FEL oscillators: Mode Locking, short Pulses and Harmonic Generation

G. Dattoli

On Behalf of

P. L. Ottaviani, S. Pagnutti, E. Sabia, V. Petrillo, P. Van der Slot,
S. Biedron, S. Milton

- Art work from

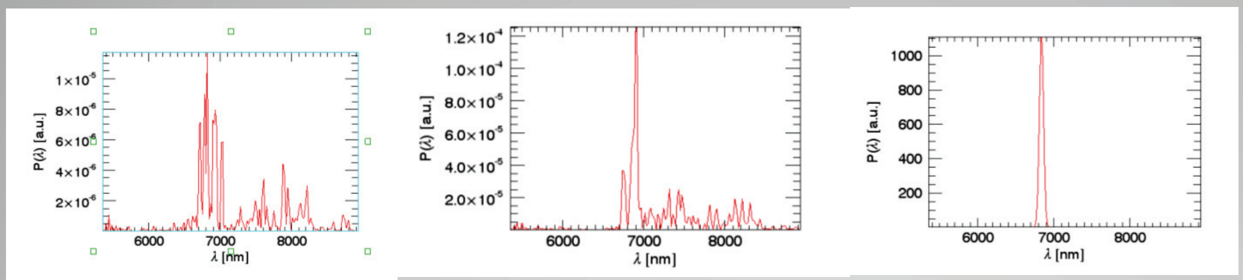
- N. Thompson Introduction to FEL

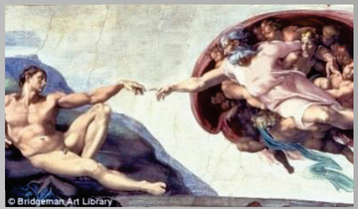
  D. Nguyen: US Particle Accelerator School (2009)

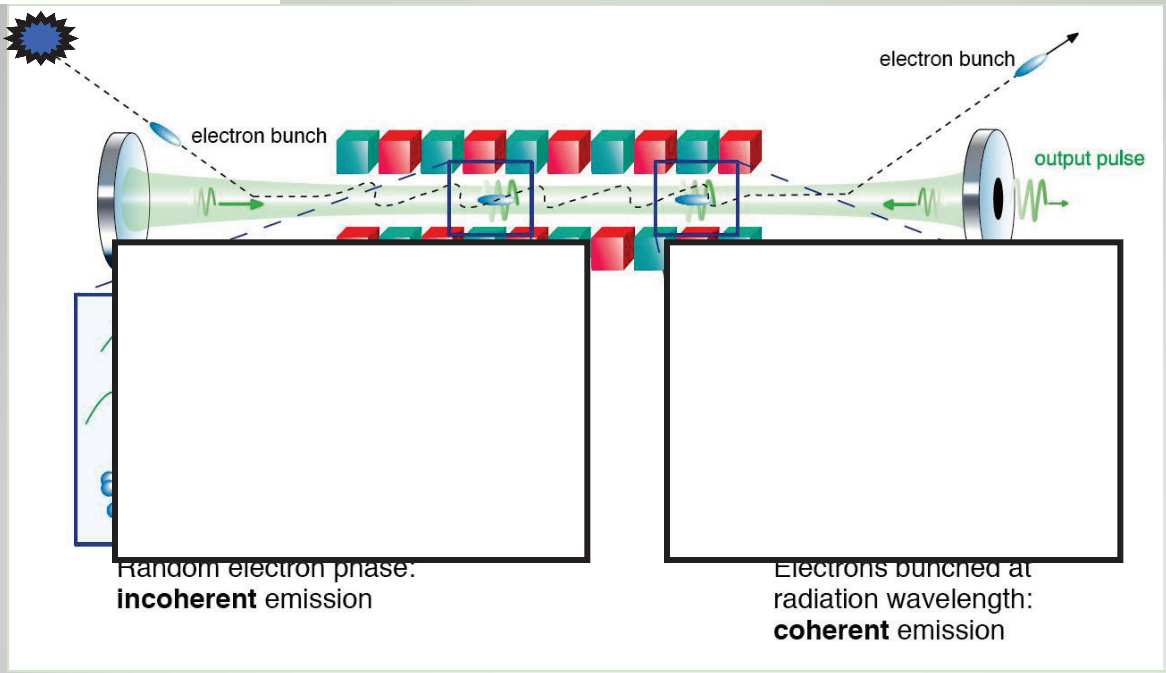
  G. Dattoli, M. Del Franco, M. Labat, P. L. Ottaviani and S. Pagnutti
- «Introduction to the Physics of Free Electron Laser and Comparison with conventional laser sources»

- Oscillator FEL’s (make any sense?)
- Pro’s
- Mode selection
- Coherence, no seeding…
- Power control, pulse shaping…
- Con’s
- Absence of optics at short wave-lengths
- Advanced solutions (Kim…)

How does a FEL oscillator work? (Genesis Simulation (V. Petrillo))







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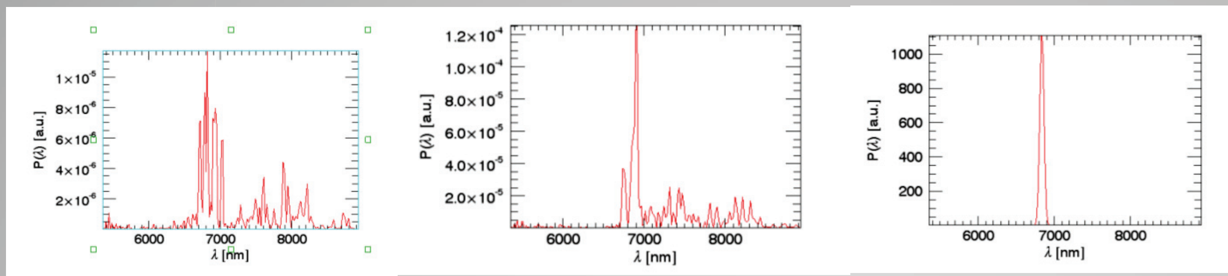

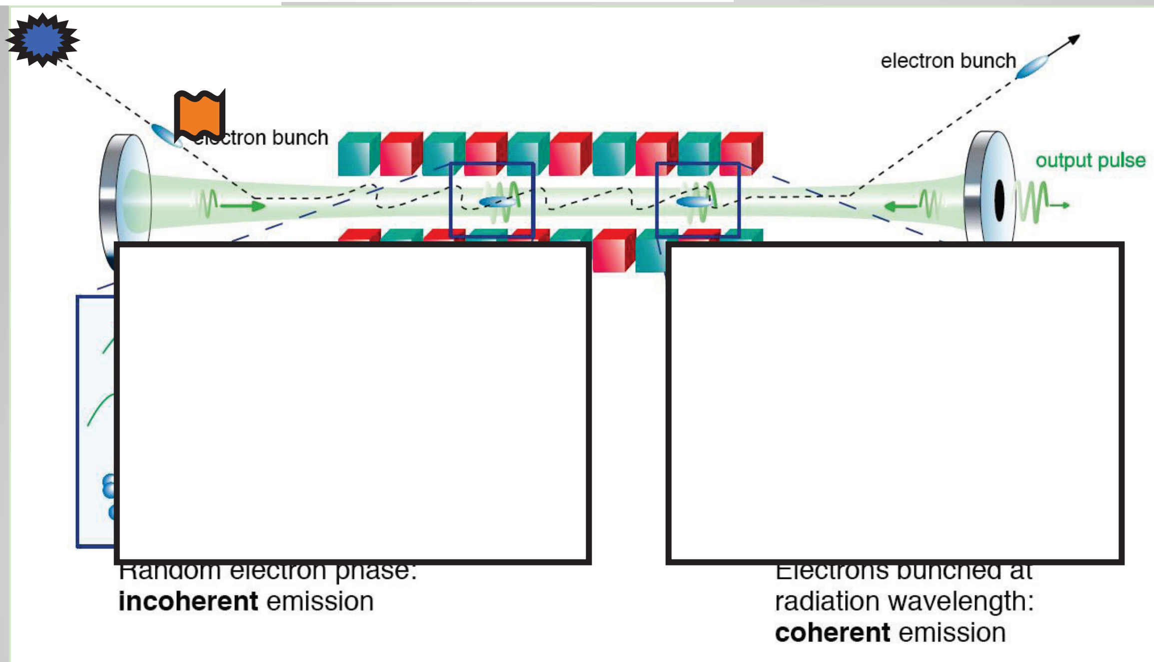

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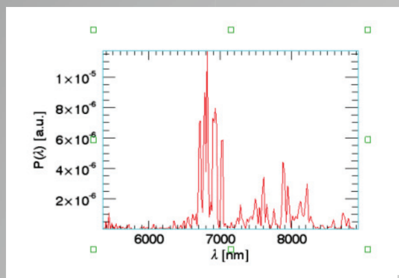

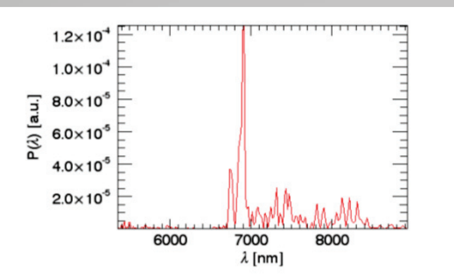

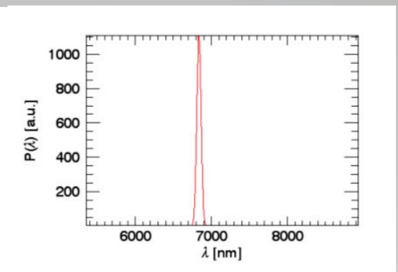

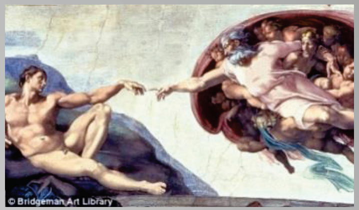

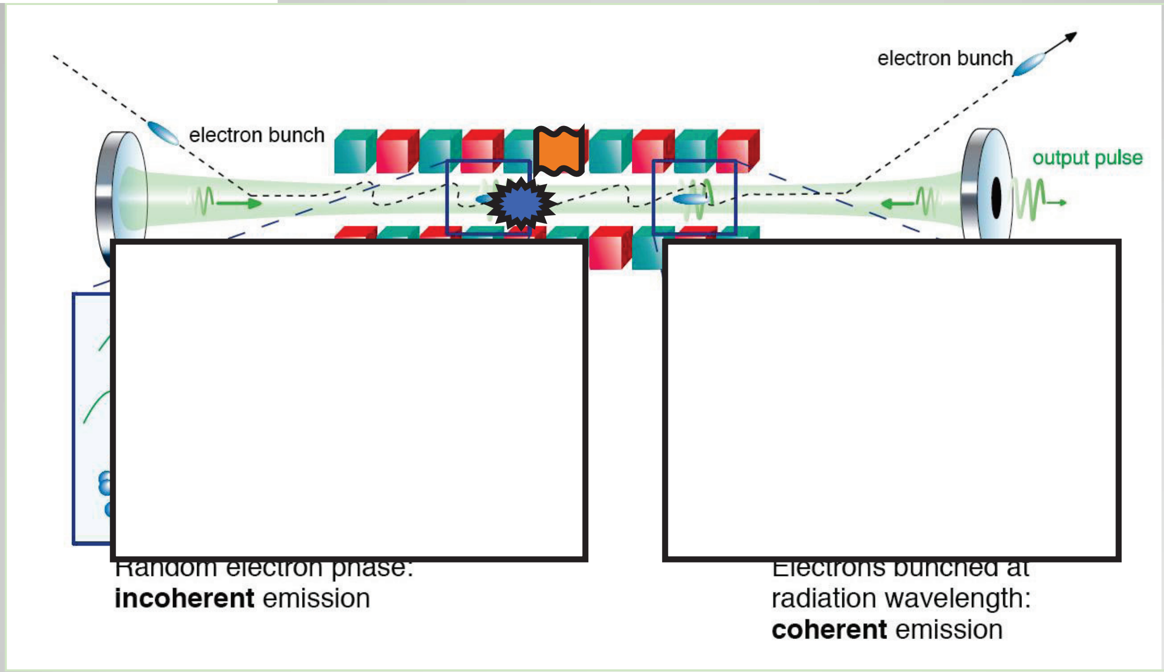

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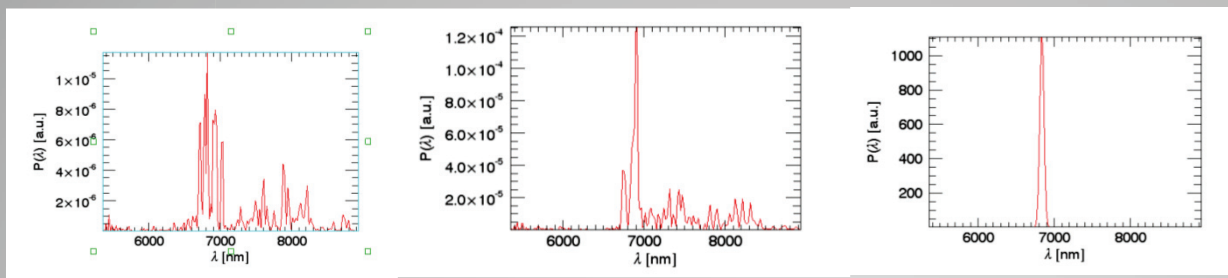

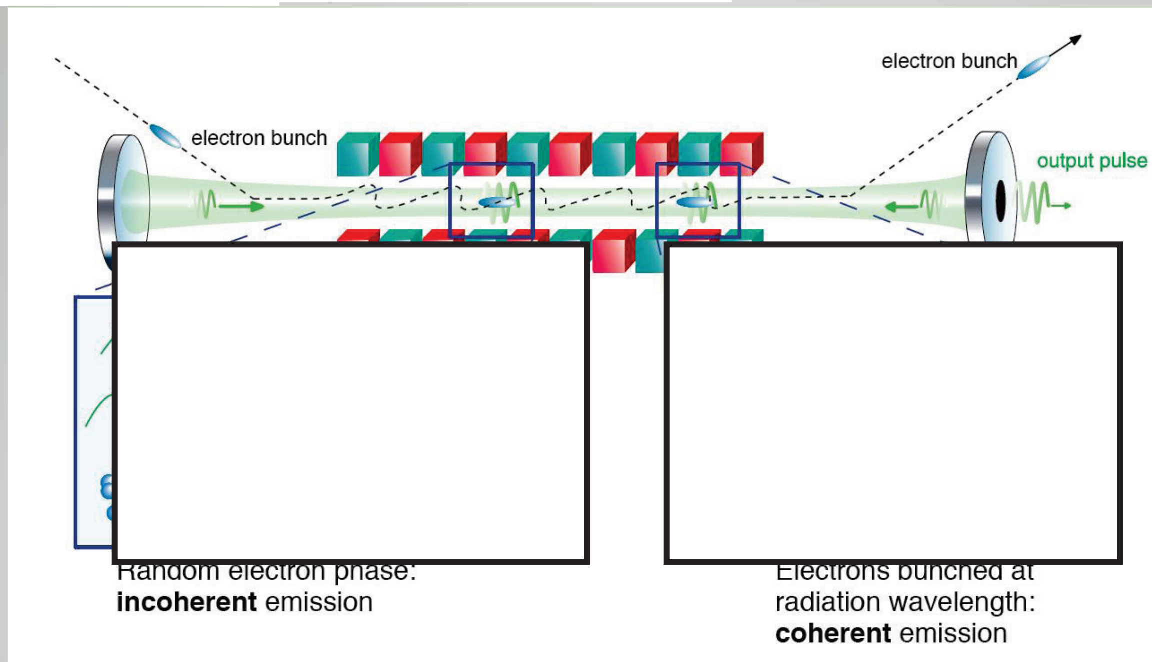

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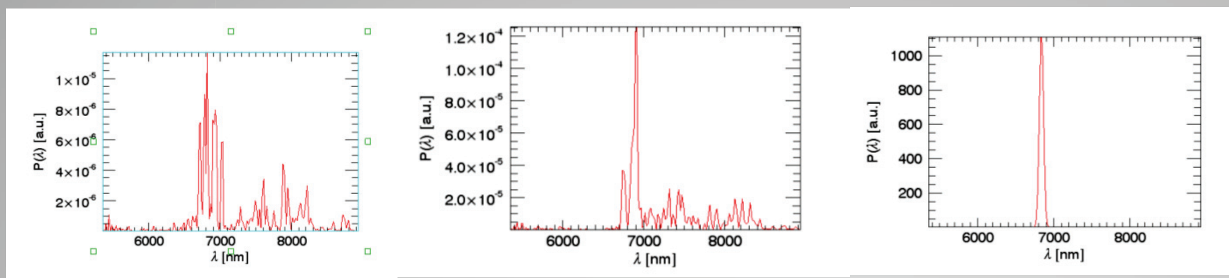
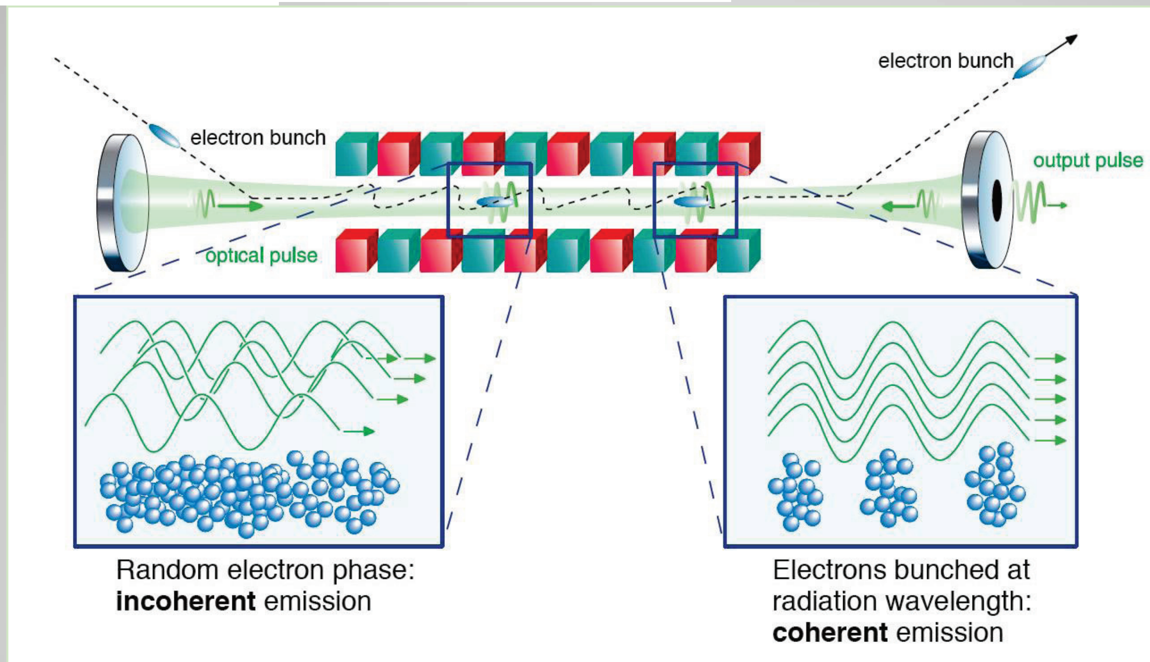




Intracavity growth

Intracavity Intensity builds up from noise to saturation in «n» passes and its growth can be modelled through the equation

$$I_{r+1} = I_0 \frac{\left[(1-\eta)(G_M+1)\right]^r}{1+\frac{I_0}{I_e}\left\{\left[(1-\eta)(G_M+1)\right]^r-1\right\}},$$

$$I_e = \left(\sqrt{2}+1\right)\left(\sqrt{\frac{1-\eta}{\eta}G_M}-1\right)I_S,$$

$\eta \equiv$ Cavity Losses (active & Passive),

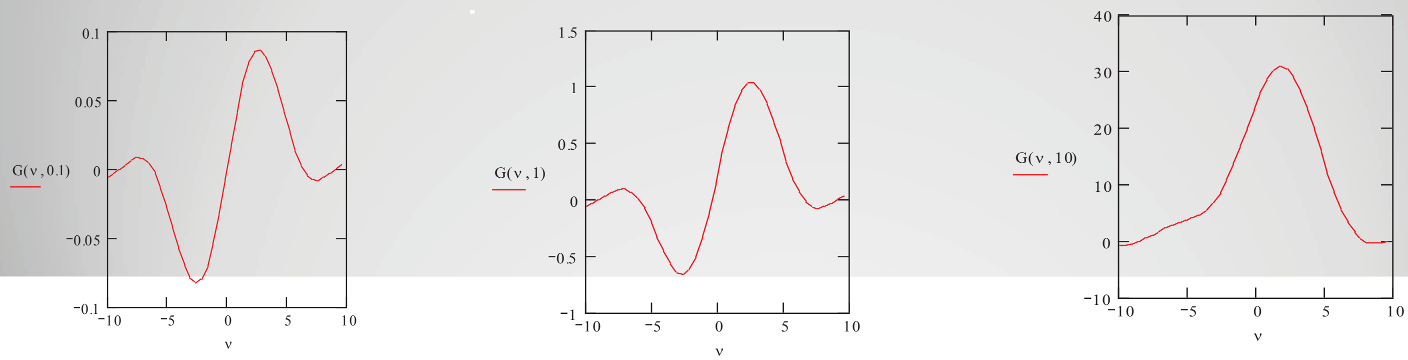
$G_M \equiv$ Maximum Gain,

$r \equiv$ Round Trip number

key parameters:
Small signal gain coefficient and saturation Intensity

- The maximum gain G_M
- Is a Function of many parameters, but primarely of the small signal gain coefficient
- $$g_0 = \frac{16\pi}{\gamma}\frac{|J|}{I_0}\lambda_0 L N^2 \xi f_b^{\,2} = 2[2\pi N \rho]^3$$

$$G_M = G(g_0) \cong 0.85 g_0 + 0.192 g_0^2 + 4.23 \cdot 10^{-3} g_0^3$$



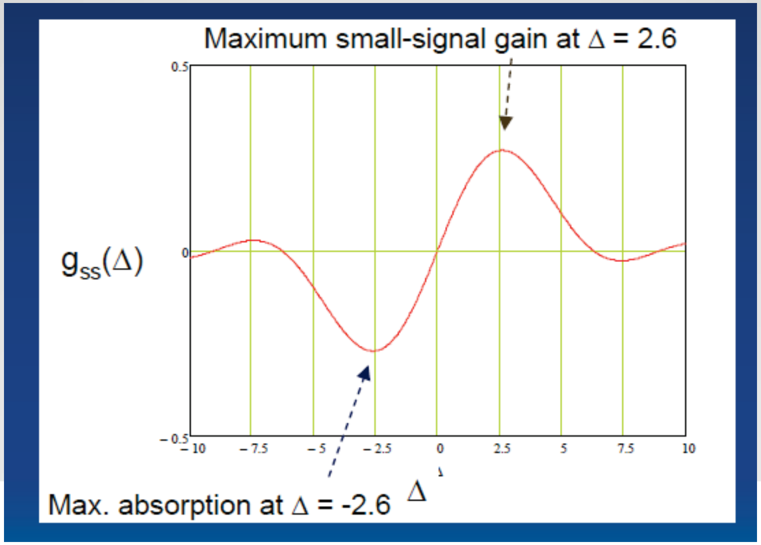
...small signal Gain Coefficient, Saturation Intensity and efficiency

$$P_E = E \cdot \frac{|J|}{e} = \frac{m_e c^2}{e}\gamma |J| = 2N g_0 I_S \qquad e-beam\ power\ density$$

$$I_S\left[\frac{MW}{cm^2}\right] = 6.9312 \cdot 10^2 \frac{1}{2}\left(\frac{\gamma}{N}\right)^4 \left(\lambda_u[cm]K^* f_b\right)^{-2}$$

$$P_E\left[\frac{MW}{cm^2}\right] = E[Mev] \cdot 10^{-4} |J|\left[\frac{A}{m^2}\right] = 0.5109989 \cdot 10^{-4}\ \gamma\ |J|\left[\frac{A}{m^2}\right]$$

$$g_0 I_S = 2\varepsilon P_E \rightarrow \varepsilon = \frac{1}{4N}$$



- The Saturation process
- Reduction of the small signal gain due to the induced energy spread

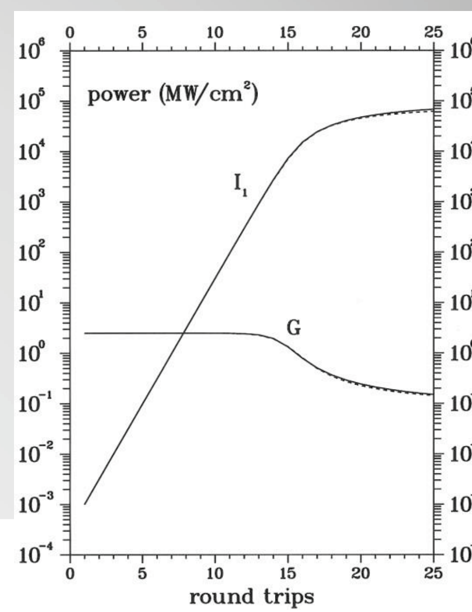
$$\sigma_i(X_r) \cong \frac{0.433}{N} \exp\left[-0.25(\beta X_r) + 0.01(\beta X_r)^2\right] \sqrt{\frac{\beta X_r}{1-e^{-\beta X_r}} - 1}$$

$$X_r \leq 10,\ X = \frac{I}{I_S}$$

$$\sigma_\varepsilon(X_r) = \left[\sigma_{\varepsilon,0}^2 + \sigma_i^2(X_r)\right]^{1/2} \qquad \textit{total energy spread},$$

$$G(X_r) = \frac{\eta}{1-\eta} \Rightarrow r^* \cong \frac{\ln(0.1 I_e/I_0)}{\ln[(1-\eta) G_M + 1]}$$

$$E = 108.24\ Mev, P_E = 2.012 \cdot 10^6\ MW/cm^2, \lambda_u = 2.8\,cm, K = 2.1, N = 50,$$
$$g_0 = 0.2, I_S = 8.08 \cdot 10^3\ MW/cm^2, \sigma_\varepsilon(0) = 10^{-4}, \lambda_0 = 1.008\ \mu m$$



That's not all

Intracavity Harmonic generation while bunching evolves

$$I_{n,r+1} = \Pi_{n,0} \frac{\left[(1-\eta)(1+G_M)\right]^{nr}}{1+\frac{\Pi_{n,0}}{I_n^*}\left\{\left[(1-\eta)(1+G)\right]^{nr}-1\right\}},$$

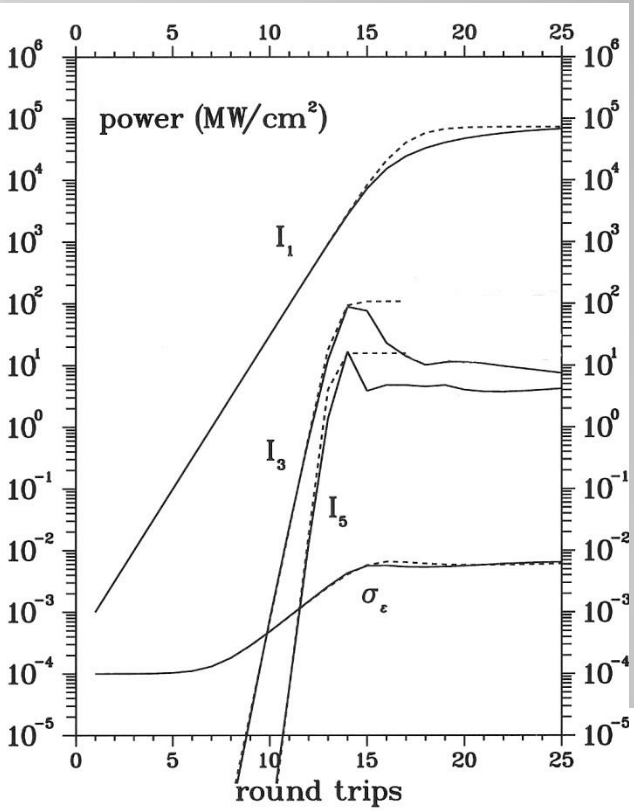
$$\Pi_{n,0} = (n-1)!\,(n-2))\sqrt{\frac{n-1}{2}}\,g_{0,n}\,\frac{P_E}{2N}\left(\frac{I_0}{I_S}\right)^n,$$

$$P_n^* = \frac{1}{4}\frac{\sqrt{n}}{n^3}\sqrt{\frac{n-1}{2}}\,g_{0,n}\,\frac{P_E}{2N}$$

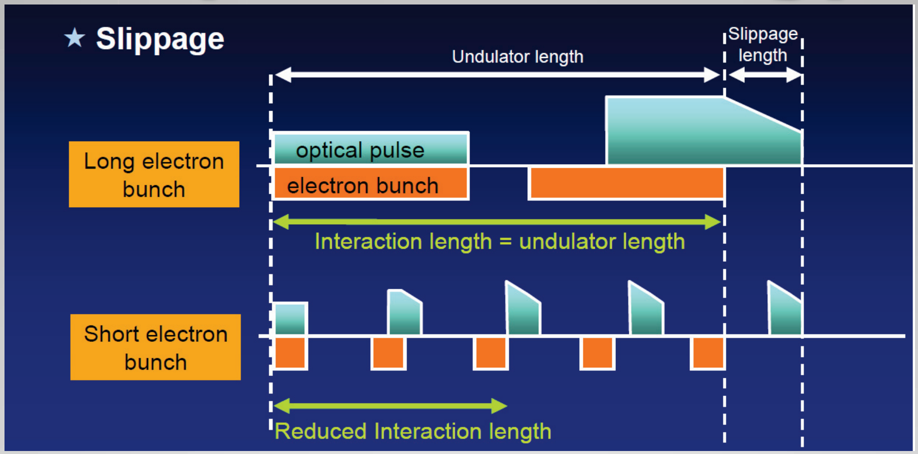
$n = 3, 5, 7$ *harmonic number*

$\Pi_{n,0}$ *harmonic seed power*

P_n^* max*imum harmonic power*



Short Pulses phenomenology

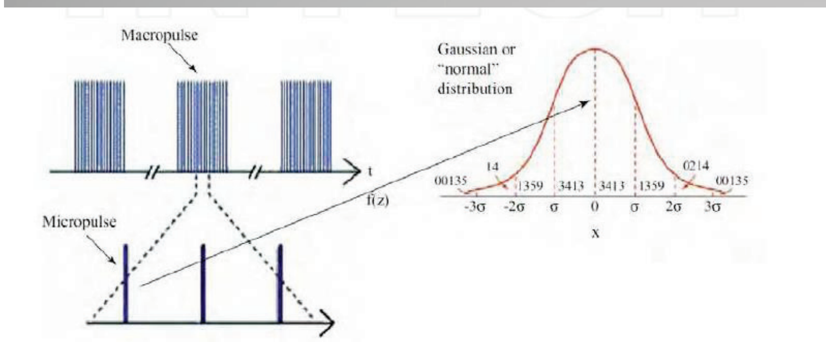


$$\Delta = (1-\beta_z)cT \cong N\lambda_0 \equiv \text{Slippage Length}$$

$$\sigma_z \equiv \text{Bunch Length}$$

$$\mu_c = \frac{\Delta}{\sigma_z}$$

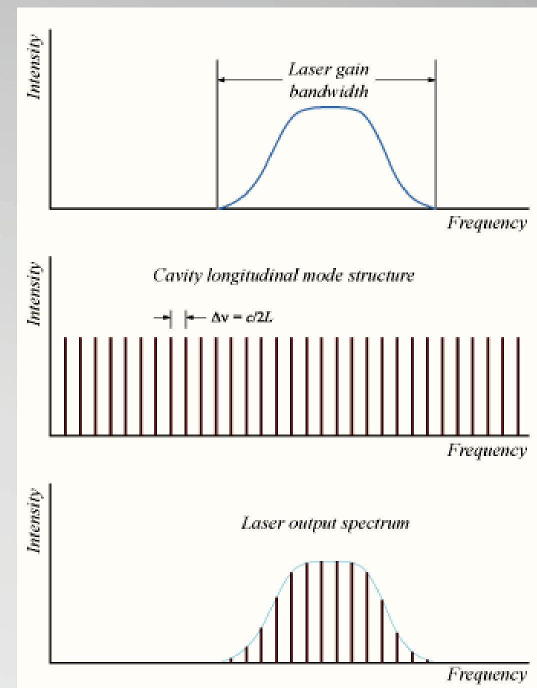
Cavity mode coupling in FEL oscillators operating with short electron pulses



$$\tilde{f} = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} f(z)\, e^{-i(k-k_0)z}\, dz \propto$$

$$\propto \int_{-\infty}^{+\infty} e^{-\frac{z^2}{2\sigma_z^2}}\, e^{-i\frac{\nu}{\Delta}z}\, dz \to \frac{1}{\sqrt{2\pi\mu_c}}\, e^{-\frac{\nu^2}{2\mu_c^2}},$$

$$\mu_c = \frac{\Delta}{\sigma_z}$$



Small Signal FEL Gain and short pulses

- Low gain regime

$$g(\nu) = \int_0^1 (1-t)\, t\, e^{-i\nu t} dt$$

$$\int_{-\infty}^{+\infty} e^{-\frac{z^2}{2\sigma_z^2}} e^{-i\frac{\nu}{\Delta} z} dz \rightarrow \frac{1}{\sqrt{2\pi\mu_c}} \int_{-\infty}^{+\infty} g(\nu - \nu_0)\, e^{-\frac{\nu_0^2}{2\mu_c^2}} d\nu_0$$

Lethargy: Slow down of the optical packet velocity

$$v_g = \frac{c}{n_g},\ n_g \equiv \text{group velocity refractive index,}$$

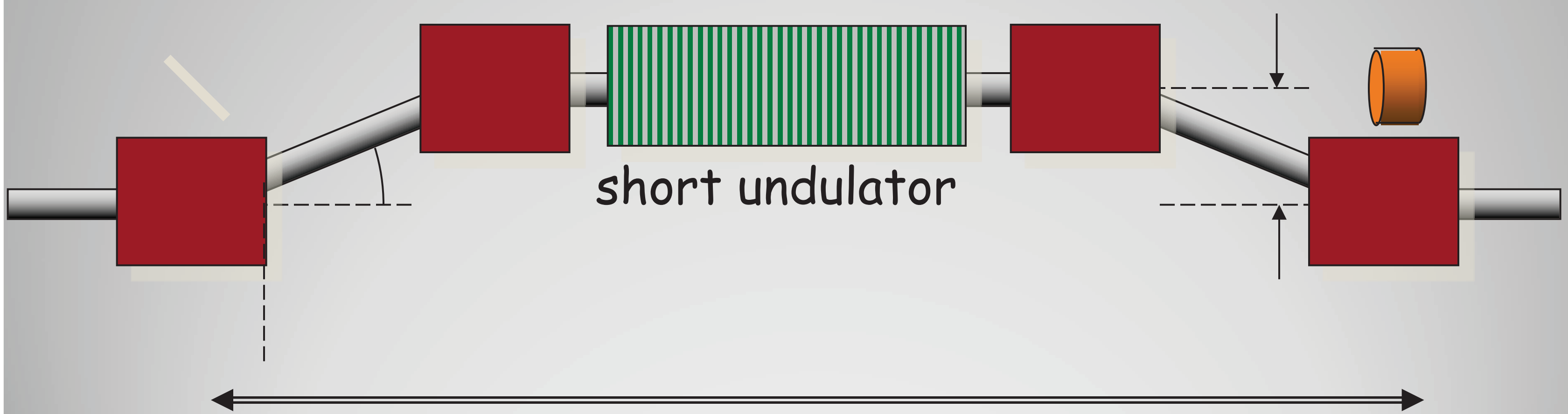
$$\delta L \qquad \textit{Cavity mismatch}\ (>0\ \textit{for cavity shortening}\ )$$

$$\theta = \frac{4\delta L}{g_0 \Delta} \qquad \textit{Cavity detuning parameter}$$

$$R = \Delta - 2\delta L \qquad \text{Delay,}$$

$$n_g = 2\frac{\delta L^*}{L} + 1$$

FEL-oscillator dynamics



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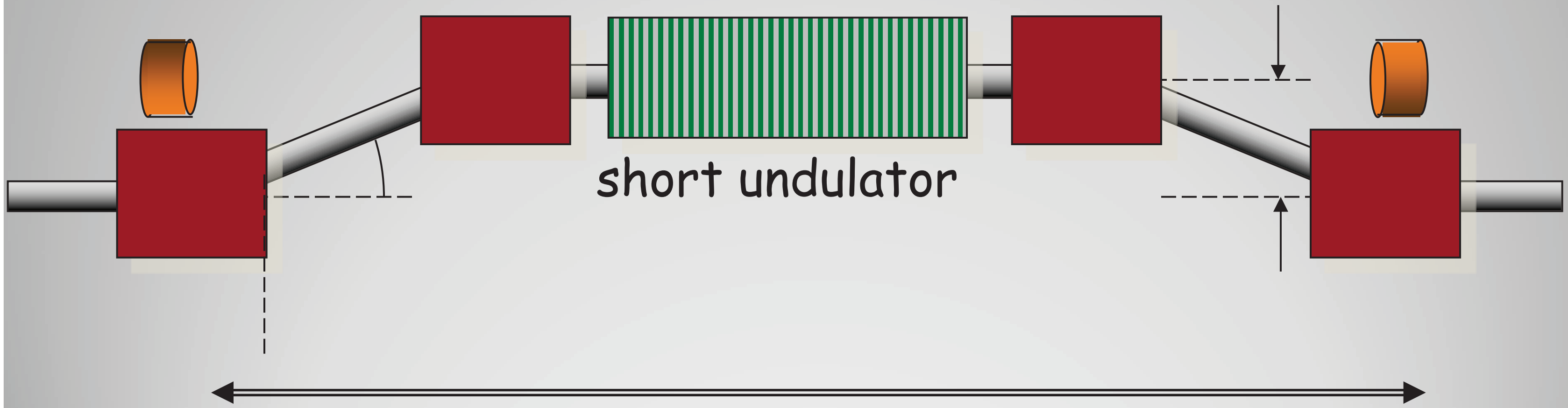

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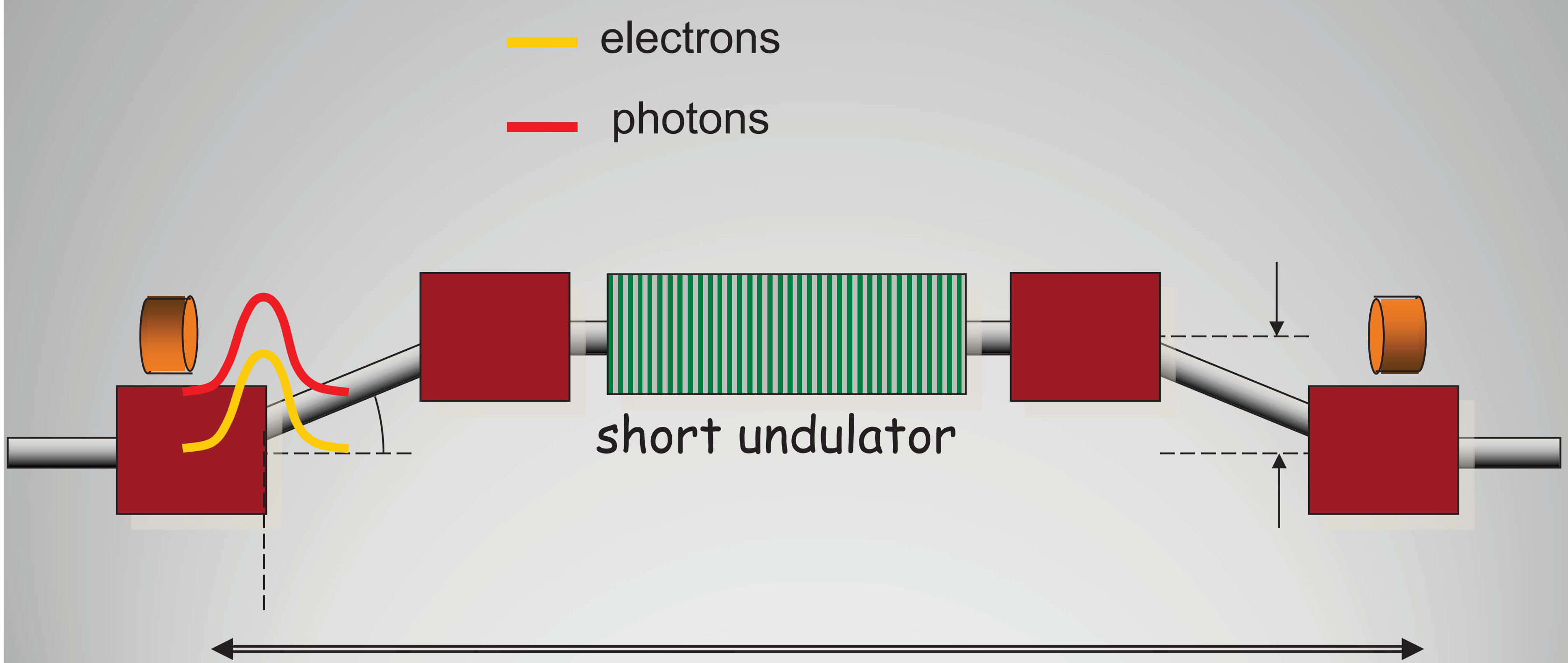

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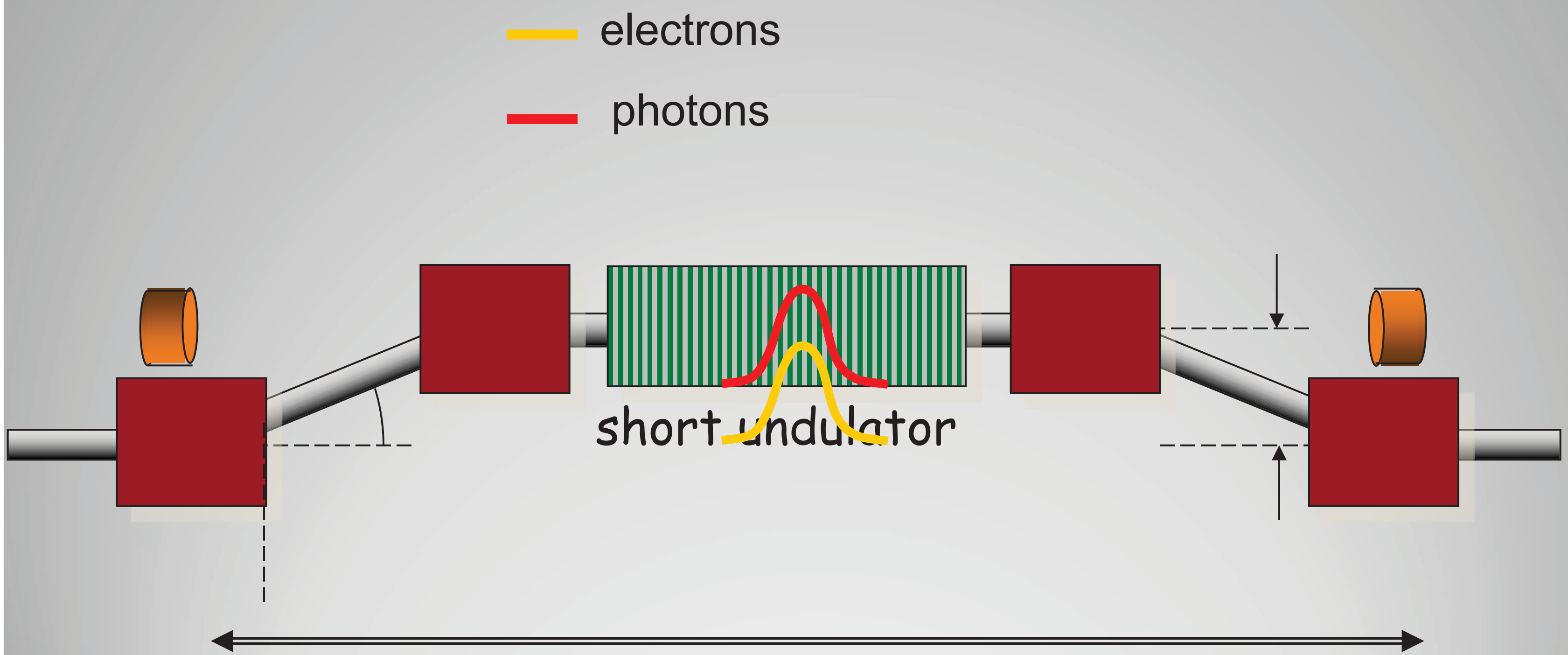

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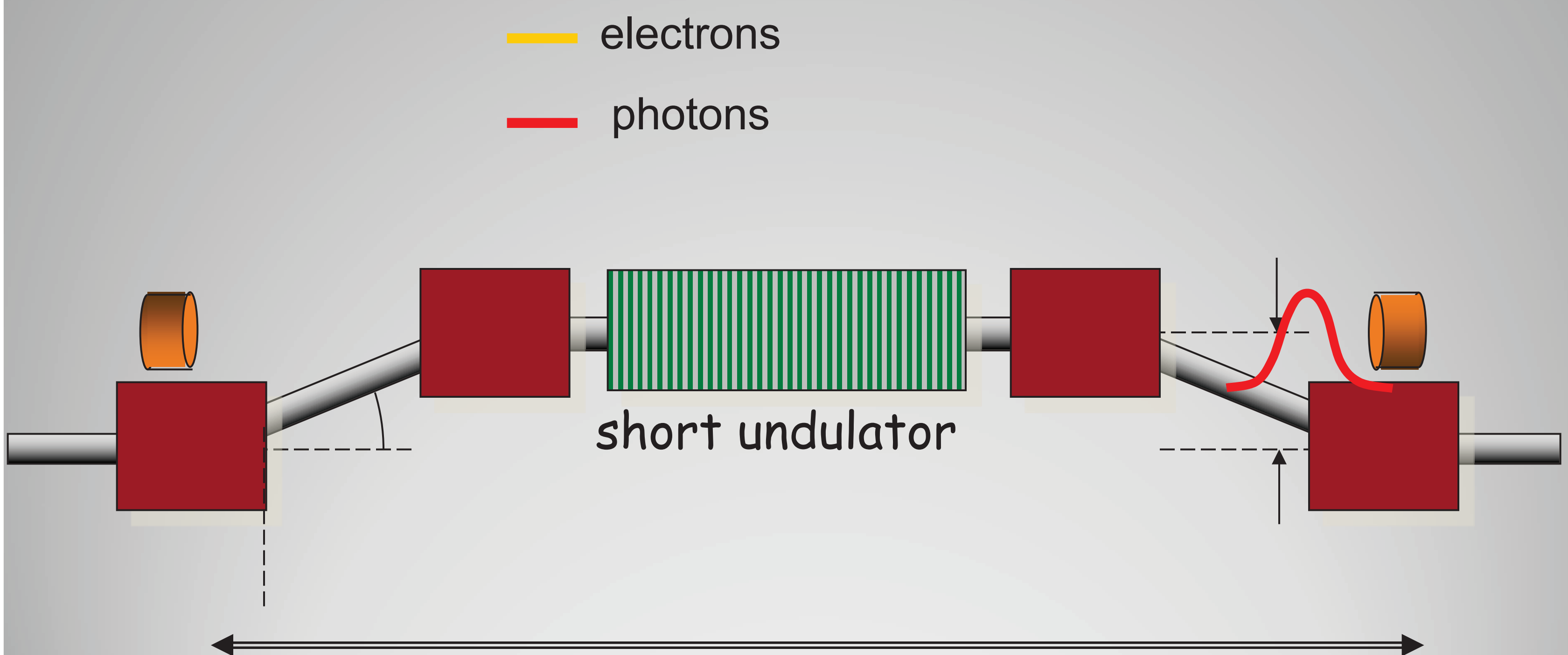

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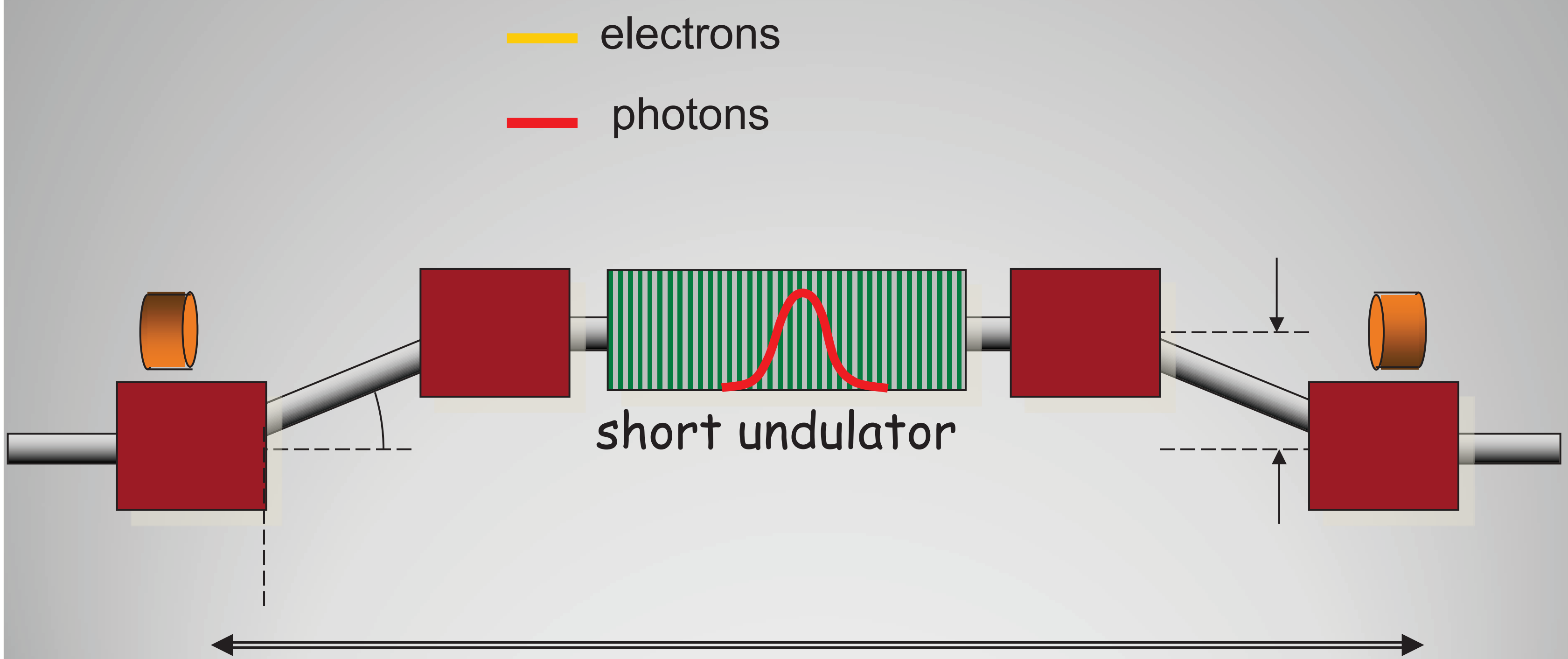

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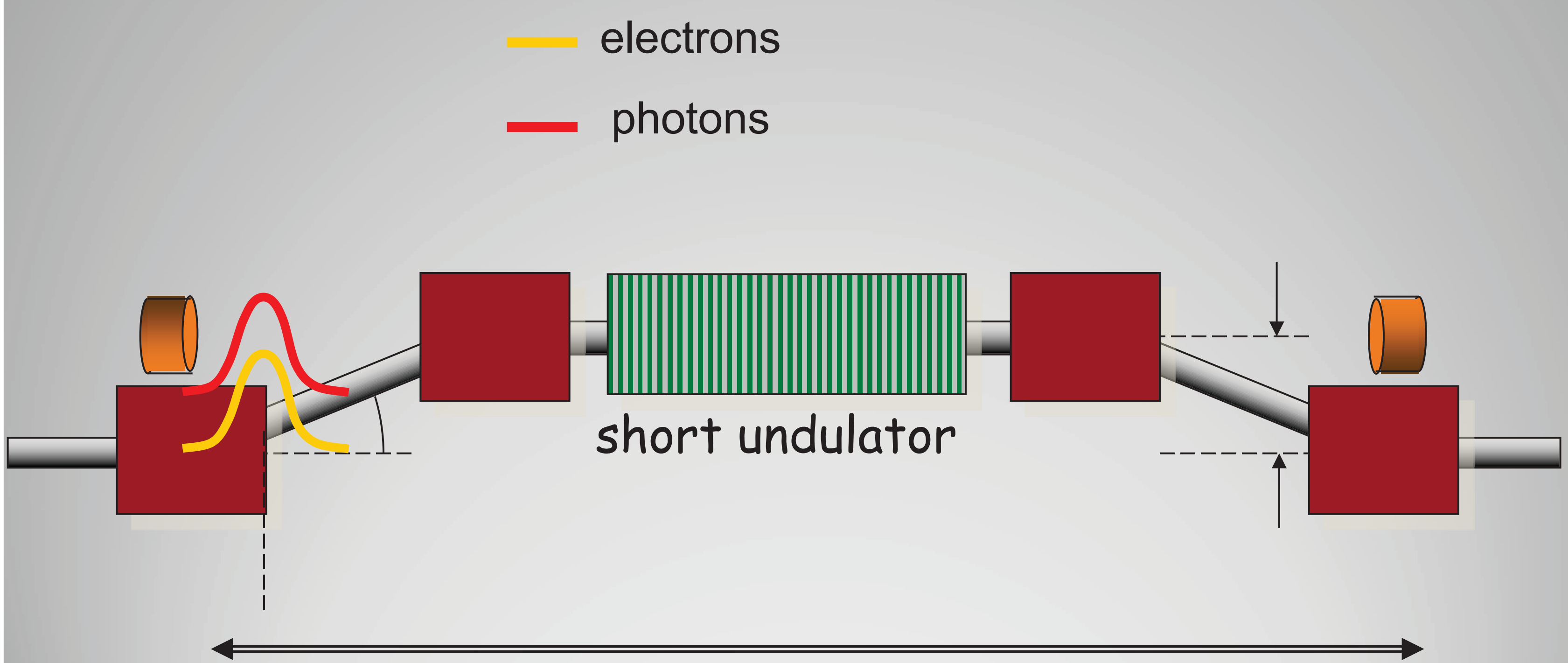

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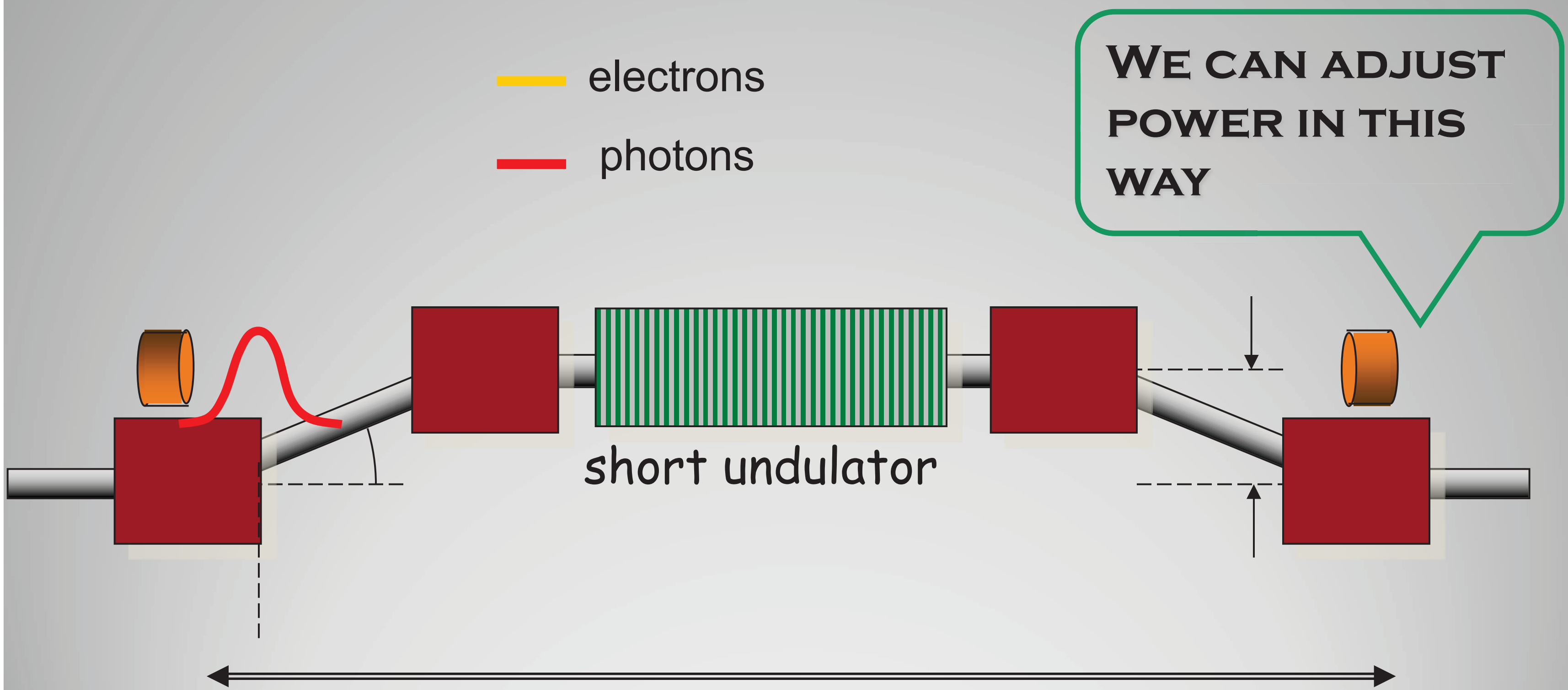

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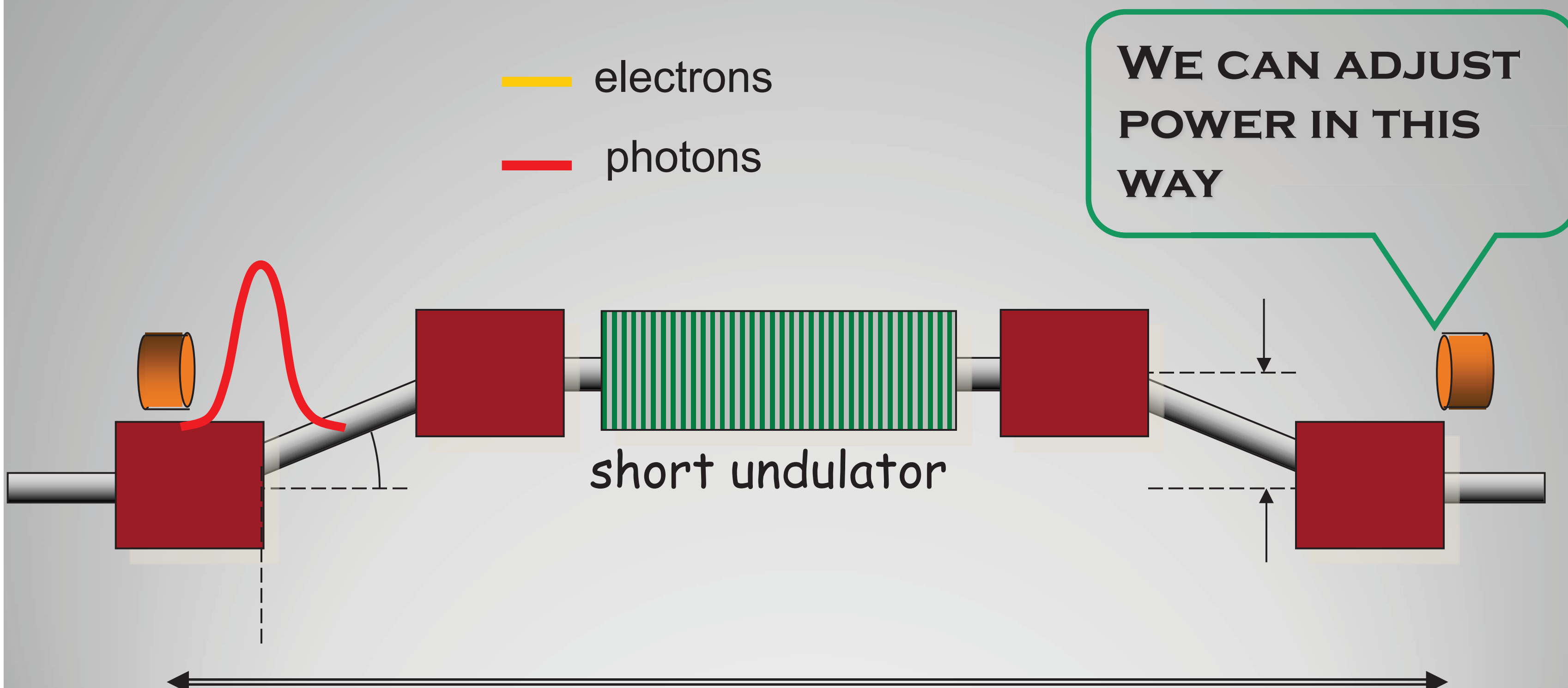


Gain and output power dependance!

Gain & Short Pulses

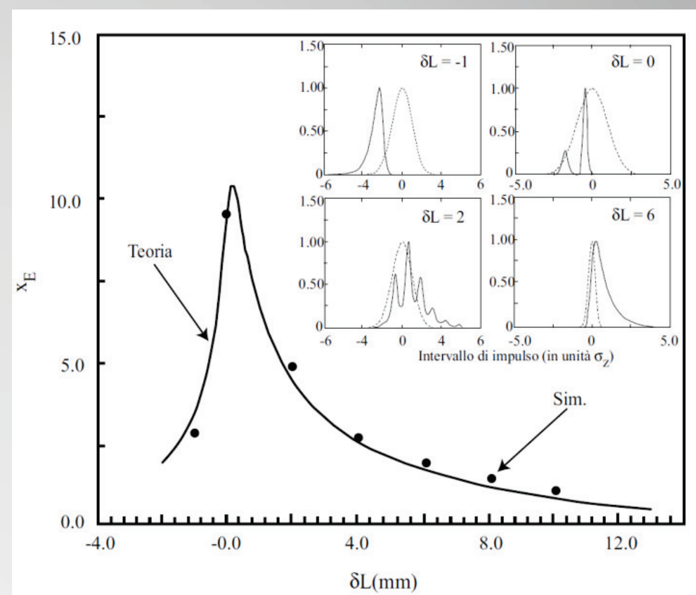
$$I_{r+1}(\theta,\mu_c) = I_0 \frac{\left[(1-\eta)\left(G(\theta,\mu_c)+1\right)\right]^r}{1+\frac{I_0}{I_e(\theta,\mu_c)}\left\{\left[(1-\eta)\left(G(\theta,\mu_c)+1\right)\right]^r-1\right\}},$$

$$I_e(\theta,\mu_c) \cong \left(\sqrt{2}+1\right)\left\{\sqrt{\frac{\theta^*}{\theta}}\exp\left[\frac{1}{2}\left(1-\frac{\eta}{(1-\eta)G^*}\frac{\theta^*}{\theta}\right)\right]-1\right\} I_s,$$

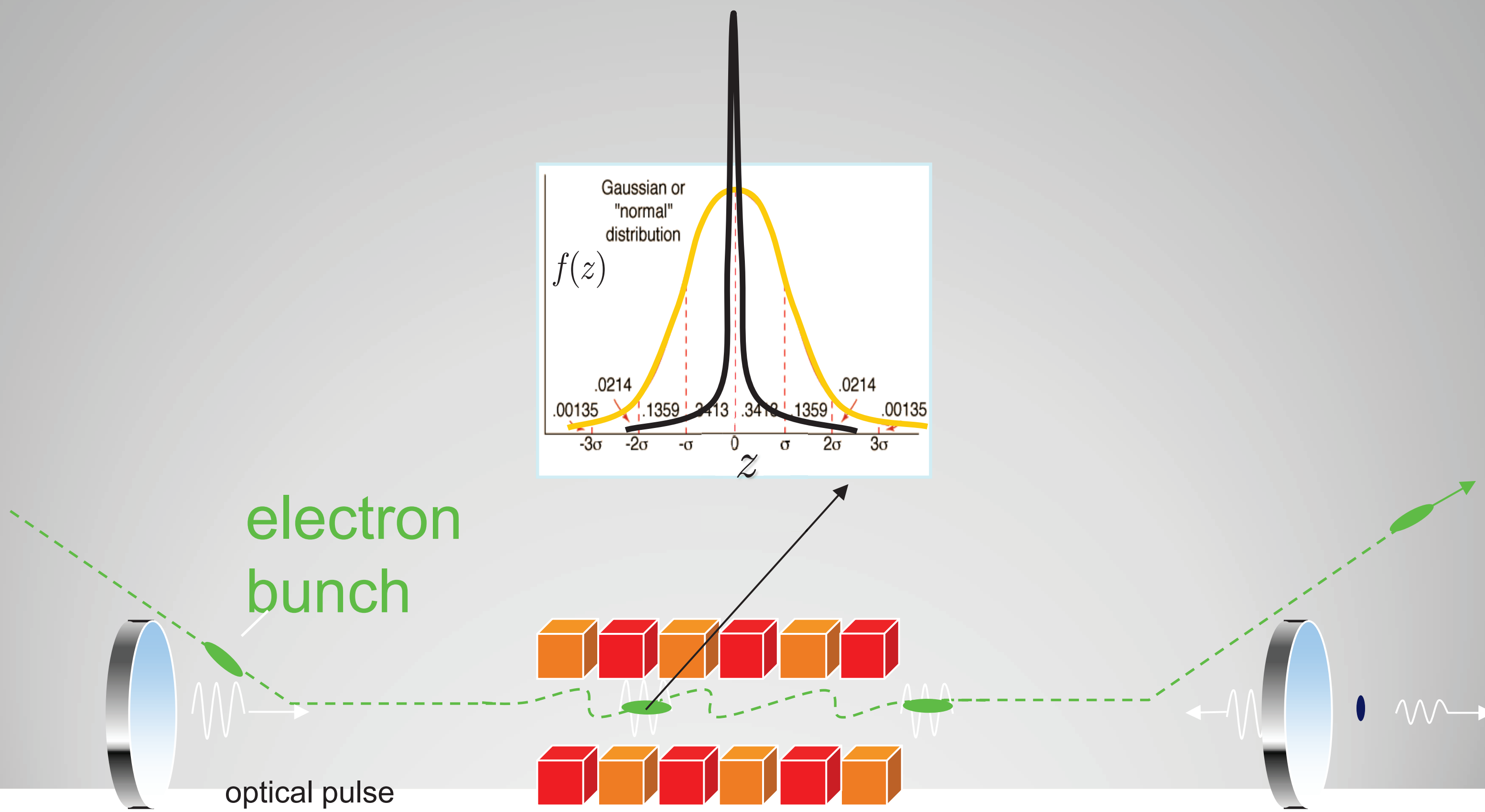
$$G(\theta,\mu_c) = G_M\frac{\theta}{\theta_s}\left[1-\ln\left(\frac{\theta}{\theta_s}\gamma_c\right)\right], \qquad G_M \cong 0.85 g_0, \qquad \theta_s \cong 0.456$$

$$\gamma_c = 1+\frac{\mu_c}{3}$$

$$0 \le \theta \le e\frac{\theta_s}{\gamma_c} \qquad G^* = \frac{G_M}{\gamma_c}, \qquad \theta^* = \frac{\theta_s}{\gamma_c}$$



The Optical beam can also be shaped



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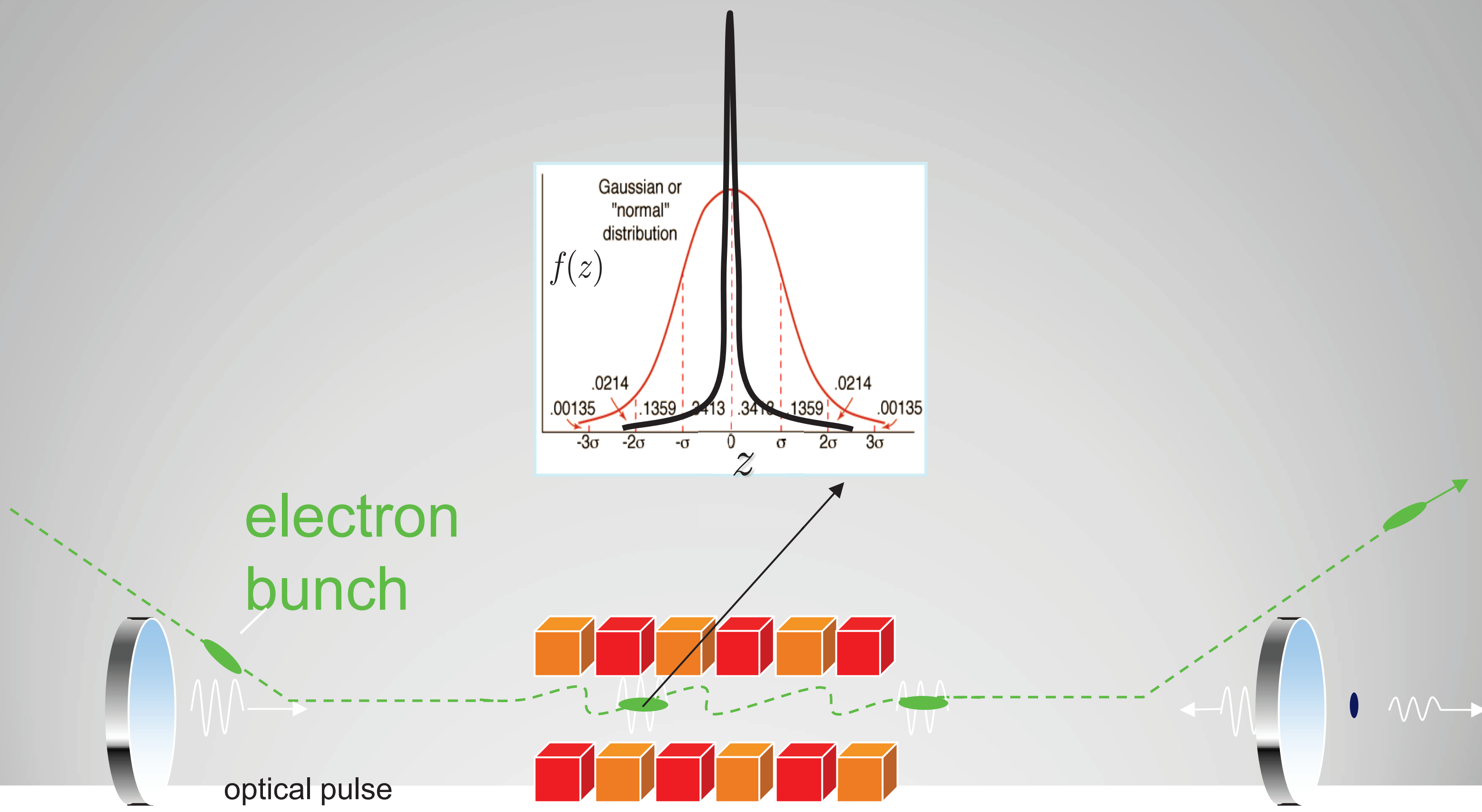

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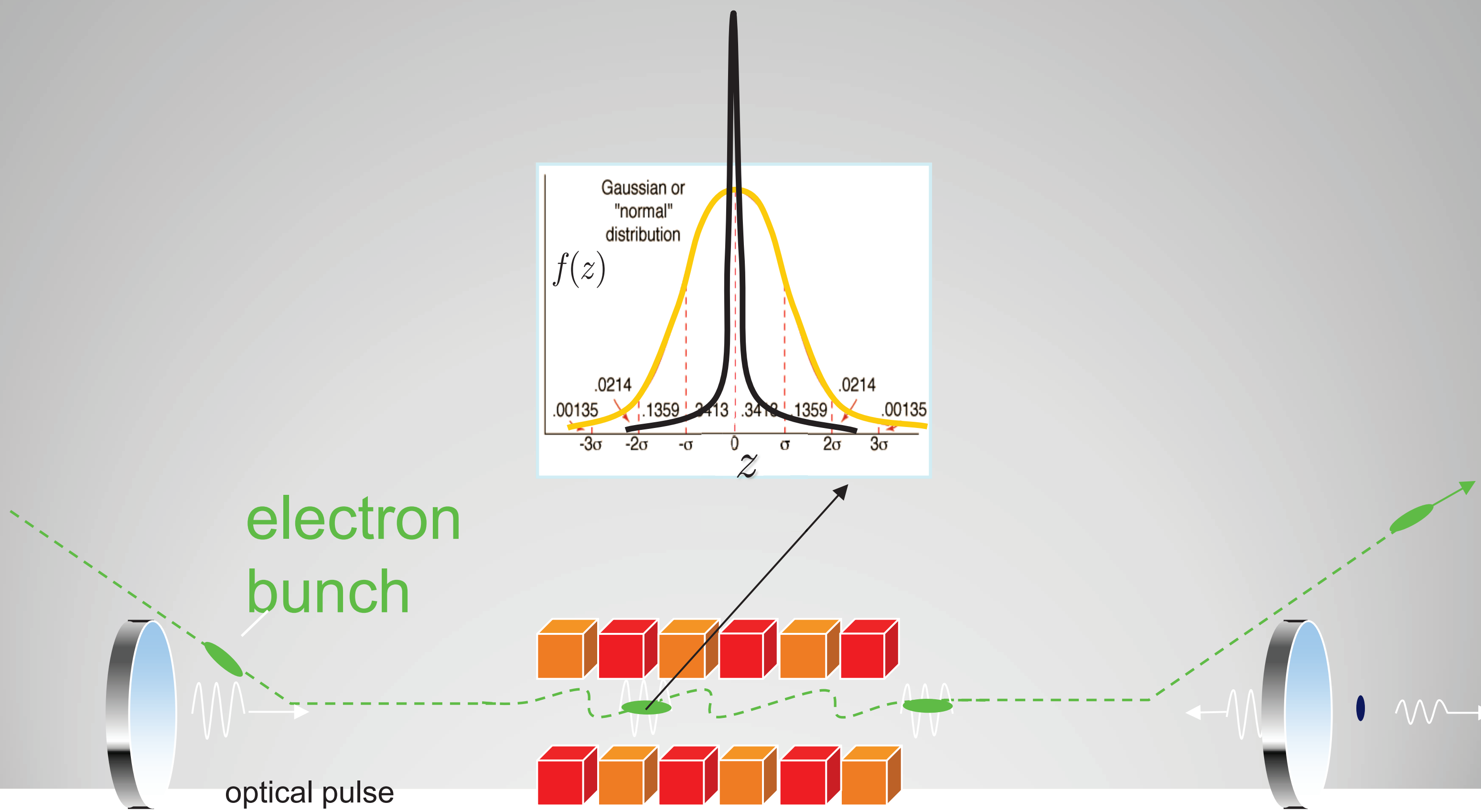

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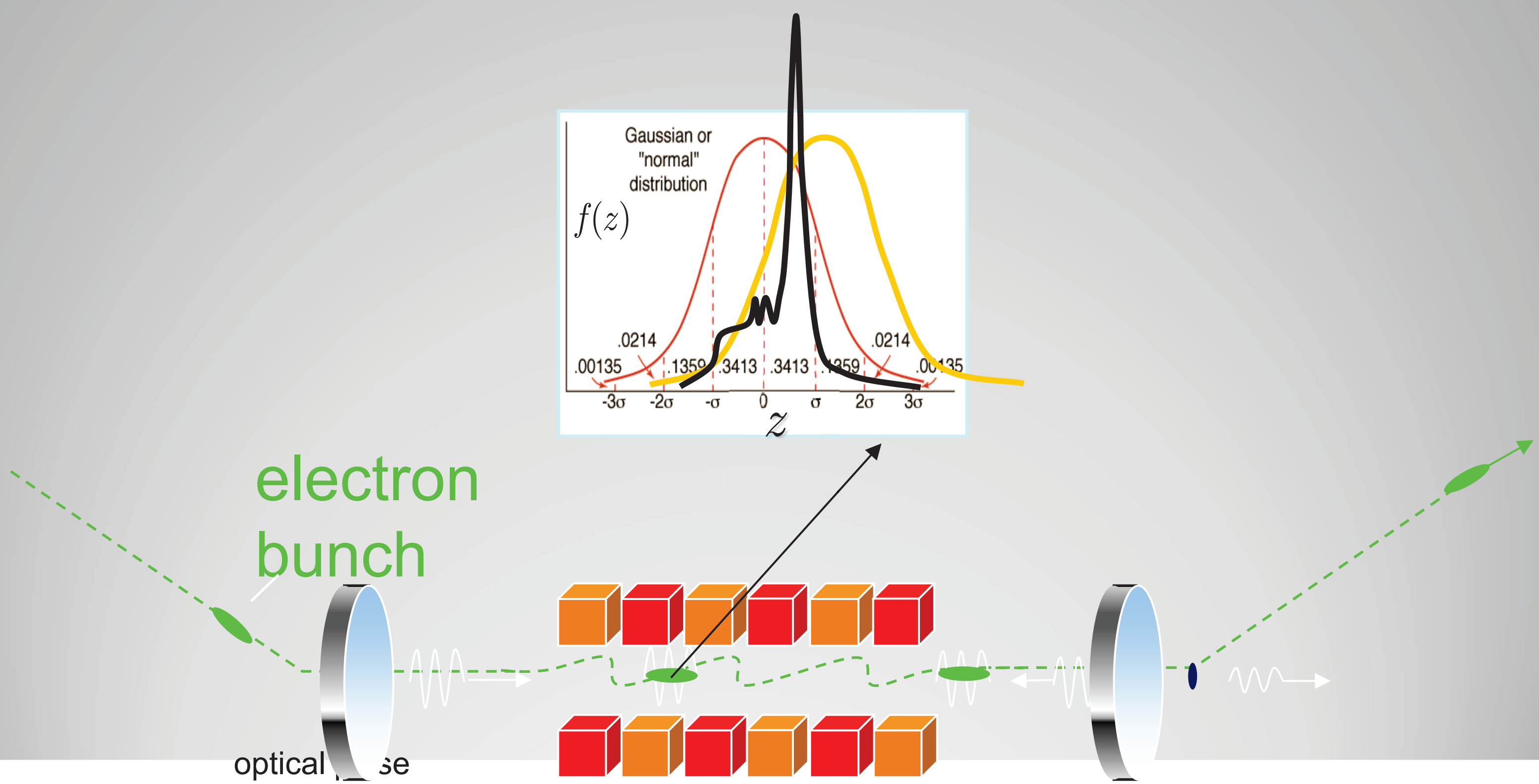

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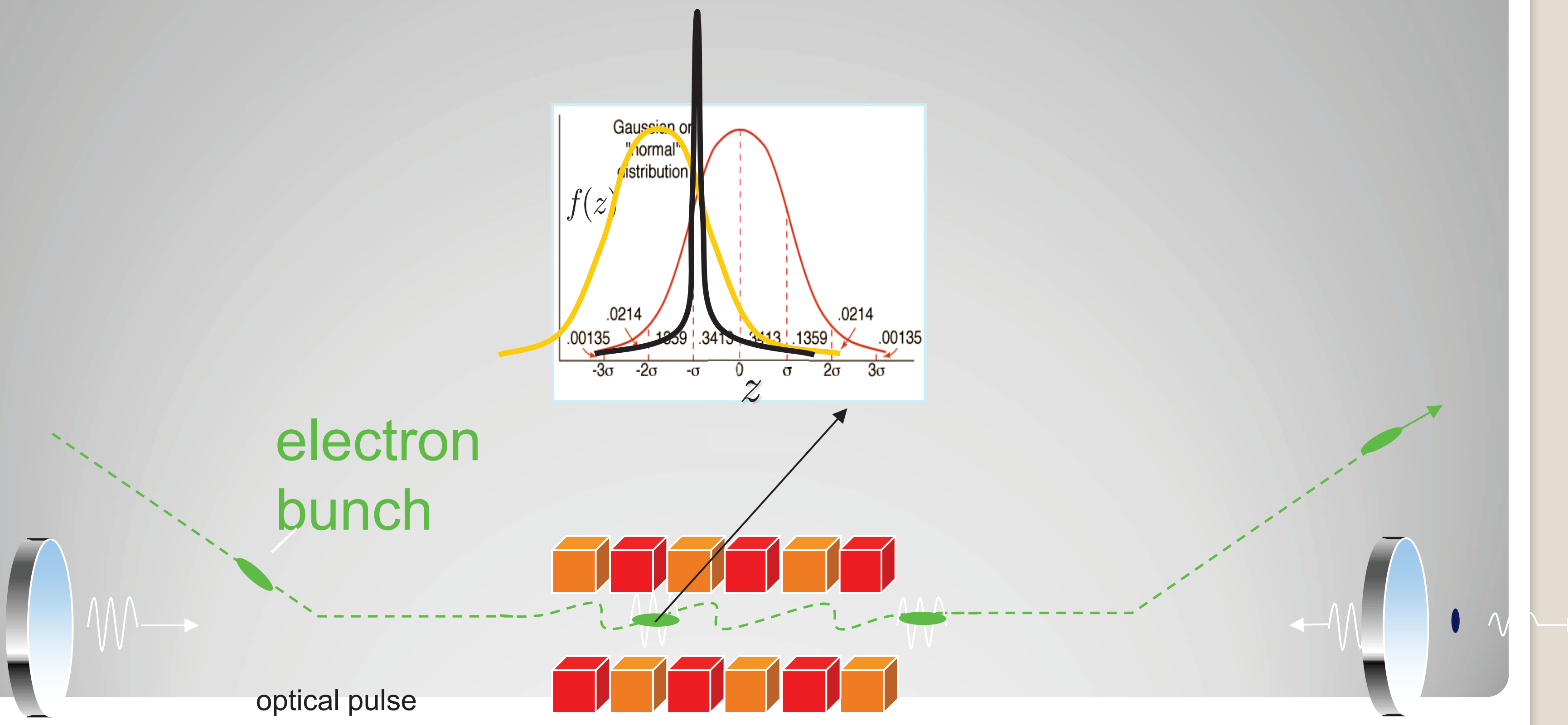
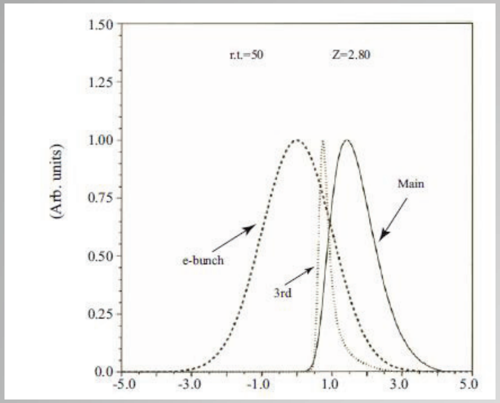
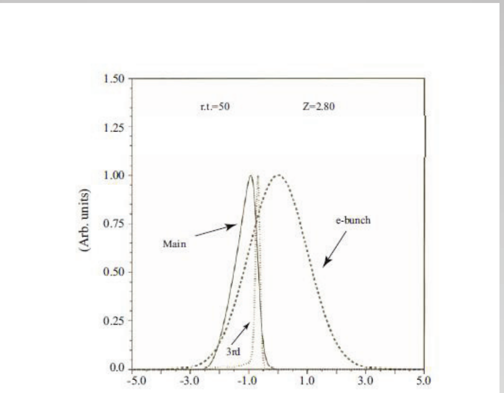
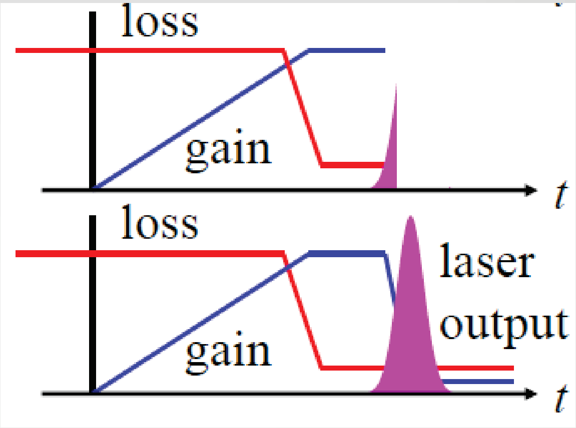


- Pulse shaping and Q-Switching by mirror positioning



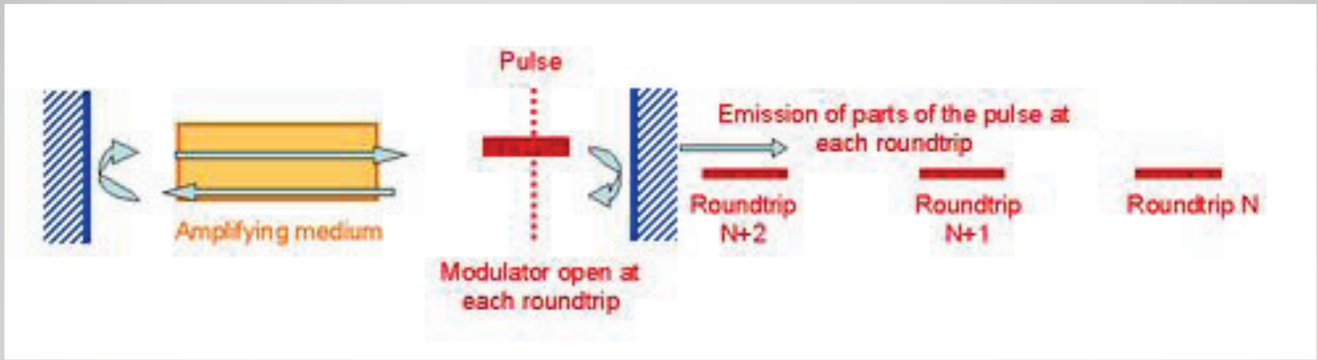


Detuned cavity

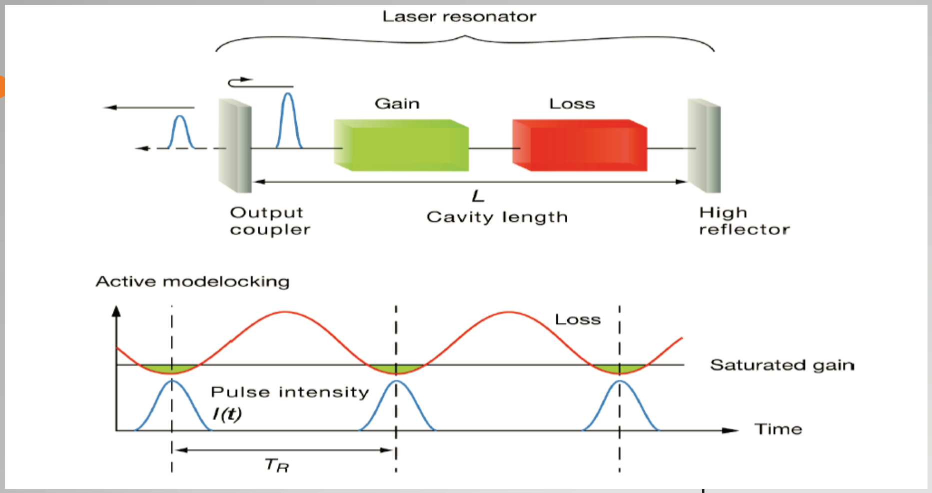


Generation of Short Pulses & mode locking

The cavity is prevented from filling with photons everywhere at the same time: only a packet of photons is allowed to propagate in the cavity. This pulse lasts for a shorter time than a round trip in the cavity. In other words, its spatial extension is markedly shorter than the length of the cavity.



Active Mode Locking: Haus Master Equation (U. Keller, L. Gallmann)



$$T_R \frac{\partial A}{\partial T} = \left[g - l + D_g \frac{\partial^2}{\partial \tau^2} - M\left(1 - \cos(\omega_M \tau)\right) \right] A \rightarrow$$

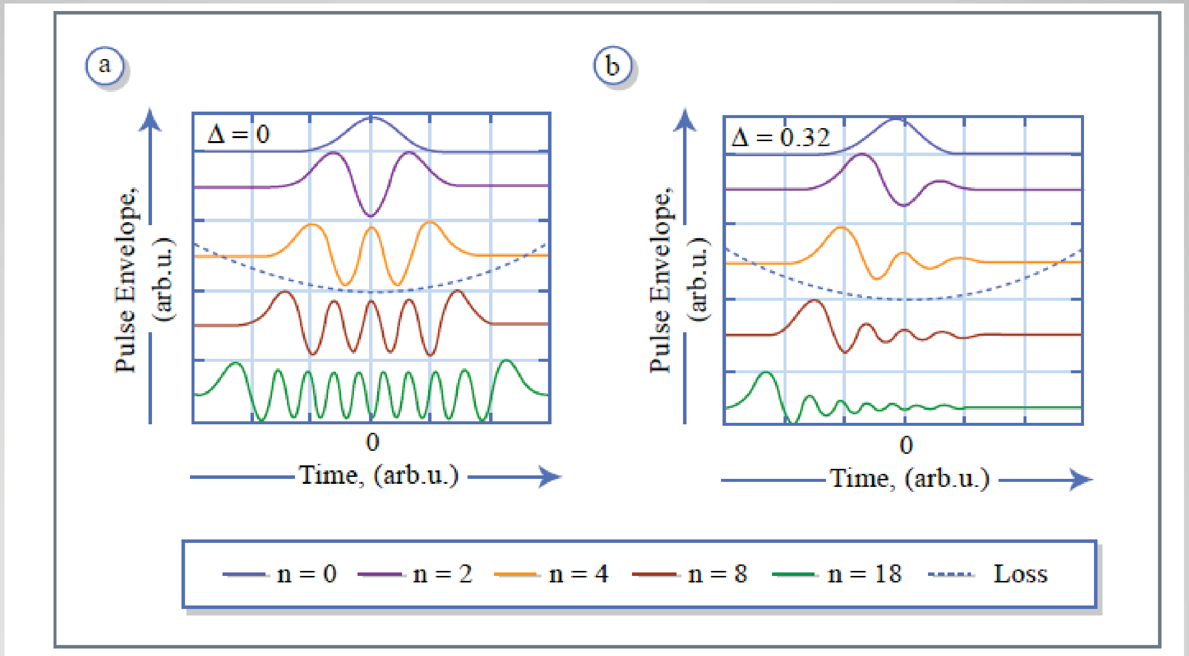
$$\rightarrow T_R \frac{\partial A}{\partial T} \cong \left[g - l + D_g \frac{\partial^2}{\partial \tau^2} - M_s \tau^2 + T_D \frac{\partial}{\partial \tau} \right] A,$$

$$D_g = \frac{g}{\Omega_g^2}, M_s = \frac{1}{2} M \omega_M^2, \omega_M = \frac{2\pi}{T_M}, T_D = T_R - T_M$$

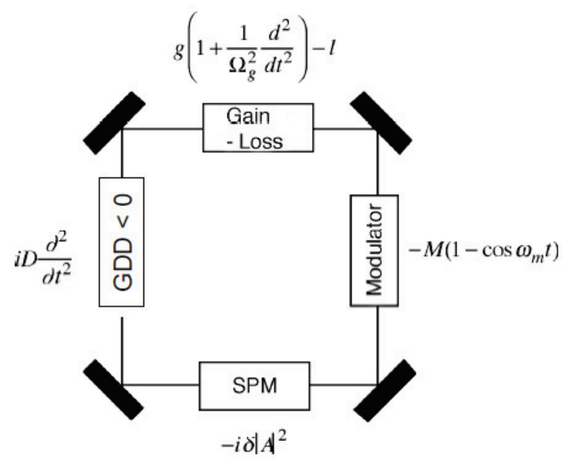
Electron bunch induced mode locking
FEL master equation: GD, A. Renieri, H. Haus
P. Elleaume, V. Litvinenko…

$$T_R \frac{\partial}{\partial T} A = \left[(G_1 - \eta) + \mu_c (G_2 - \Theta) \frac{\partial}{\partial \tau} + \frac{1}{2} G_3 \frac{\partial^2}{\partial \tau^2} - \frac{1}{2} G_1 \tau^2 \right] A$$

$$\tau = \frac{z}{\sigma_z}$$



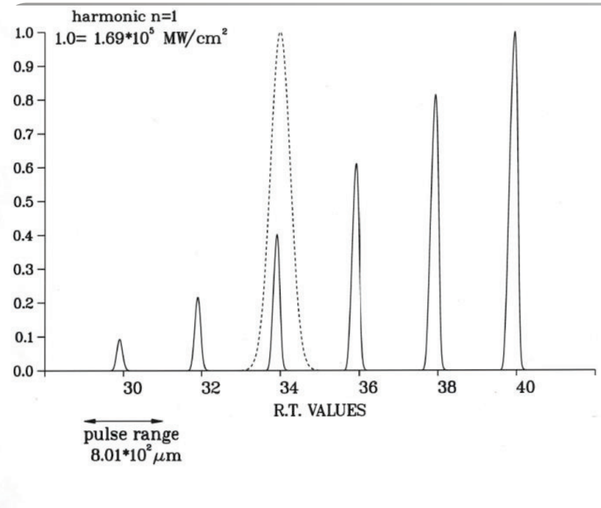
Mode Locking in FEL is the result of different mechanisms: Active & Passive, FEL SOLITONS (????)

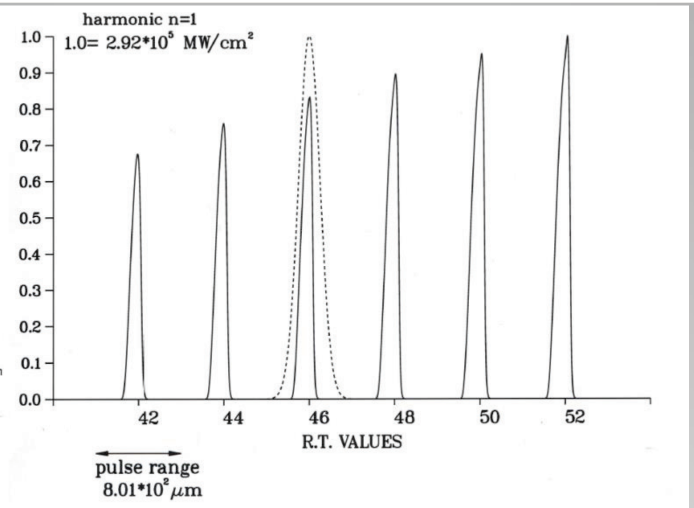


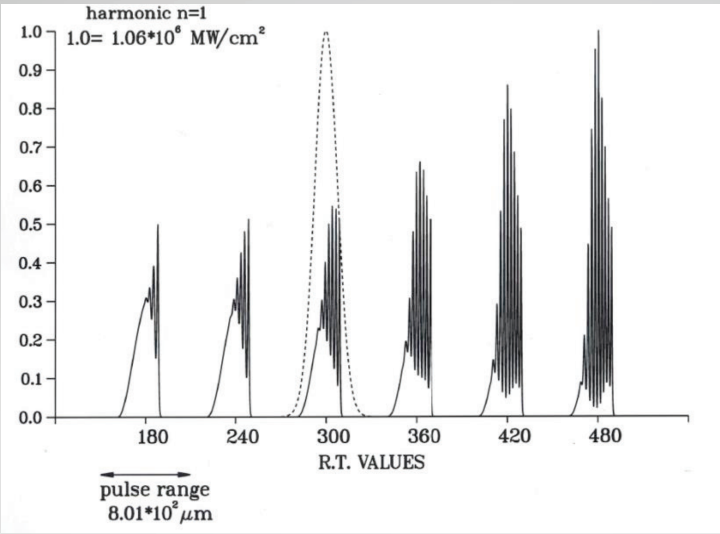
$$g = \frac{g_0}{1+\frac{I}{I_S}}$$

$$T_R \frac{\partial}{\partial T} A = \left[(G_1 - \eta) A + \frac{1}{2} G_3 \frac{\partial^2}{\partial \tau^2} A + \gamma |A|^2 \right]$$

$$A = A_0 \sec h\left(\frac{\tau}{\tau_0}\right)$$



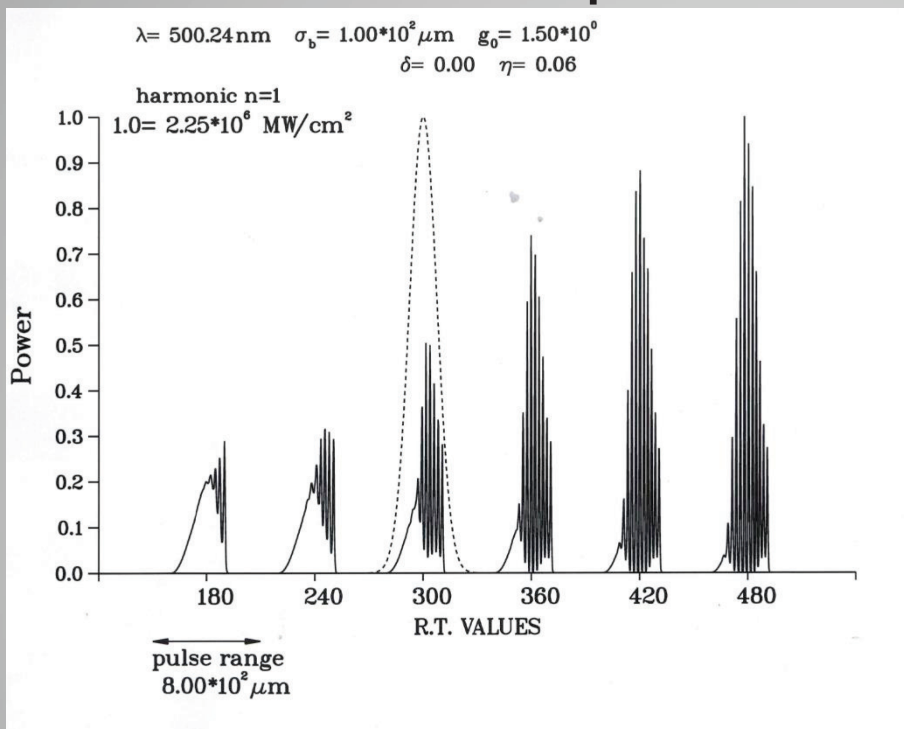


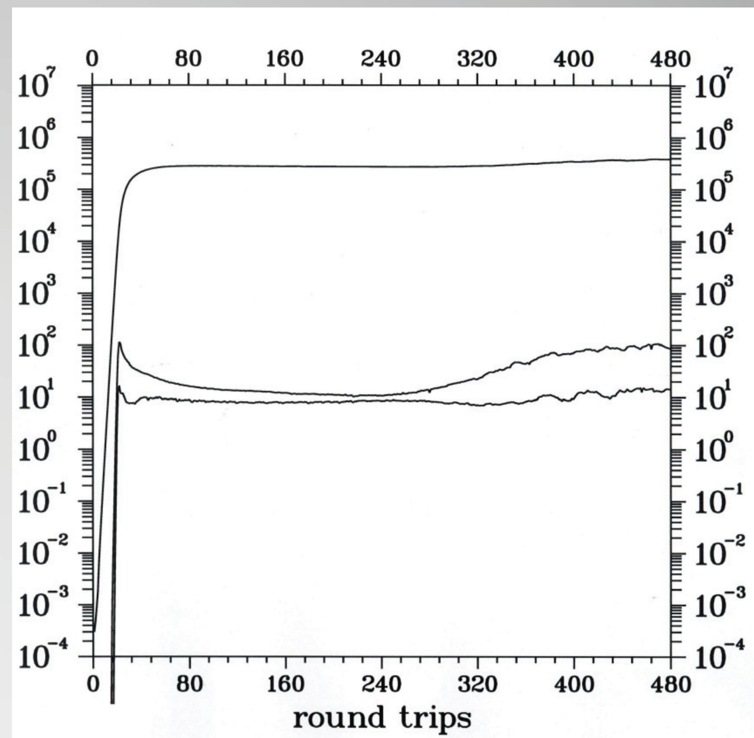


Pulse evolution per round trip (PROMETEO)

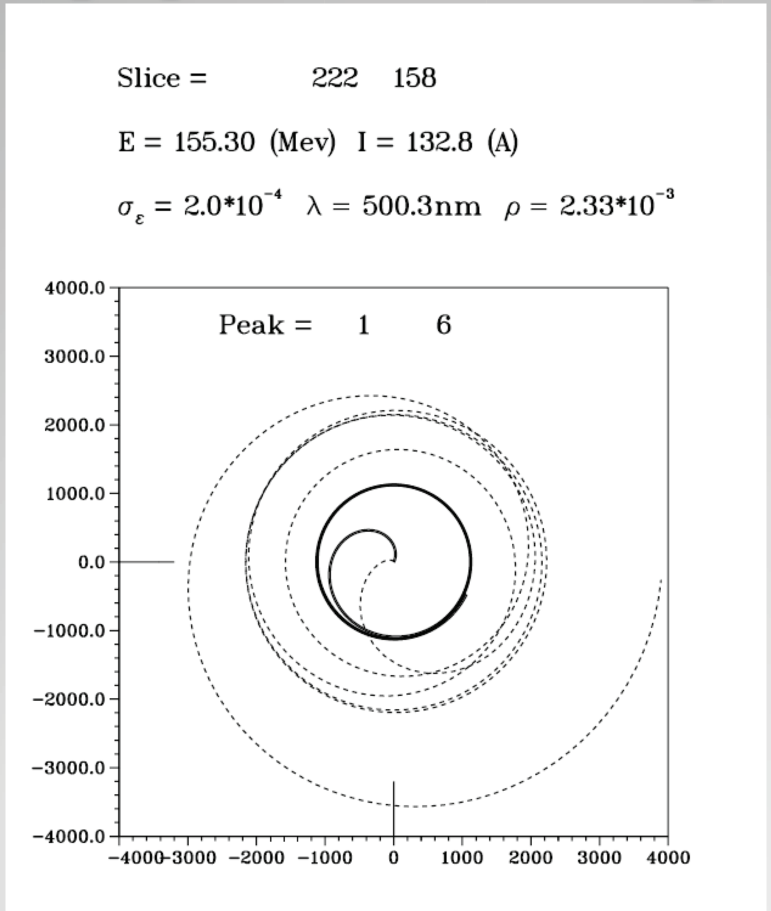
Pulses in deep saturation: Synchrotron side bands

- Last round-trip 480

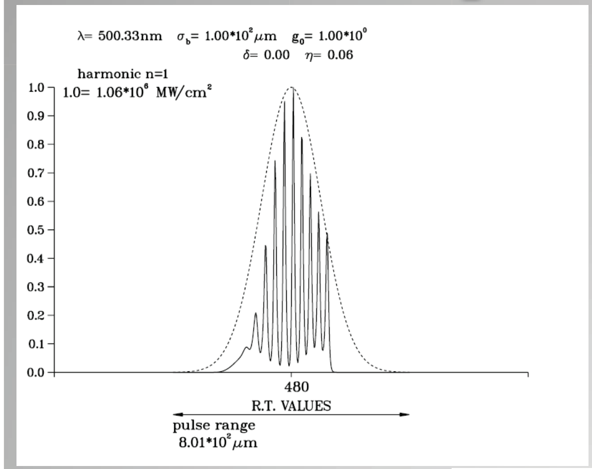


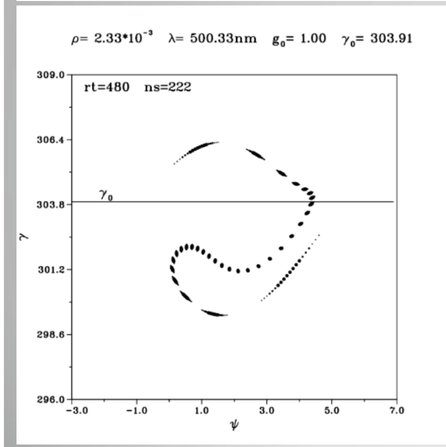


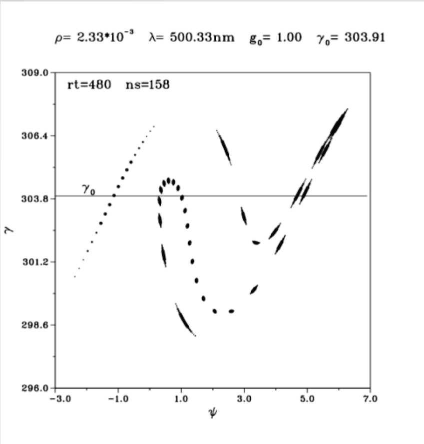
Slice Nyquist Plot (1-6)



Phase Space Plot

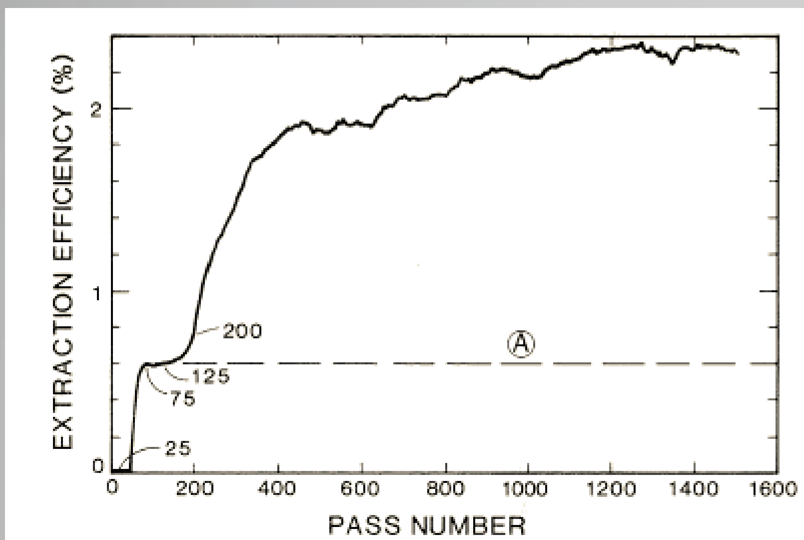


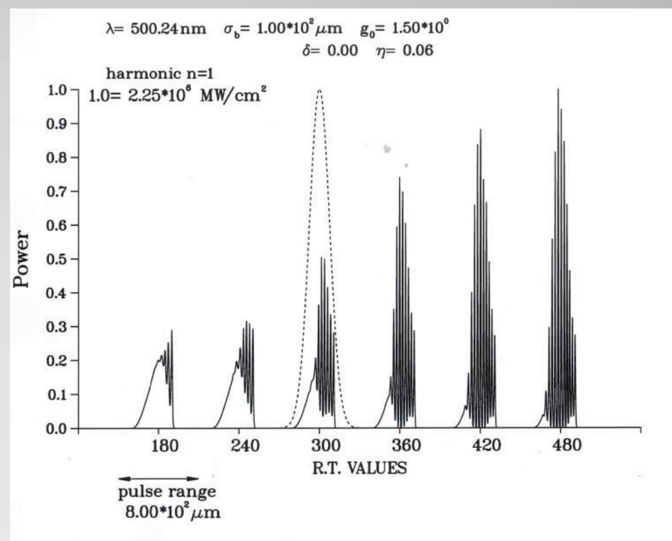


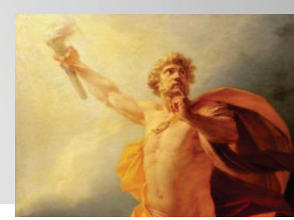


Slice number	Power $\left(\frac{MW}{cm^2}\right)$
112	5.17 (+5)
124	5.95 (+5)
135	7.37 (+5)
147	8.71 (+5)
158	1.06 (+6)
170	1.01 (+6)
183	7.85 (+5)
196	4.71 (+5)
209	2.21 (+5)
222	9.32 (+4)

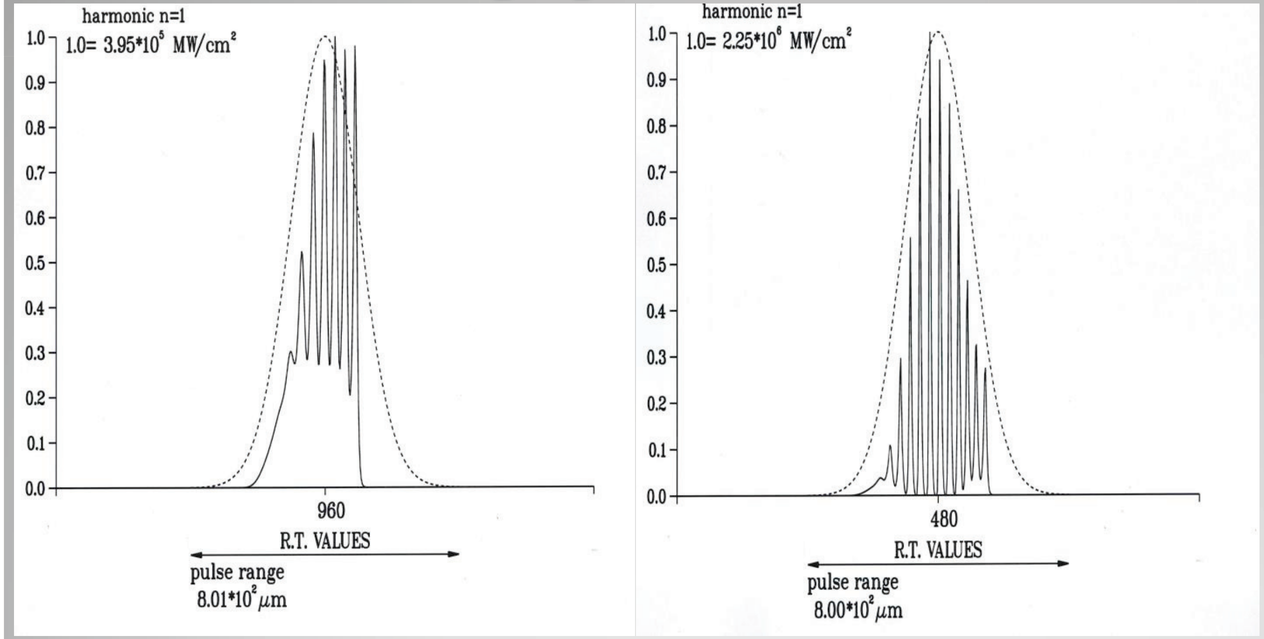
- Side Band and efficiency







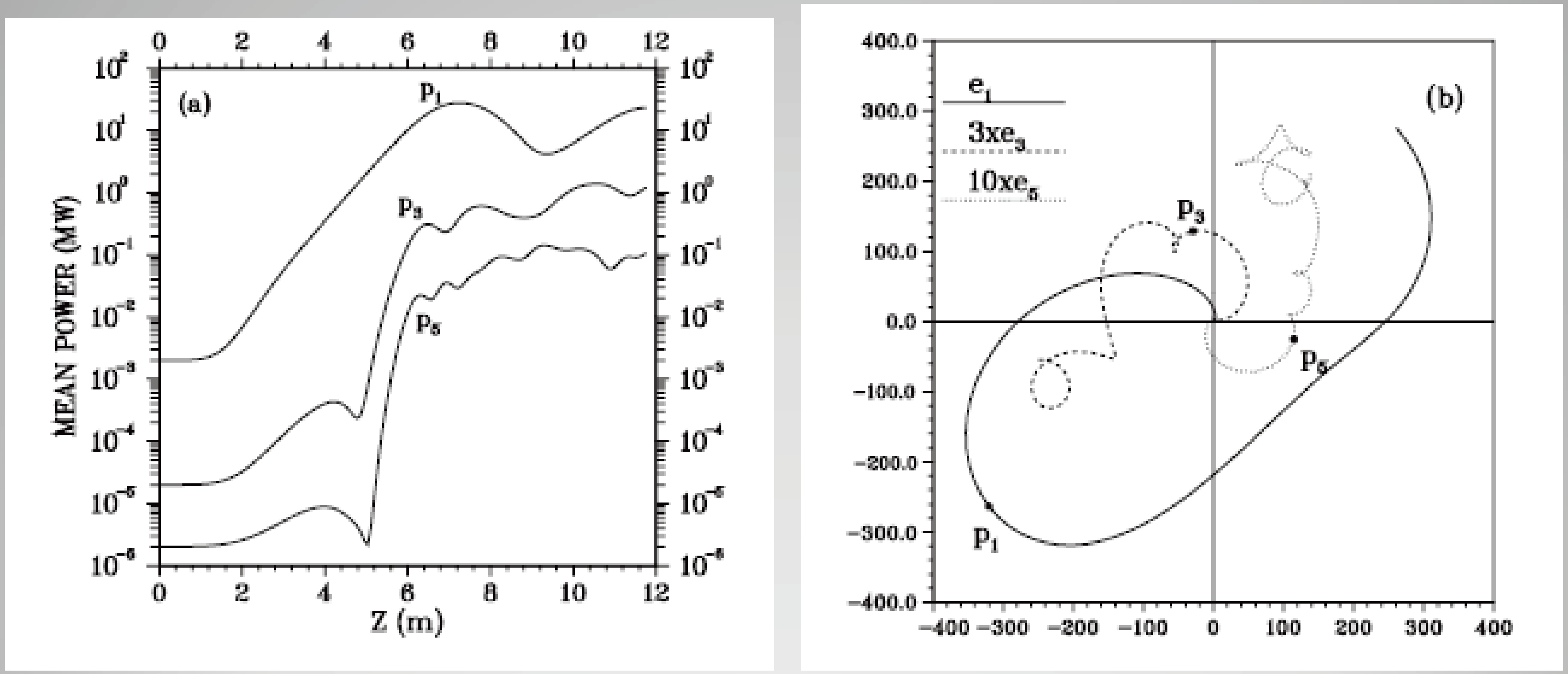
How many peaks? Frozen SPIKES



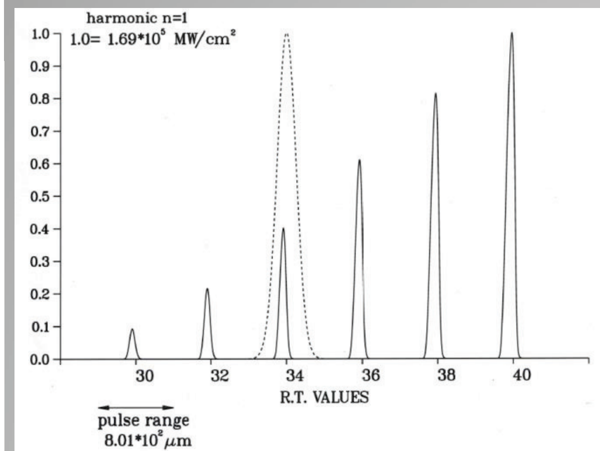
$$n_{SB} \cong \frac{\sigma_z}{l_c},$$

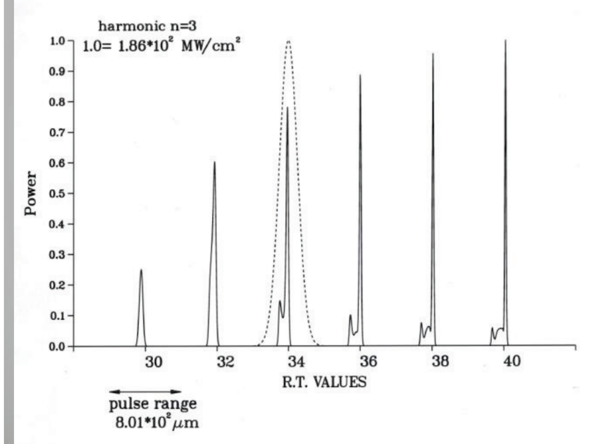
$$l_c = \frac{\lambda}{4\pi\sqrt{3}\,\rho}$$

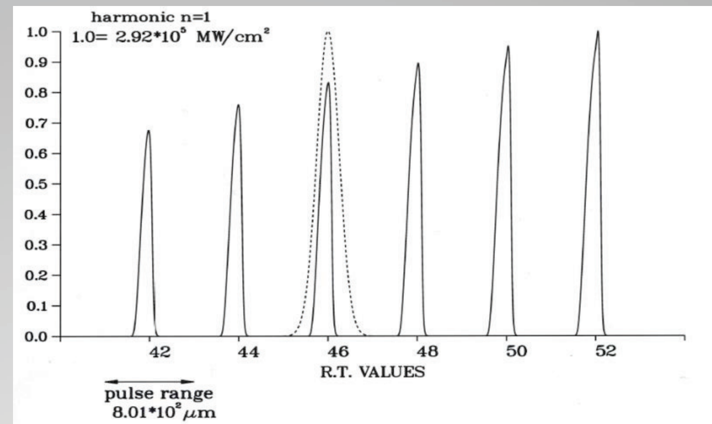
Harmonic generation Nyquist diagram

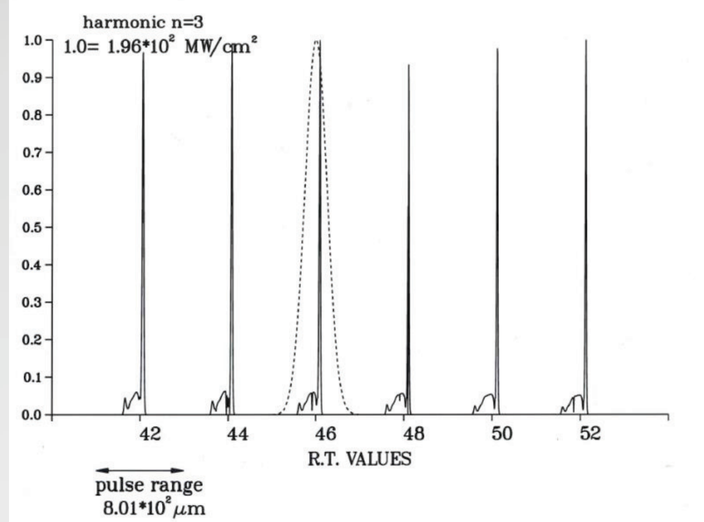


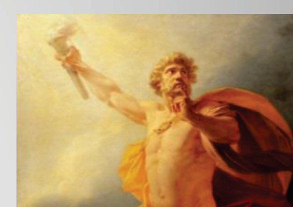
R.T. and Intracavity Harmonic Generation





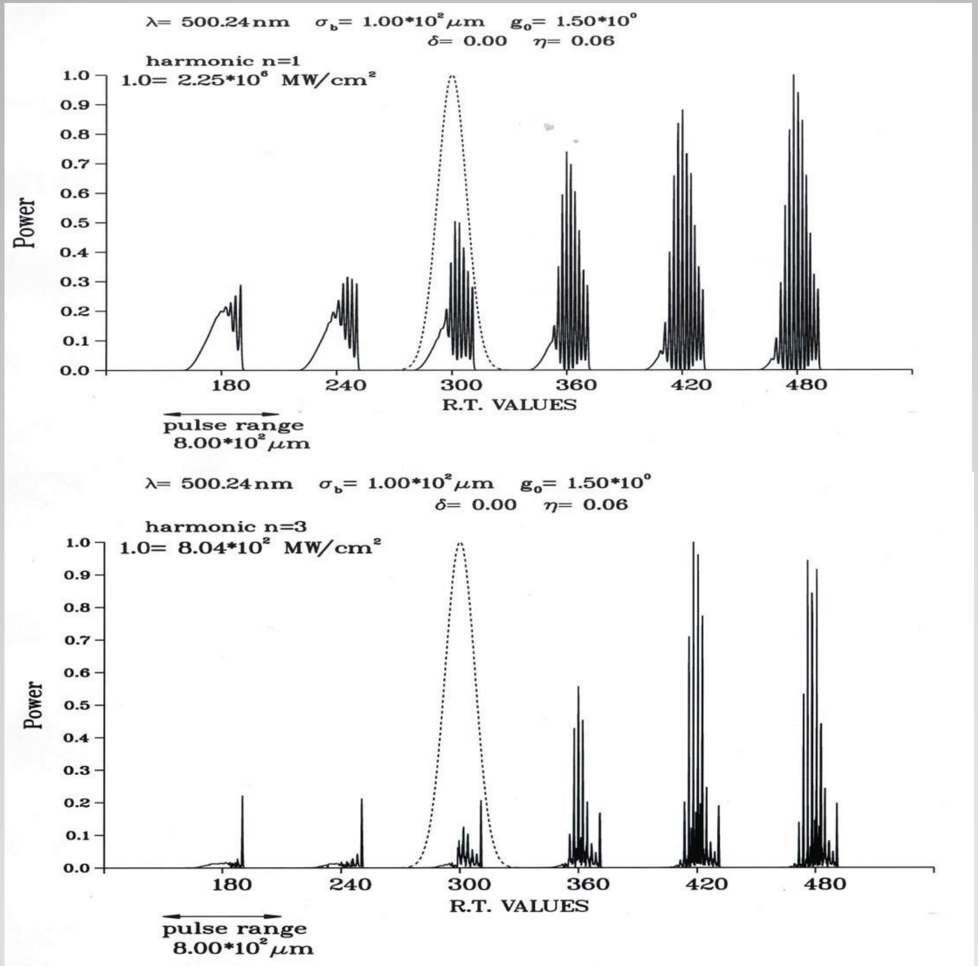


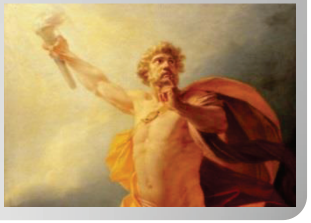




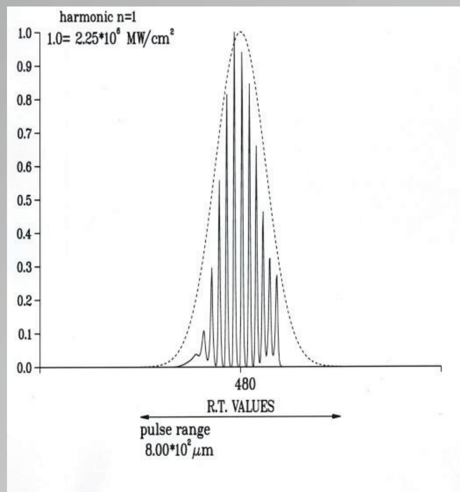
Non Linear Harmonic generation and Side Bands-III

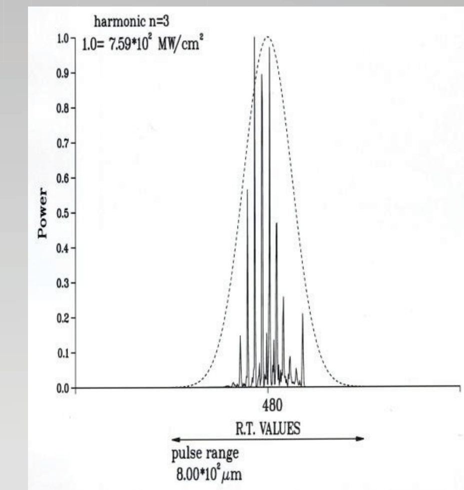
- 3-rd

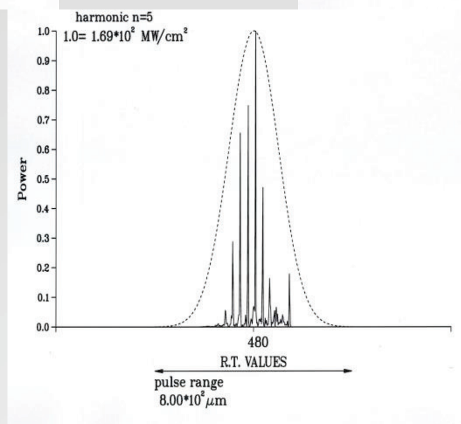




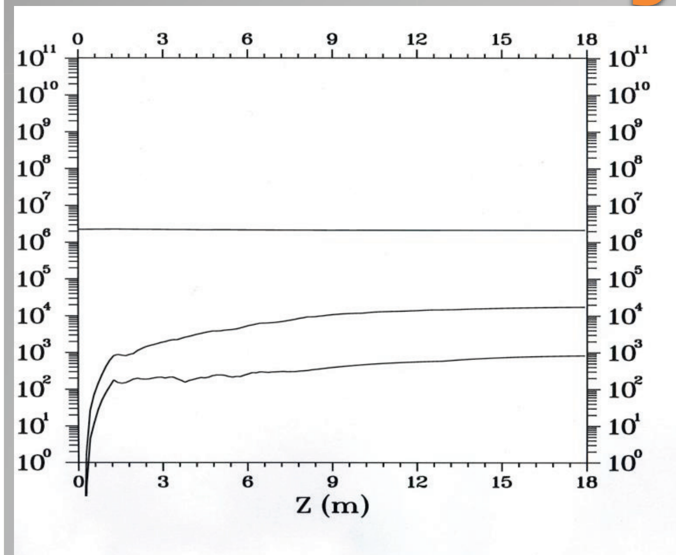
I-III-V

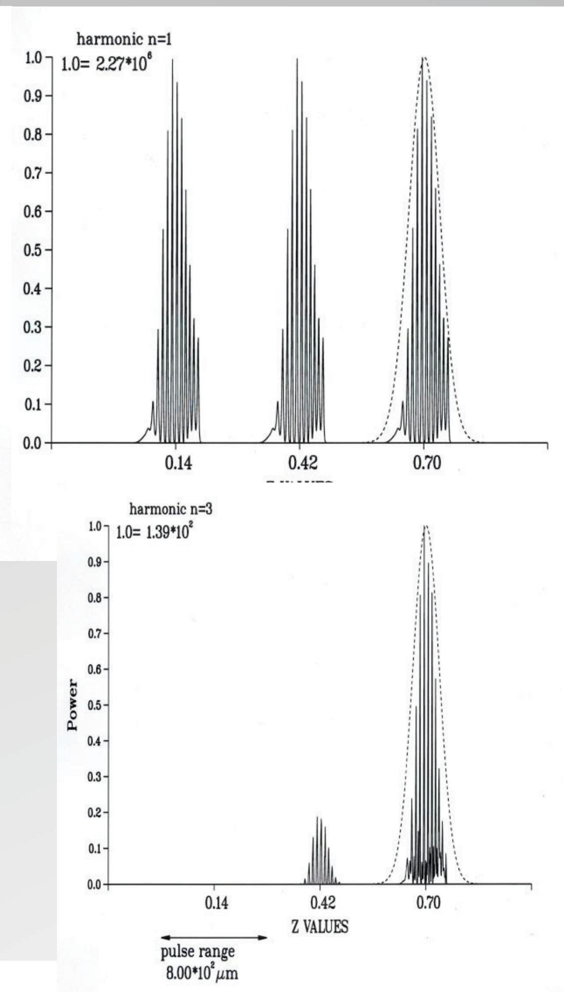

harmonic n=1
1.0= 2.25*10^6 MW/cm^2
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
480
R.T. VALUES
pulse range
8.00*10^2 μm

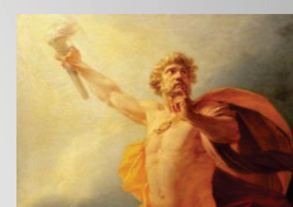

harmonic n=3
1.0= 7.59*10^2 MW/cm^2
Power
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
480
R.T. VALUES
pulse range
8.00*10^2 μm


harmonic n=5
1.0= 1.69*10^2 MW/cm^2
Power
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
480
R.T. VALUES
pulse range
8.00*10^2 μm

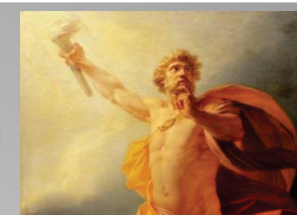
External Injection



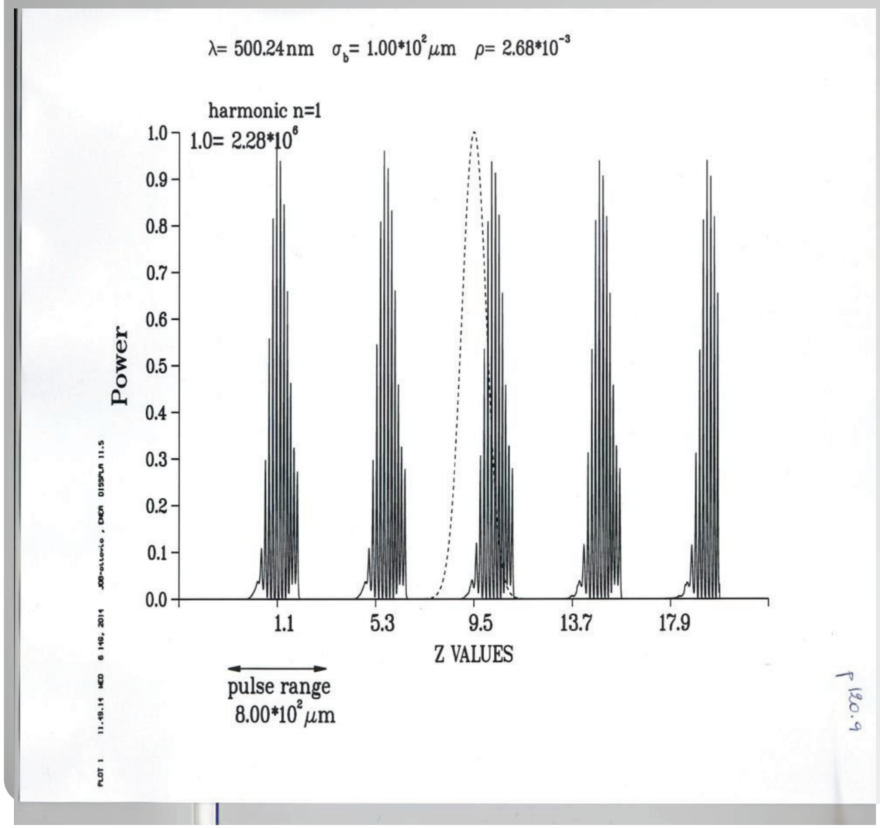


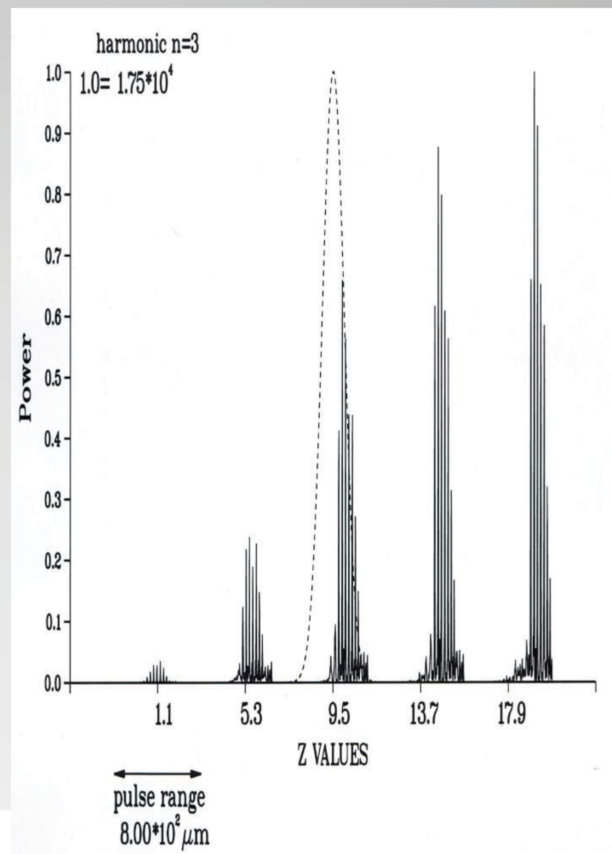


Superrandiance-Amplifier

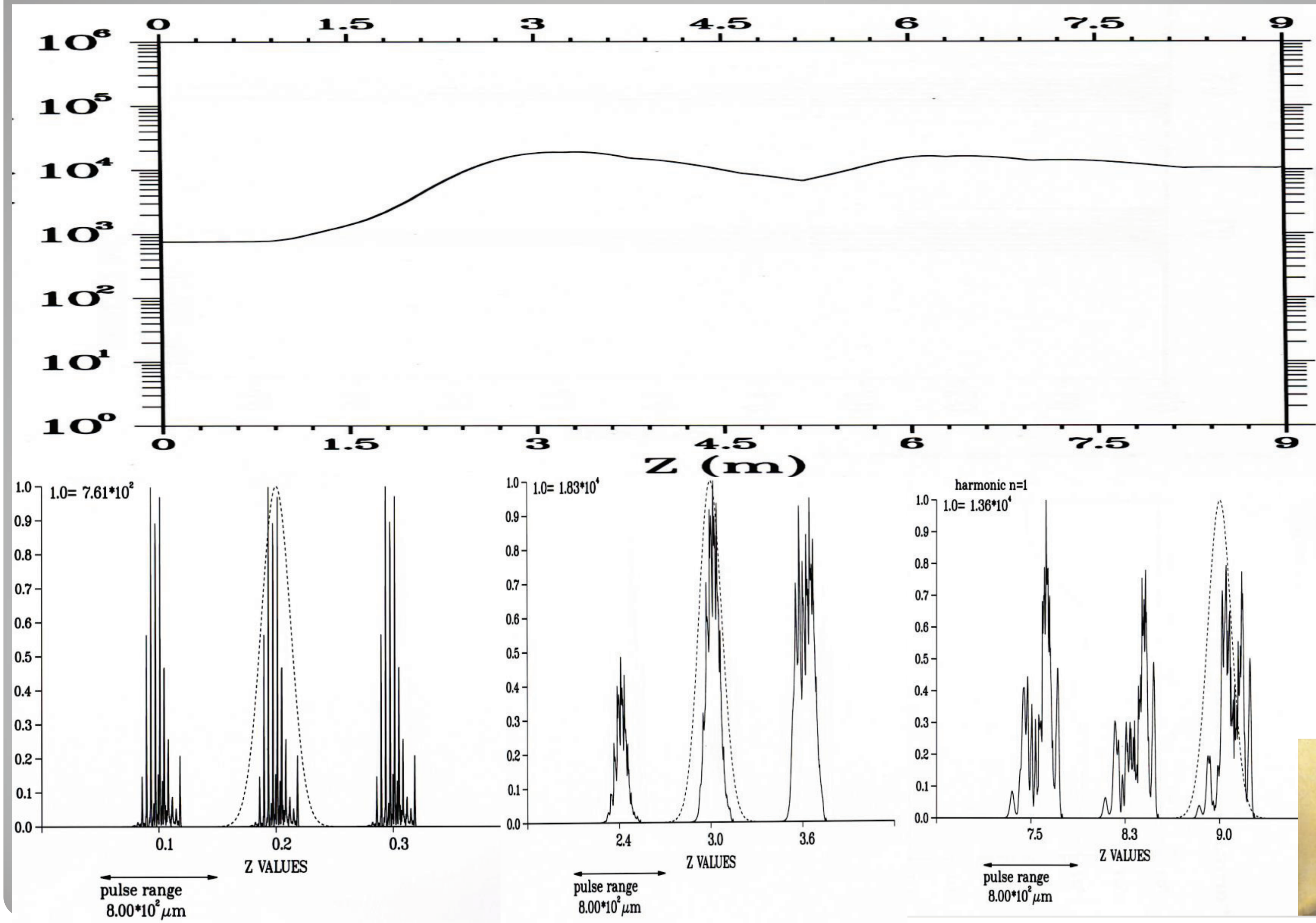


- Identical undulator and electron bunch





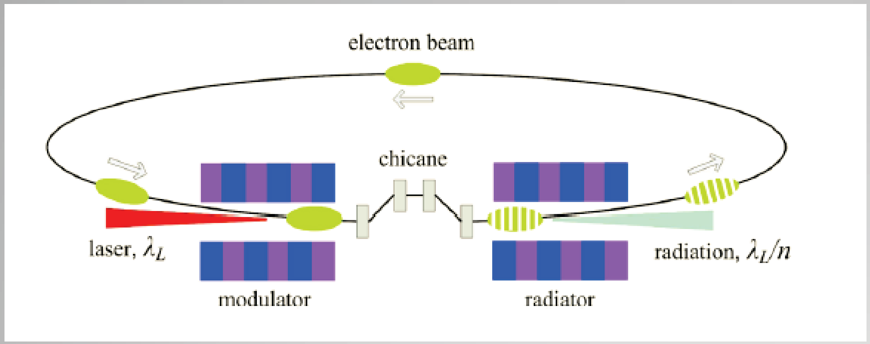
Third Harmonic amplification

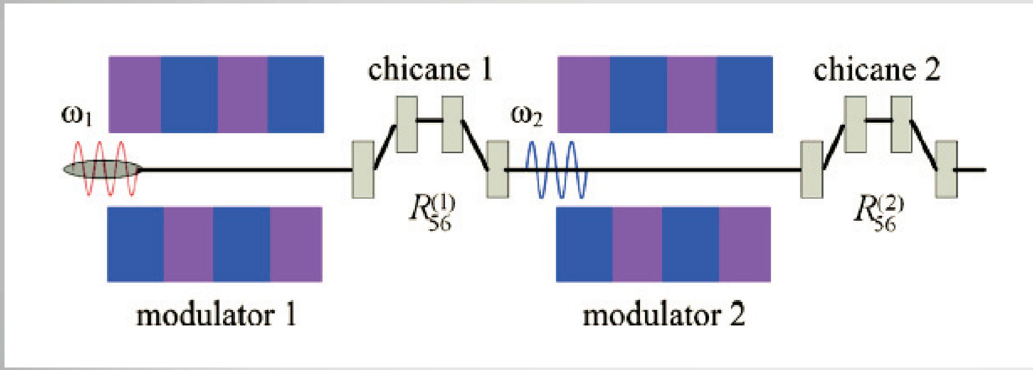


Can all this be useful?

- Probably yes
- A) Output coupler for intracavity harmonic generation
- B) Photocathode trigger to produce comb electron structure with fs slicing (???..)
- C) Use as external seed with beam manipulation scheme

Helpful with other schemes (Hemsing, Stupakov, Xiang, Zholents Rev. Mod. Phys. (2014)

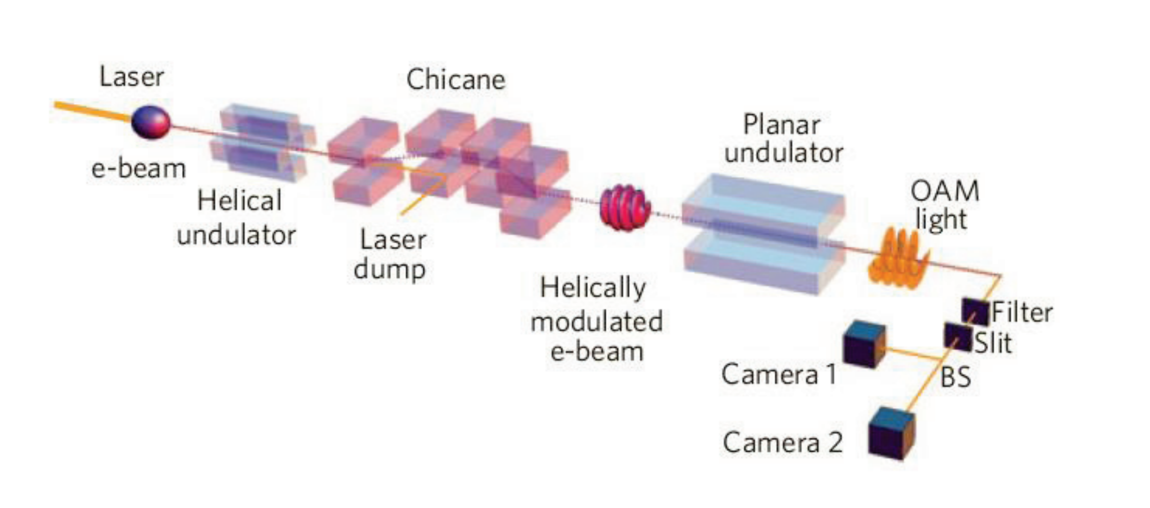




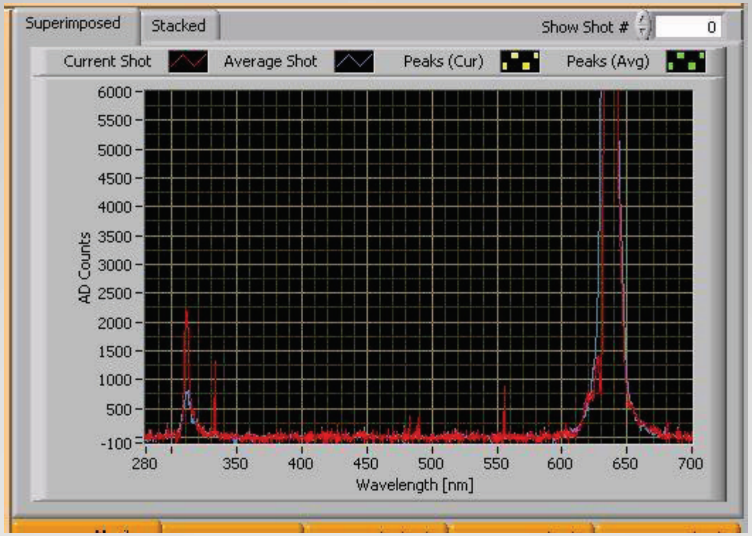
Optical vortex generation:

Hemsing, Knyazik, Dunning Xiang, Marinelli, Hast, Rosenzweig
Nature Photonics (2013)

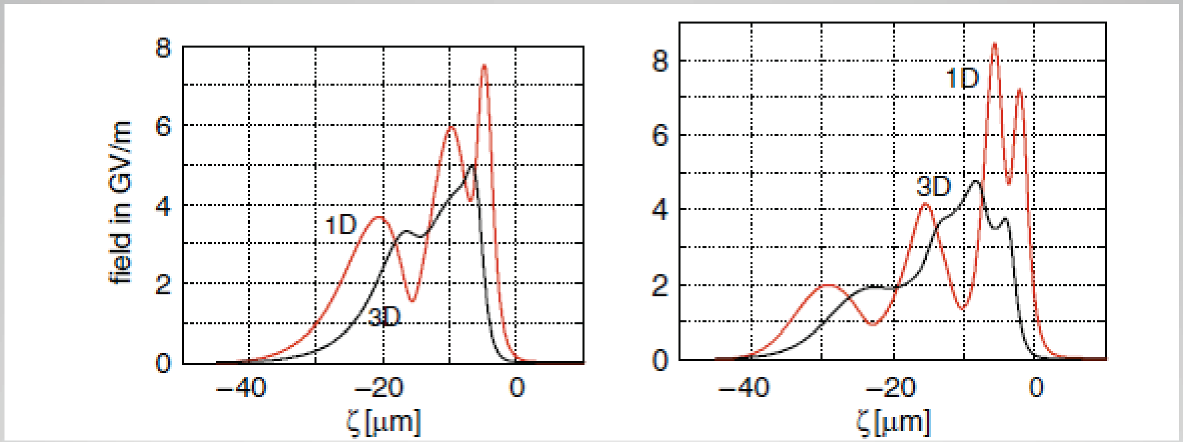
- The Laser could be generated in an oscillator driven by a helical undulator and then injected into a linear undulator

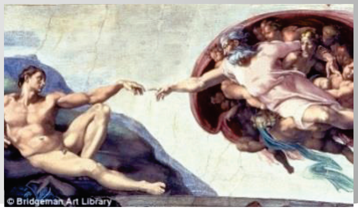


Or as recently done at SPARC (Segmented Undulators, Yurkov et al. 2013)

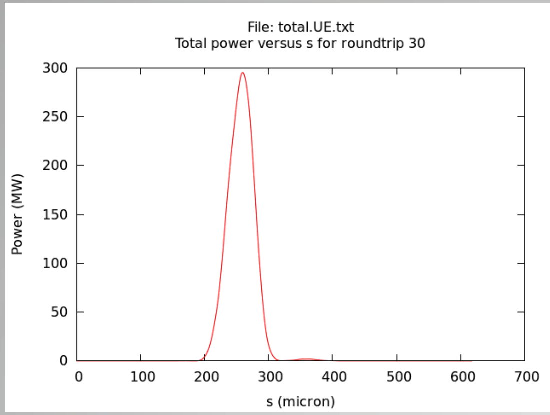


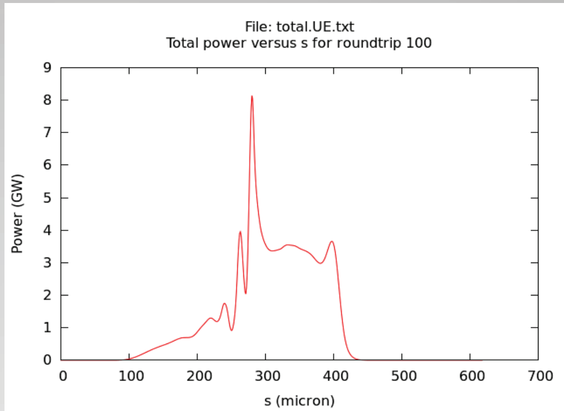
1D-Vs. 3D
P. Schmuser, M. Dohlus, J. Rossbach
(UV and Soft X-Ray FEL-Springer 2008)

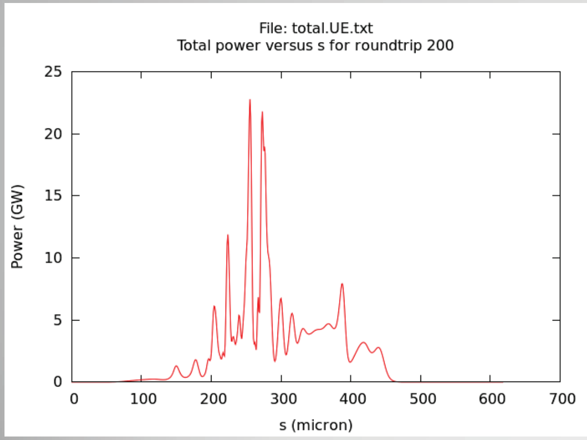


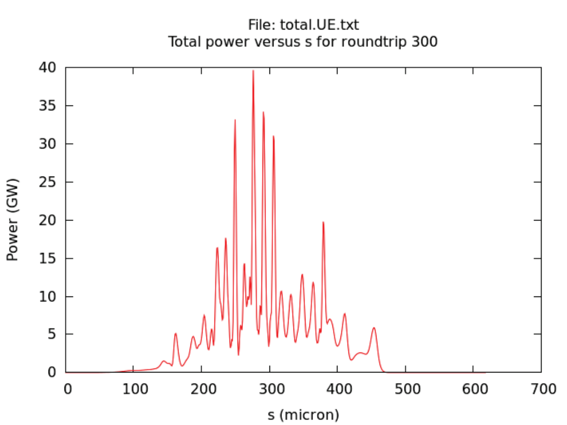


Is 1-d Appropriate to study these effects? P. Van Der Slot (Genesis)



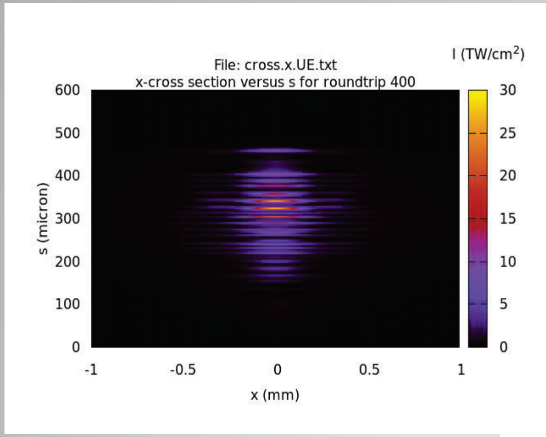


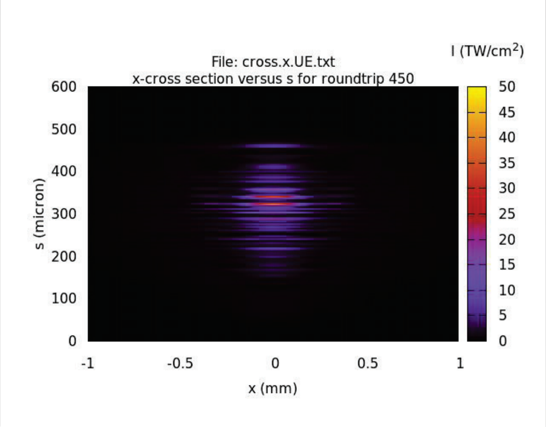


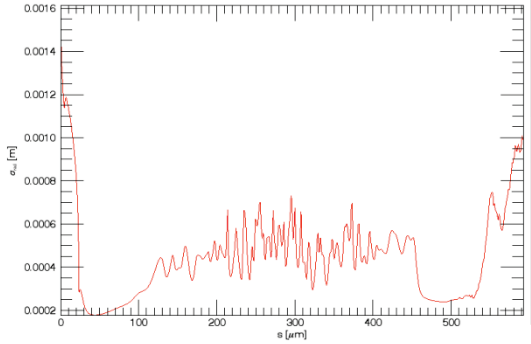


What is new

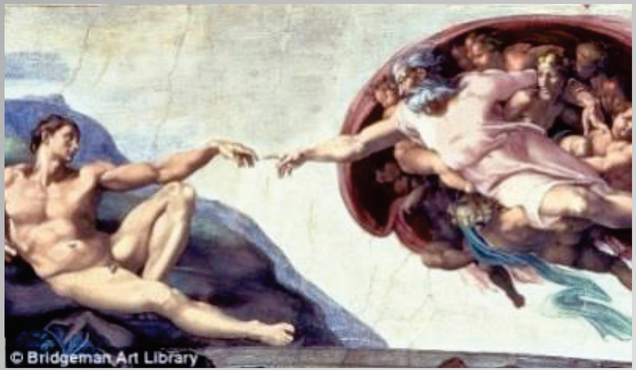
- Longitudinal transverse coupling?







??????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Perseo? Why not?
May be some conflict of interest may arise!!!