became relatively straightforward to calculate the requirements for an OAM mode to be amplified. One striking result was that, even in the presence of averaged betatron motion and energy spread for a round e-beam, the OAM modes in the FEL are orthogonal. Modes of a given l do not couple to each other unless the cylindrical symmetry is broken. This is related to the fact that the spatial structure of the bunching in the e-beam during amplification corresponds with the phase structure of the emitted light. As such, the bunching factor for OAM light should be written in a modified form to incorporate the helical phase,

$$b_{l_b}(k) = \frac{1}{N}\sum_{j=1}^{N} e^{iks_j - il_b\phi_j}. \tag{2}$$

When coupled to a radiation field of the form $e^{iks-il\phi}$, there is a direct relationship between the l_b mode in the e-beam and the l mode in the field at the fundamental lasing frequency,

$$l_b = l. \tag{3}$$

This means that, in an ideal system, an OAM mode can be amplified independent from other modes in the system and they will not cascade down to the fundamental $l = 0$ mode because they are tied to the helical e-beam distribution. Thus, an OAM mode with a sufficiently large head start in the amplification process will dominate up to saturation. But this also means that it has to be externally seeded in order to dominate over the SASE-driven fundamental mode that has the highest gain. This isn't necessarily a problem because there are numerous ways in which OAM modes can be produced to act as an EM seed. Mode conversions have been performed with conventional lasers by shaping the laser pulse front with dedicated optics [31–33]. But such methods are not optimal from the standpoint of harnessing the extreme wavelength tunability of the FEL down to x-ray wavelengths, where can also be difficult to obtain a sufficiently intense x-ray OAM seed.

FEL TAILORING FOR OAM

Without an external EM seed that carries OAM to jumpstart the FEL, one is left to rely on two possible options, vis-a-vis Figure 1. One is beam shaping, the other is radiation shaping. For beam shaping to work, the OAM seed comes from the electron beam itself. Because of the helical phase structure of the OAM modes, this option requires a precise helical manipulation of the e-beam structure at the level of the lasing wavelength as given by the bunching in Eq. 2. The e-beam must be helically bunched so that the EM emission in the undulator has the helical phase structure of the desired OAM mode, thereby initiating the FEL process [25]. But how does one make a spiral electron beam density distribution at the lasing wavelength?

A clue came from a 2008 paper by Sasaki and McNulty [34] where they showed that the phase structure of harmonic radiation from helically polarized undulators carries an azimuthal component that is characteristic of OAM light. It had in fact been known for some time that all harmonics $h > 1$ from helical undulators have an off-axis annular intensity profile (see e.g., [35]), which is characteristic of OAM modes. This profile structure was confirmed for coherent emission by Allaria et al [36]. That the associated phase was helical, however, had apparently gone previously unnoticed. Only recently was the predicted helical phase structure confirmed in experiments on second harmonic incoherent undulator radiation from a 3rd generation synchrotron light source [37].

The intimate connection between the helical phase and the harmonic radiation fields thus provided a beam-shaping mechanism to tailor the FEL for OAM amplification at the fundamental lasing frequency. A subsequent revisiting of the spatial coupling between the e-beam and the resonant undulator harmonics suggested that the correspondence between the helical bunching mode and the radiation l modes should be modified [38],

$$l_b = l \pm (h - 1), \tag{4}$$

where the + or − sign indicates the right or left handedness of the helical undulator field. From the harmonic interaction it was shown that there is a way to couple azimuthal modes in the e-beam to different azimuthal modes in the fields. This feature essentially provided a solution to the problem of jumpstarting the FEL to emit an OAM mode without an OAM EM seed, namely, through harmonic coupling.

From these realizations came two different methods by which coherent OAM light can be produced in an FEL, illustrated in Figure 3. Though they appear similar in layout, the first method A) is essentially a beam shaping method whereas B) is a radiation shaping method. The interaction in each undulator is effectively governed by the coupling in Eq. 4, although the bunching is not revealed until the beam exits the chicane. In the beam shaping case A), the electron beam is energy modulated by the simple $l = 0$ laser (e.g, a Gaussian profile) at the second harmonic resonance of the helical undulator, assumed here to be right handed. In other words, the resonant frequency of the helical undulator is $\omega_{h,r} = \omega_L/2$ where ω_L is the laser frequency. This generates a spatially dependent energy modulation that, after passage through the simple chicane produces $l_b = 1$ helical bunching at the frequency ω_L. The bunched beam

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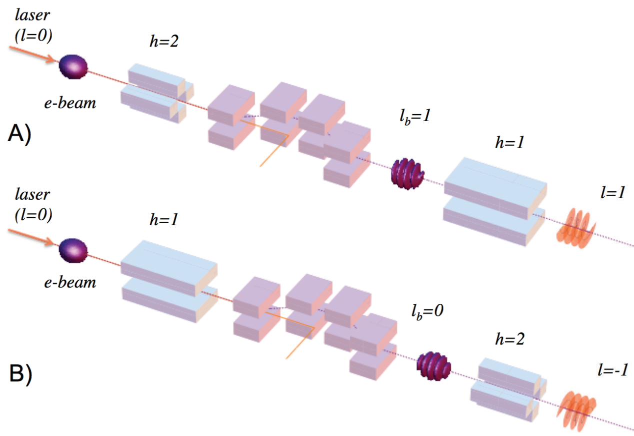

Figure 3: Two different OAM production methods in FELs. A) Beam shaping with helically microbunched e-beam. B) Radiation shaping with regularly microbunched e-beam radiating at 2nd harmonic.

then enters an FEL planar undulator tuned so that it's fundamental frequency is the same as that of the laser, $\omega_{p,r} = \omega_L$. Because the beam is also bunched at the fundamental frequency, the FEL lases and produces an $l = 1$ OAM mode at the frequency ω_L, which matches the $l_b = 1$ bunching (Eq. 3). The system effectively acts like a mode convertor and amplifier, where the initial $l = 0$ laser mode is converted to an $l = 1$ OAM mode at the same frequency.

In the radiation shaping method shown in B), the helical and planar undulators are simply swapped. No other tuning is changed, but the result is different. In this case, the $l = 0$ laser modulates the beam in a simple fashion, again with $\omega_{p,r} = \omega_L$. The resulting bunching at the chicane exit is of the ordinary sort and has no azimuthal dependence, so $l_b = 0$. In the helical undulator downstream, the beam, which is again bunched at ω_L, radiates at the second harmonic because $\omega_{h,r} = \omega_L/2$. By Eq. 4, this generates $l = -1$ OAM light at the frequency ω_L. Note that the OAM mode has reversed sign between case A) and case B), while the frequency of the light stays the same. This 'radiation shaping' scheme also acts like a mode convertor of the $l = 0$ laser, but in this case the OAM light is generated by exploiting the spatial features of the harmonic radiation rather than the beam distribution.

EXPERIMENTS

Several experiments designed to investigate the different OAM generation techniques followed. The first, performed at the Neptune Laboratory at UCLA in 2011 called HELiX [39], was a test of the helical microbunching concept proposed in [38] required by the beam shaping method of OAM generation. The setup of the experiment is shown in Figure 4. The 12-12.5 MeV electron beam was modulated by a 10.6 μm wavelength CO_2 laser in a short helical undulator, which was tuned to be resonant at 21.2 μm. The e-beam and the laser thus interacted at the second harmonic resonance, which generated a single twist helical energy modulation.

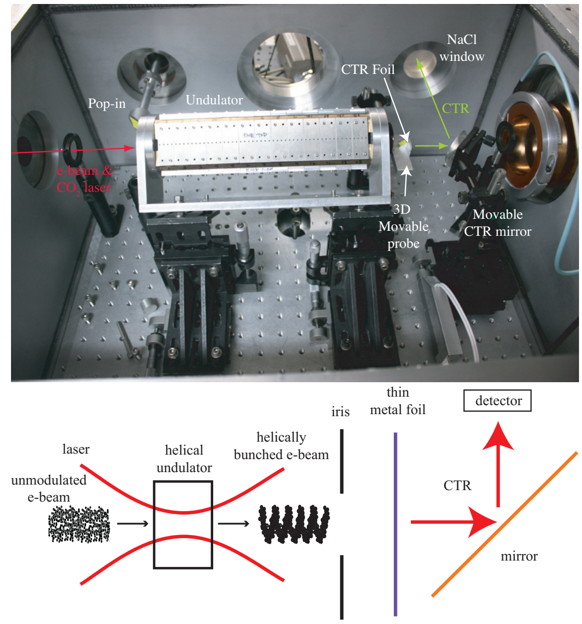

Figure 4: Picture (above) and cartoon (below) of the HELiX helical microbunching experiment at UCLA.

At these low e-beam energies, the longitudinal dispersion through the undulator was sufficient to turn the energy modulation into a helical density modulation, so no chicane was needed. The beam then hit a thin aluminum foil and emitted coherent transition radiation (CTR) with an imbedded helical phase. The integrated CTR signal intensity was measured and matched well with theoretical expectations for the CTR energy of a helically modulated beam [40], but direct experimental evidence for the helical phase was lacking.

A direct measurement of the helical phase in the radiation emitted by a helically microbunched beam was performed at the SLAC Next Linear Collider Test Accelerator (NLCTA) in 2013 [41]. The layout was identical to the beam shaping scheme in Fig. 3. Piggybacking on the infrastructure used for the echo enabled harmonic generation (EEHG) program [42–45], in this experiment, a helical undulator built by Andrey Knyazik at UCLA was used to modulate the 120MeV e-beam through the second harmonic interaction with an 800 nm laser [46]. Helical bunching was produced as the beam transited the chicane, and the coherent OAM light emitted in the following planar undulator was sent through a beam splitter and captured with two cameras set to image different focal planes. This enabled the phase to be determined by an iterative phase reconstruction algorithm from the measured intensities. Results showed that 85% of the radiation power was contained in the $l = 1$ azimuthal mode. Further direct evidence of the helical phase structure was obtained from images of the far field diffraction pattern of the light as it passed through a narrow slit, which showed clear evidence of a π phase difference across the beam and a shearing pattern that is the hallmark of traverse energy flow.

Following the beam shaping OAM experiment, the radiation shaping technique was then also tested at NLCTA by

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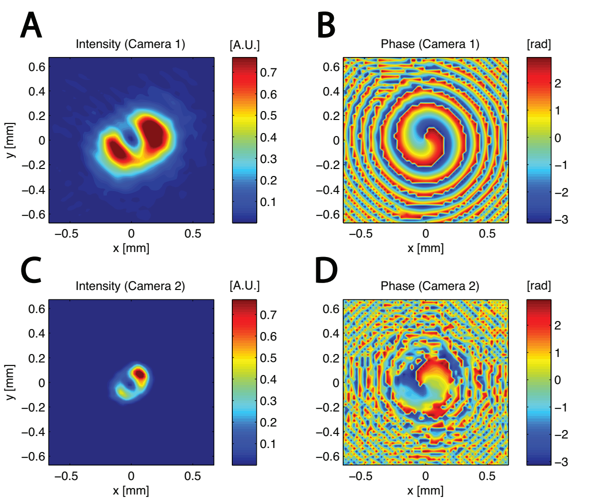

Figure 5: From [41]. Measured undulator radiation intensities (left) and reconstructed $l = 1$ OAM phases (right) from two cameras positioned to view the undulator radiation profiles at different planes. Intensity (A) and phase (B) at camera 1 are shown, and correspond to the intensity (C) and phase (D) at camera 2.

exchanging the positions of the helical and planar undulators [47]. Again, the phase reconstruction algorithm showed the presence of a helical phase in the coherent harmonic emission from the helical undulator, only this time with the opposite $l = -1$ handedness, as predicted by Eq. 4. Over 91% of the mode power was in this mode. The observed pattern of transverse interference between the coherent undulator radiation (CUR) and the CTR from the ejection screen also confirmed the helical phase via the presence of a forked pattern that is the signature of a phase singularity (See Figure 6).

Together, these two experiments confirmed the basic physics of in situ OAM light production in FELs using the e-beam as the mode conversion medium. In the beam shaping experiment, the beam radiates OAM light at the fundamental frequency of the undulator. This has the advantage that in principle the FEL can lase up to saturation to produce OAM light up to the GW level. A drawback of this technique is that the helical e-beam distribution is sensitive to asymmetries or transport effects that can wash out the carefully prepared structure and reduce the purity of the radiated OAM mode. In contrast, the radiation shaping technique, which relies primarily on the radiation emission profile of the helical undulator, is much less sensitive to these effects because the e-beam bunching does not have a helical structure. The result is higher mode purity, but because the harmonic emission is less intense than the fundamental, this scheme may produce less peak power. Even so, this technique is simpler, likely more robust in practice, and could be used in a simple FEL afterburner arrangement [48] where a helical undulator is placed downstream of an FEL to radiate coherent OAM light from the spent beam. This opens up new possibilities for pump-probe experiments with two pulses that carry different values of OAM, i.e, one $l = 0$ mode from the upstream FEL and one with $l \neq 0$ from the helical afterburner.

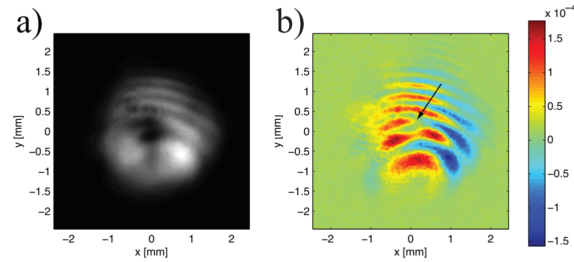

Figure 6: From Ref. [47]. Interference between CUR and off-axis CTR reveals the signature forked pattern (black arrow, right image) of a $l = -1$ vortex from the second-harmonic undulator emission. Image b) is the difference between image a) and an image taken with the e-beam is steered to remove the CTR interference (not shown).

FUTURE POSSIBILITES

Building on these concepts, new and more advanced schemes have been proposed. In a scheme dubbed Echo-v for vortex [49], the beam shaping method is integrated with an EEHG arrangement. In contrast to standard EEHG where two pairs of laser modulators and chicanes are used to generate high harmonic bunching [50, 51], in Echo-v, one or both of the modulators also imprints a helical energy modulation. Through the EEHG process the final helical bunching in the beam transforms just like the harmonics as $l_b = nl_1 + ml_2$ where n and m are the frequency harmonics of each laser and l_1 and l_2 are the helical modulations imprinted by the first and second lasers, respectively. With different combinations of l_1 and l_2, both the frequency and l_b can be up-converted simultaneously or independently to produce high harmonics at soft x-rays with either a large or small output l. Another scheme suggests the use of a simple transverse mask to seed the amplification of tunable OAM in a soft x-ray HGHG FEL [52]. The concept of OAM has also been applied directly to coherent electron beams where the quantum mechanical electron wave functions have a helical phase [53], as distinct from the classical helical bunching described here. This has become a rapidly advancing field of study in its own right.

CONCLUSION

It has become clear that FELs are extraordinarily powerful machines for discovery. They are highly flexible in that the character of the light they produce can in many ways be precisely tailored to suit an ever growing set of needs. The landscape of 'beam by design' techniques used previously to tailor the temporal profile of the light can now begin to include transverse tailoring schemes that open exciting new regimes of study in accelerator and photon science.

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