Quieted HGHG FELs: Correlated energy spread removal with space charge for high gain harmonic generation

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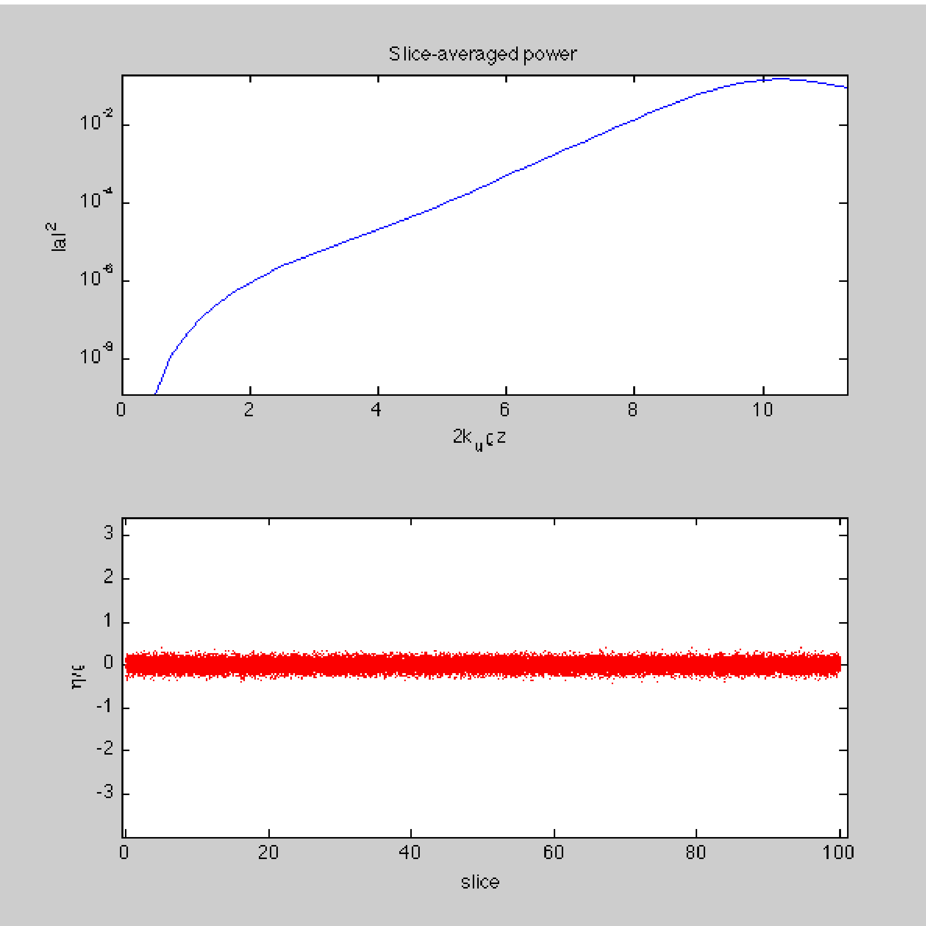
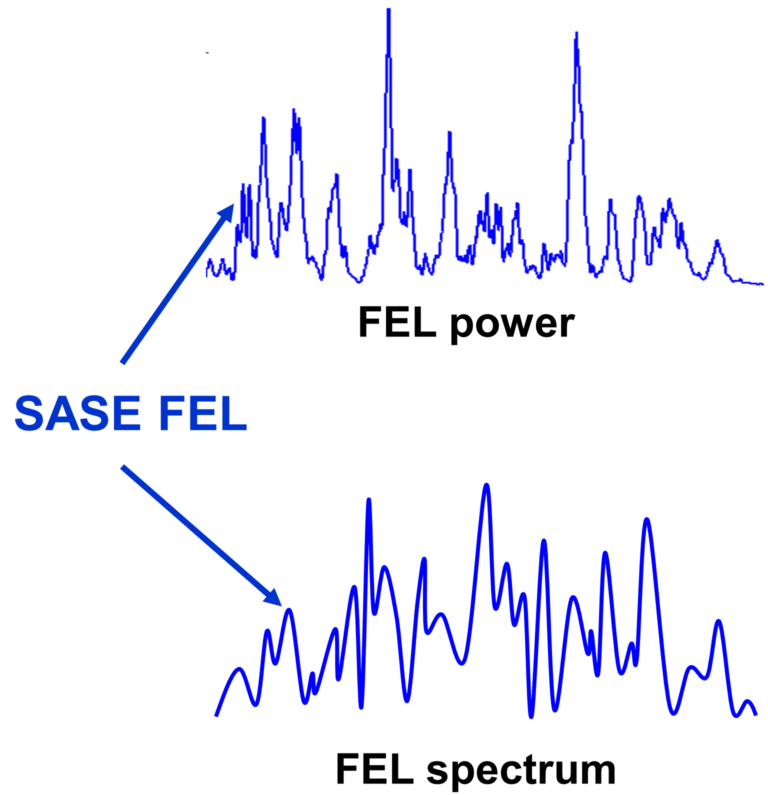
The Need to Seed

FEL output power



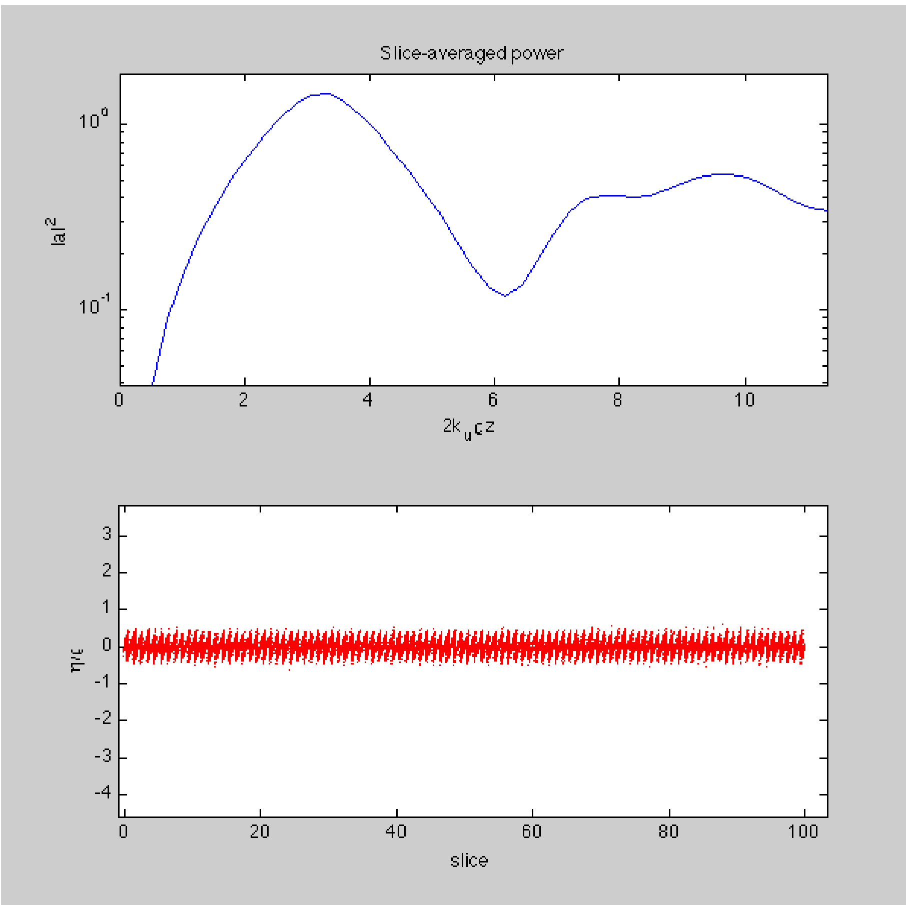
e-beam energy vs time space (many slices)



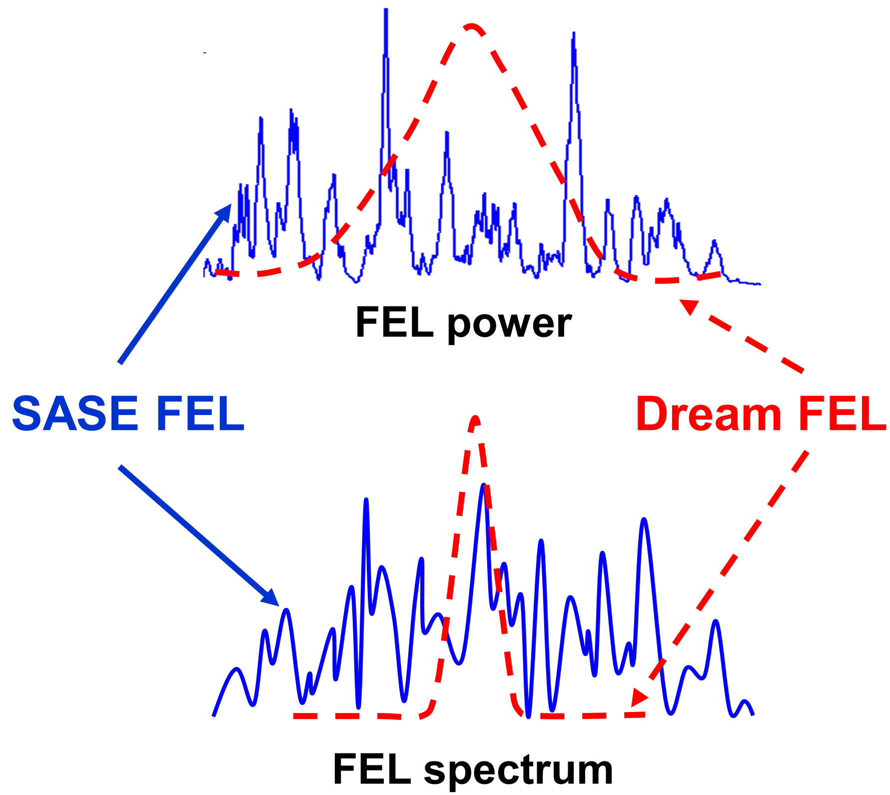
Improving the temporal coherence in the free electron laser (FEL)

The effect of coherent seeding

FEL output power



e-beam energy vs time space (many slices)



Seeding would make an FEL an extraordinarily good laser

Seeding Methods

Ultimate goal: Seeding to generate transform limited x-ray pulses

Several coherent seeding approaches:

External EM wave:

- High Harmonic Generation (HHG)

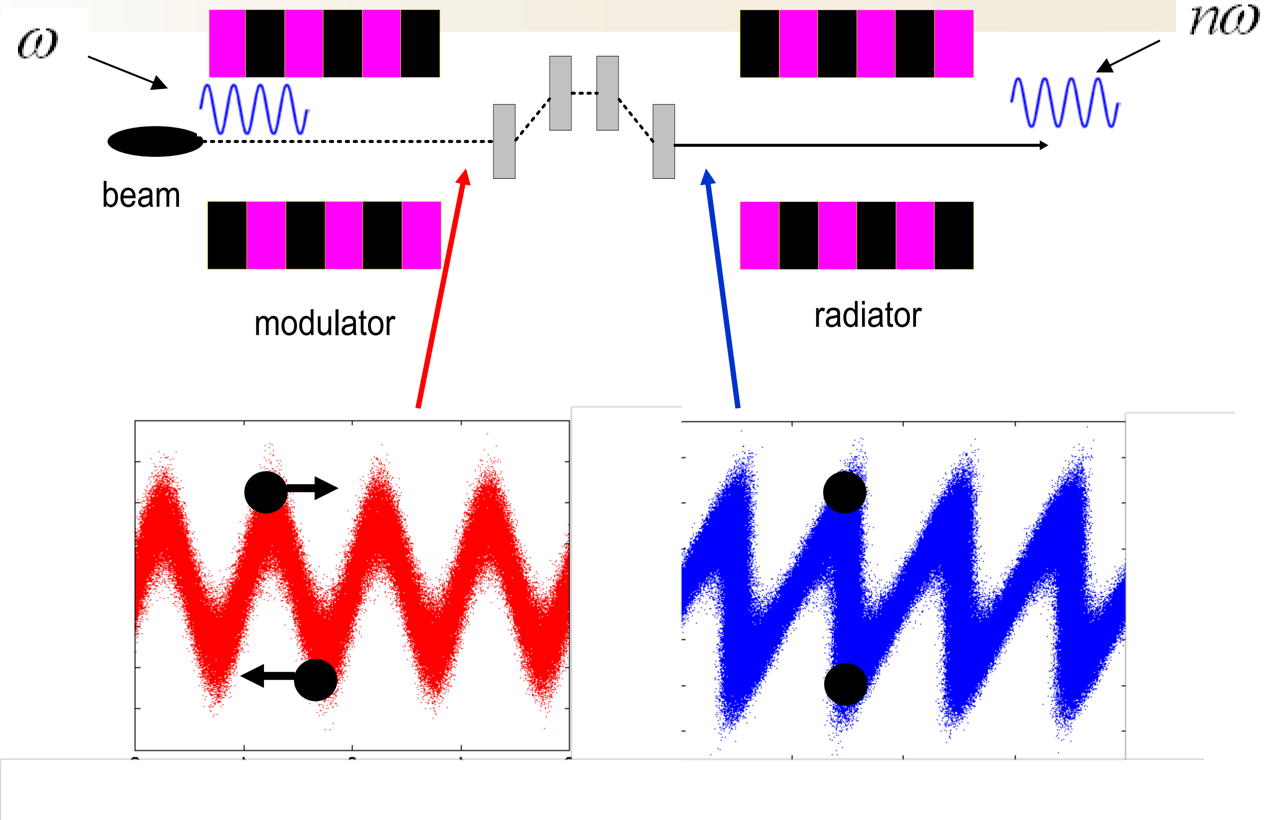
FEL-generated EM wave

- Various Self-seeding techniques (HXRSS and SXRSS)

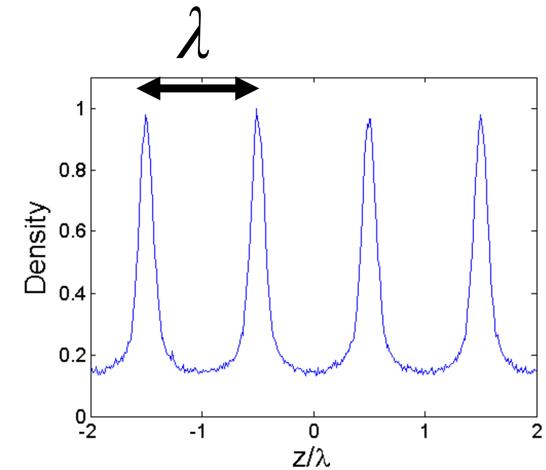
E-beam current modulation

- Echo-Enabled Harmonic Generation (EEHG)
- High Gain Harmonic Generation (HGHG)

Classical external seeding with HGHG



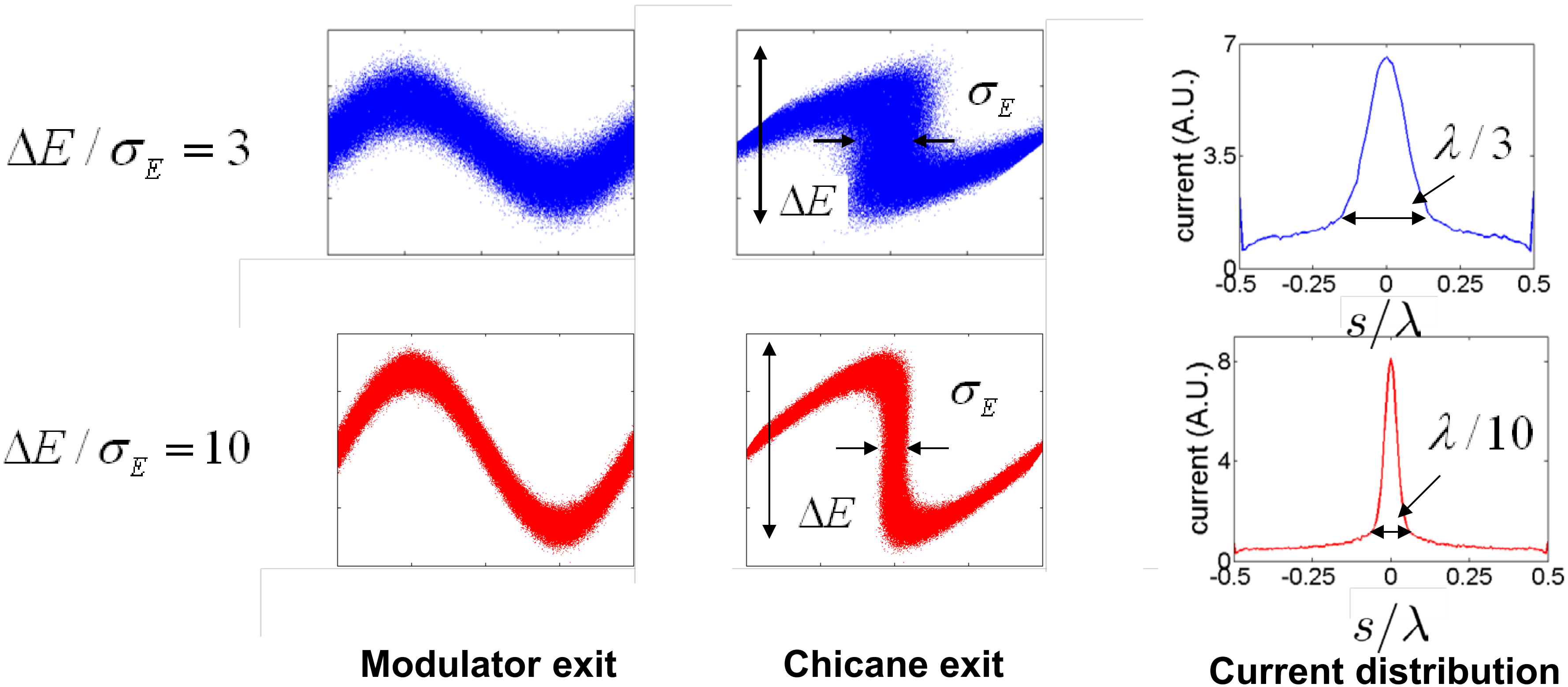
$$R_{56}\frac{\Delta E}{E}=\frac{\lambda}{4}$$



- **Energy modulation converted to density modulation with a chicane**
- **Coherent radiation at $n\omega$ amplified to saturation in a radiator**

Harmonics determined by width of current spikes

Single stage HGHG

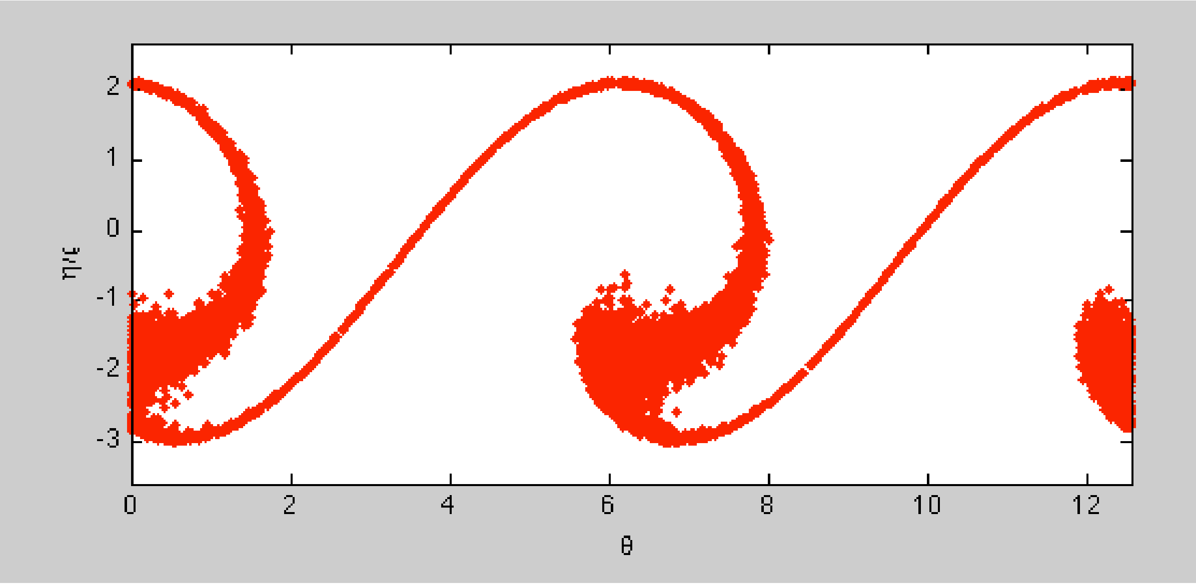


Width of spike determined by amplitude of energy modulation

$$\Delta s \simeq \lambda \frac{\sigma_E}{\Delta E}$$

Limitations on single stage HGHG

- **Harmonic up-frequency conversion efficiency:** $\Delta E/\sigma_E \simeq h$
- **FEL saturates when the energy spread reaches the bandwidth ρ**



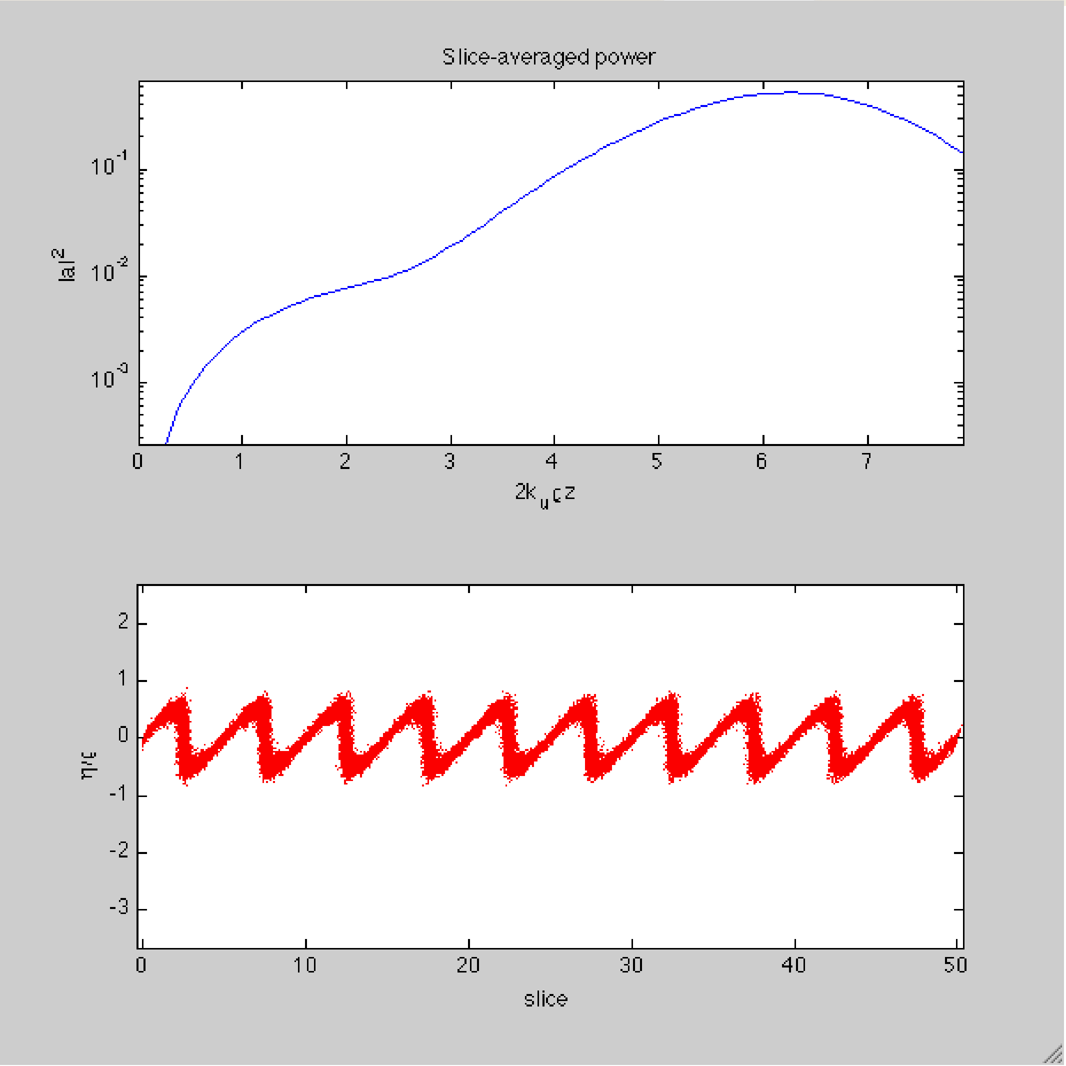
$$\Delta E/E \sim \rho$$

Laser modulation cannot be too large!

- **In modern machines:** $\rho \simeq 10\sigma_E/E$
- **Largest harmonic is therefore** $h \simeq 10 - 15$

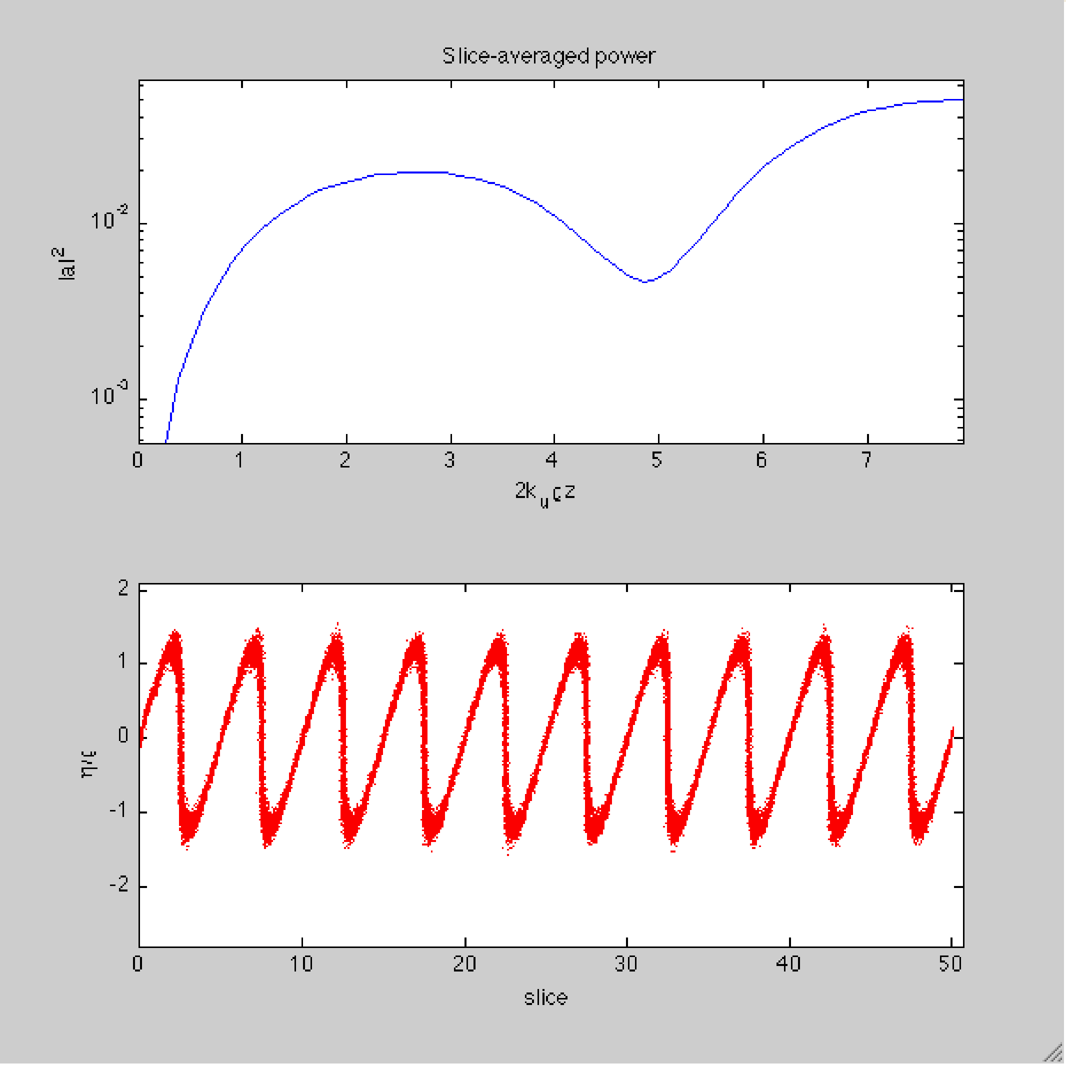
Example: 5th harmonic, small modulation

$$\Delta E/E < \rho$$



Example: 5th harmonic, large modulation

$$\Delta E/E > \rho$$



Reduced gain, loss of temporal coherence, poorer performance

New approaches

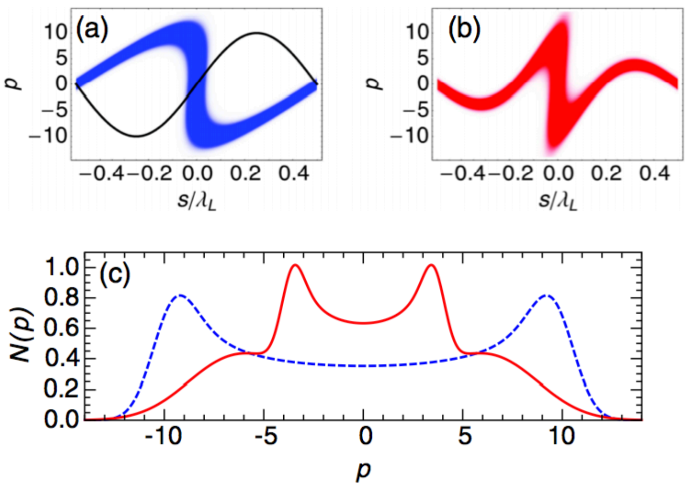
Unmodified HGHG is limited in harmonics by the relation

$$\frac{\rho}{\sigma_\eta} \simeq 10$$

Without some way of reducing slice energy spread, some other techniques are here needed.

- Avoid laser heater?
  - Reversible heater
- HGHG Cascading
- Partially remove residual energy modulation

B. W. J. McNeil, G. R. M. Robb, and M. W. Poole, PAC (2005)
E. Allaria and G. De Ninno, PRL. 99, 014801 (2007)
Q. Jia, APL 93, 141102 (2008).



What about collective effects?

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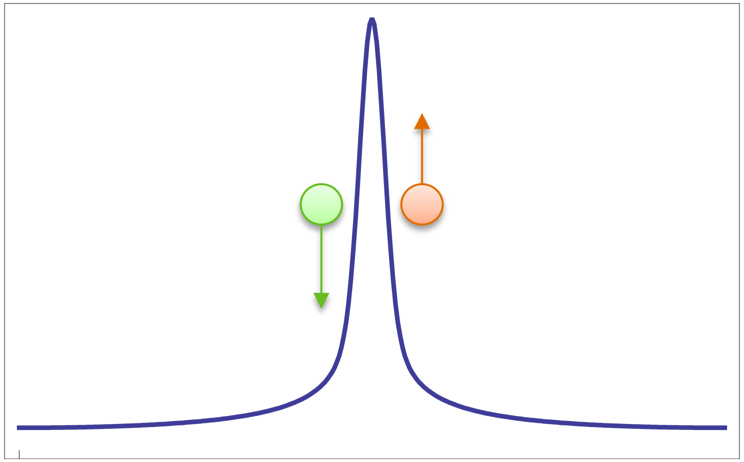
• Sharp density spikes generate strong local space charge fields on the scale of the laser wavelength and at harmonics

electrons in back of the spike feel a force pushing them backward, ***decreasing* *their energy***



electrons in front of the spike feel a force pushing them forward, ***increasing* *their energy***

The phase space transformations that generate bunching for HGHG generate an ideal initial distribution to take advantage of the energy kick from longitudinal space charge (LSC) fields

Shape of the LSC forces

Beam distribution function

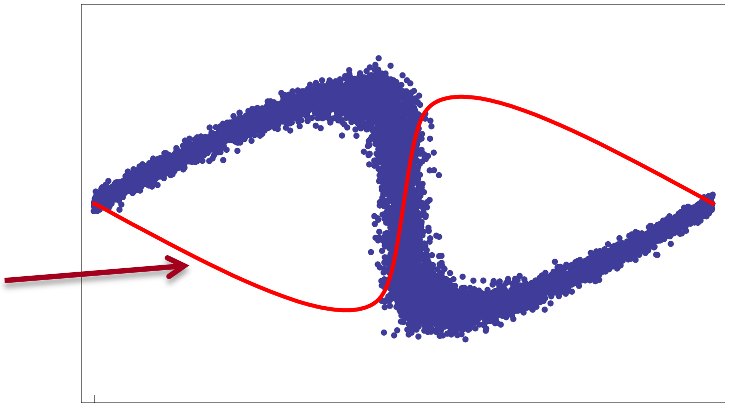
$$f_s(s) = 1 + 2\sum_{n=1}^{\infty} b_n \cos(nks)$$

Longitudinal space charge field

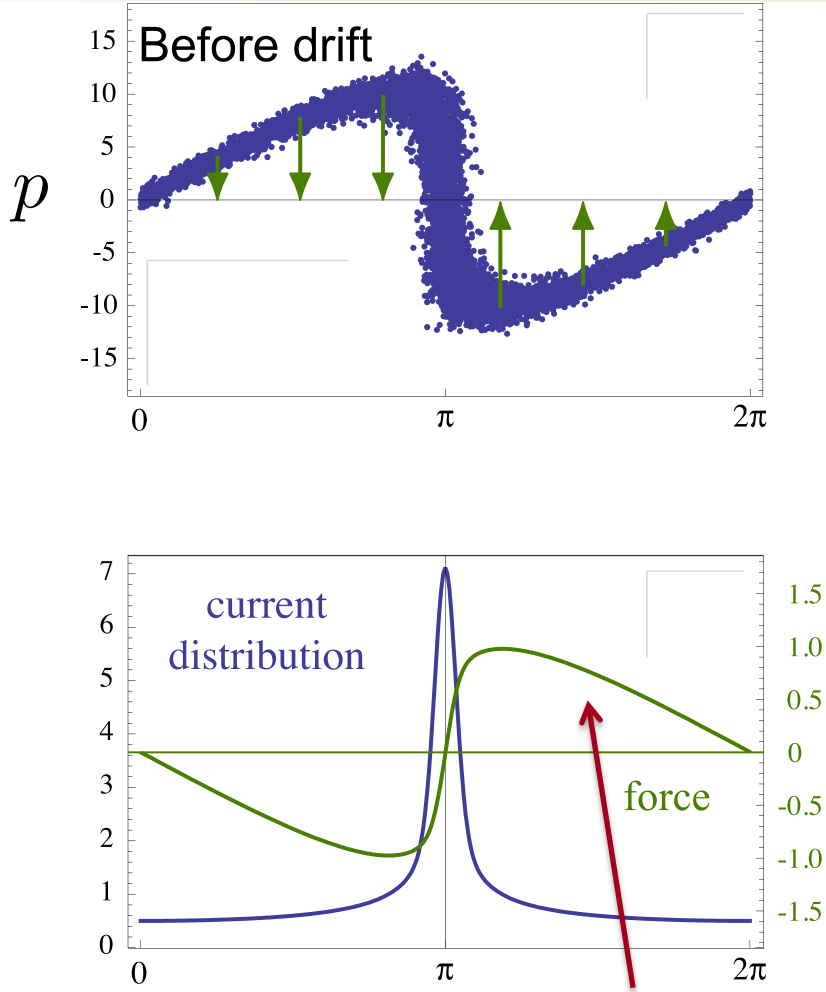
$$\frac{dE_z}{ds} = \frac{qn_0}{\epsilon_0} f_s(s)$$

Longitudinal space charge force

$$F_z = \frac{2q^2 n_0}{\epsilon_0 k} \sum_{n=1}^{\infty} b_n \frac{\sin(nks)}{n}$$

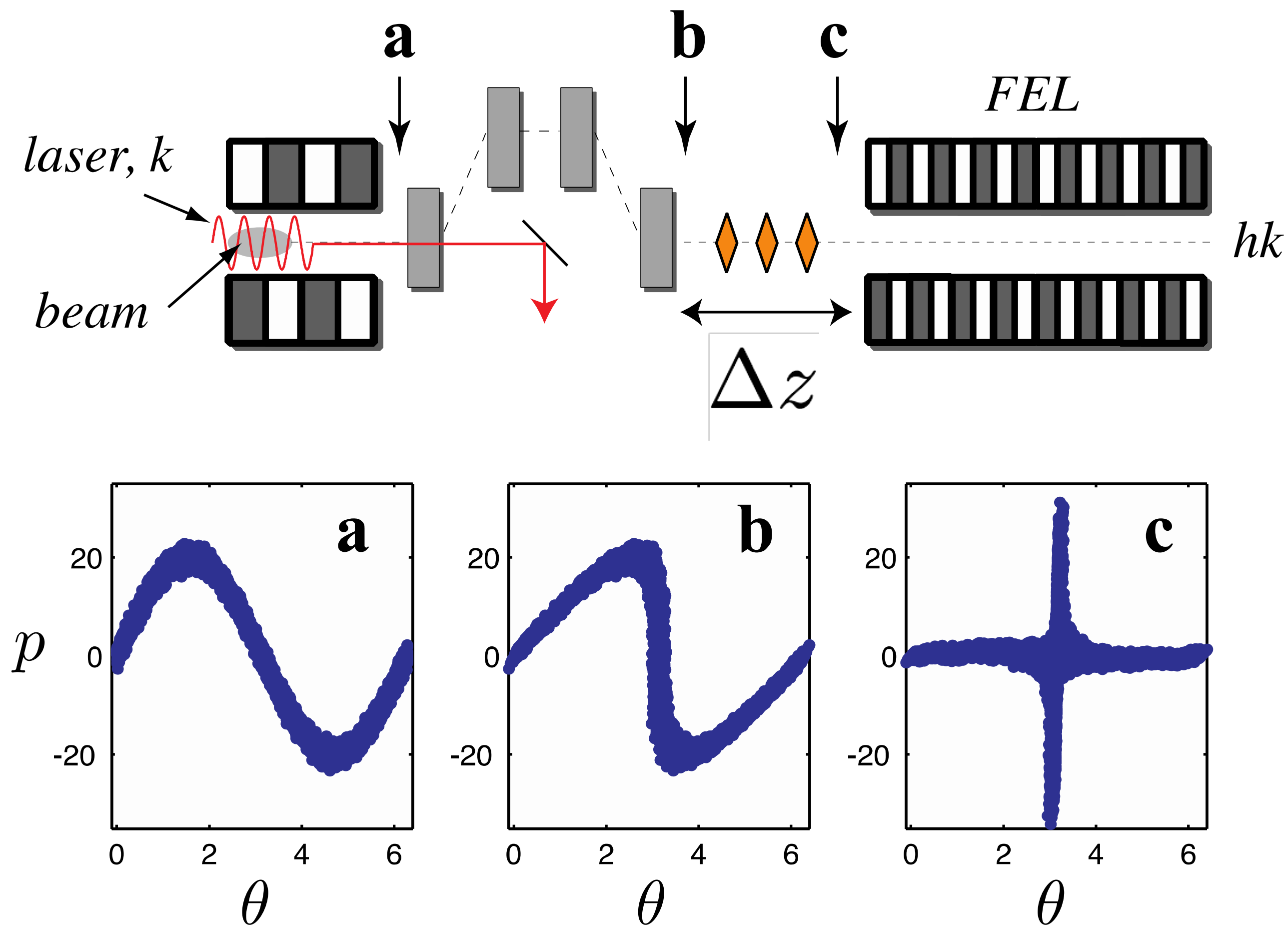


Quieting the energy modulation



LSC force distribution is a mirror of the initial phase space distribution

Quieted Harmonic Generation (QHG) Layout



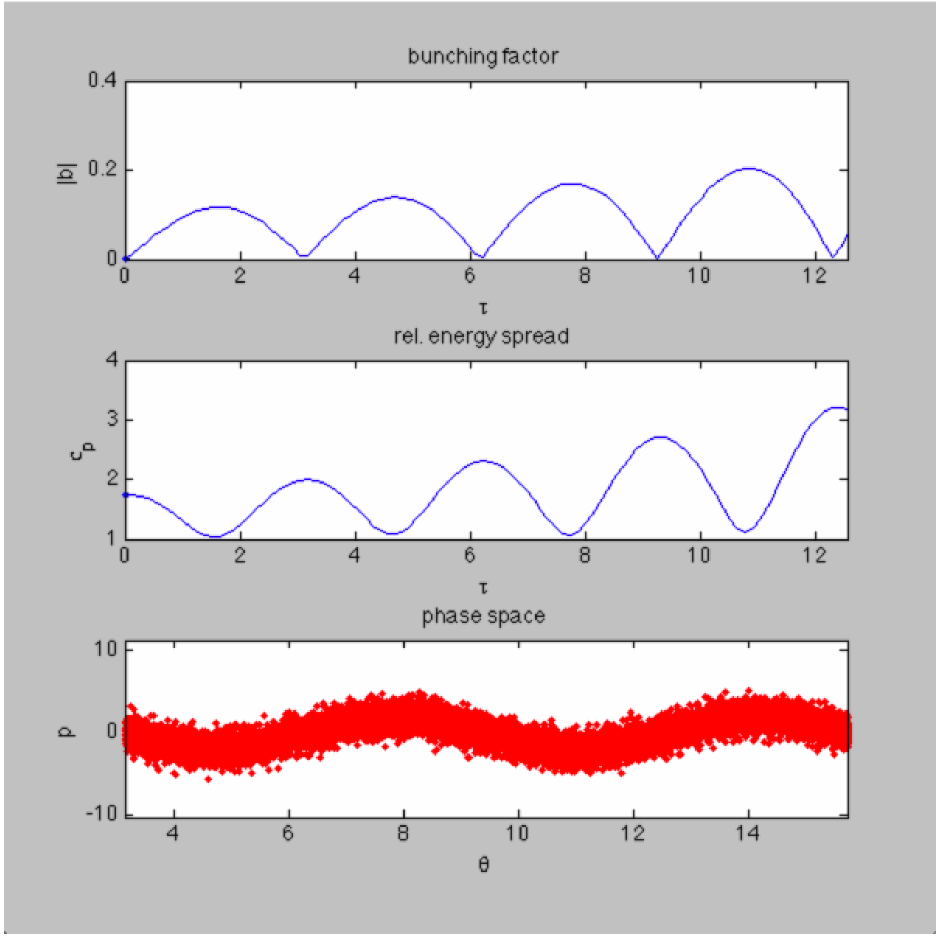
LSC fields naturally phase locked to microbunching structure

Comparison with linear plasma oscillation

What about longitudinal motion?
Need to preserve bunching factor.

- In simple linear LSC oscillations, particles change in energy and position (simple harmonic oscillator)
- Energy modulation is converted to density modulation over $\pi/2$ plasma phase advance

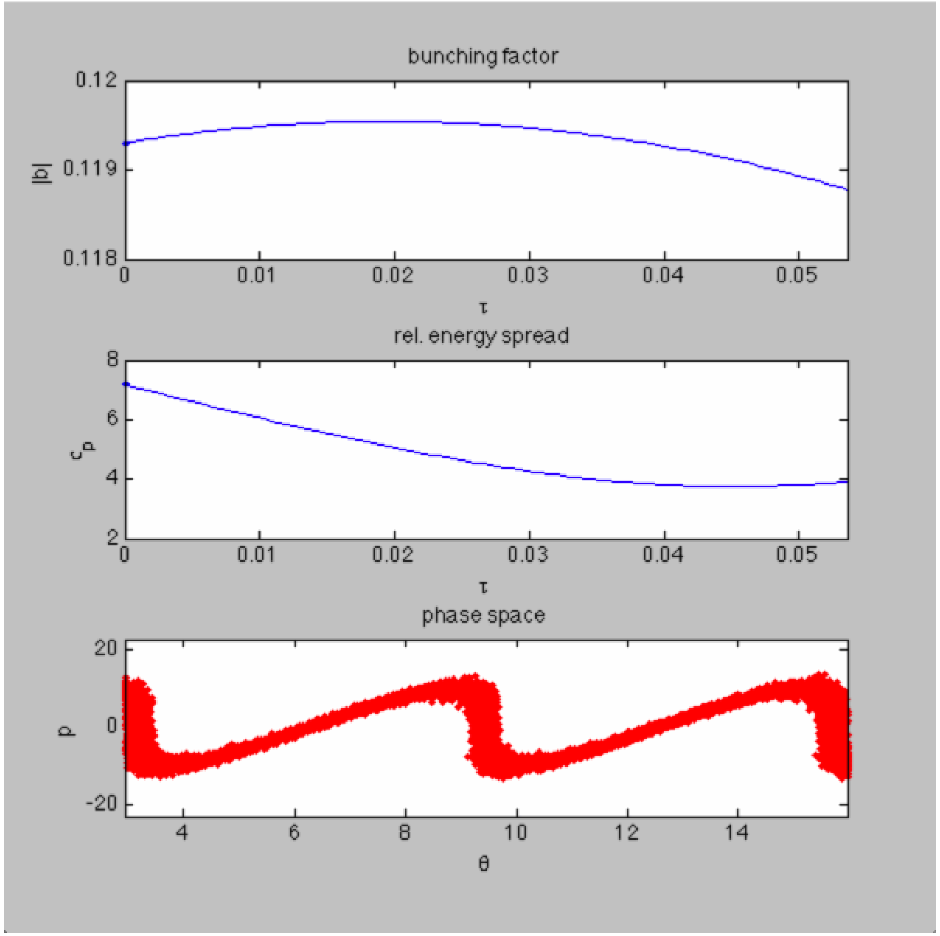
2 plasma periods ($k_p z=4\pi$)



Energy modulation ⇔ Density modulation

Comparison with linear plasma oscillation

$k_p z << 1$



In QHG, the bunching is strong enough that the motion of the particles in phase space is dominated by a change in energy, rather than in position

=> bunching can be preserved

For QHG we only need a small fraction of the plasma period $k_p z$.

1D description

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Coupled phase space equations

$$\frac{d\eta}{dz} = \frac{q}{\gamma m c^2} E_z$$

$$\frac{ds}{dz} = \frac{\eta}{\gamma^2}$$

$$\frac{dE_z}{ds} = \frac{q n_0}{\epsilon_0} f_s(s)$$

Relative energy

$$\eta = \frac{\gamma - \gamma_0}{\gamma_0}$$

Longitudinal position

$$s = z - \beta c t$$

Nonlinear LSC oscillation in scaled variables

$$\frac{dp}{d\tau} = \frac{2}{\alpha} \sum_{n=1}^{\infty} b_n \frac{\sin n\theta}{n},$$

$$\frac{d\theta}{d\tau} = \alpha p.$$

Scaled relative energy

$$p = \frac{\eta}{\sigma_\eta}$$

Phase position in bunched beam

$$\theta = ks$$

Plasma phase

$$\tau = k_p z$$

Dynamical evolution parameter

$$\alpha = \frac{k\sigma_\eta}{k_p \gamma^2}$$

(α is the ratio of the longitudinal displacement due to thermal motion in a plasma period to the laser wavelength)

QHG Scaling from linearized model

Required plasma phase advance

$$\Delta\tau \simeq \alpha h$$

Longitudinal motion constraint…

$$\Delta\theta < 1/h$$

…gives plasma advance constraint…

$$\Delta\tau < \sqrt{1/h}$$

… which defines required beam quality:

$$\alpha < \sqrt{1/h^3}$$

Relative energy modulation

$$A = \Delta E/\sigma_E$$

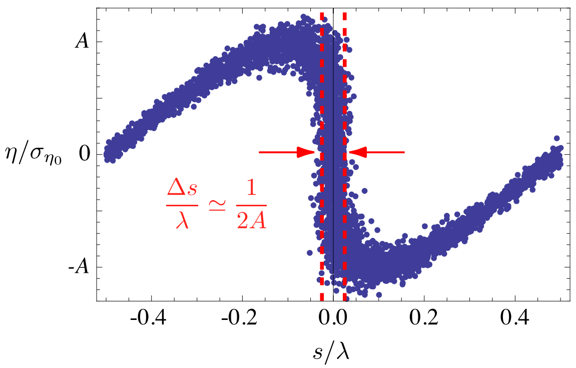
Scaled dispersion

$$B = kR_{56}\sigma_E/E_0$$

Bunching factor

$$b_h = e^{-(hB)^2/2} J_h(-hAB)$$

Bunching optimized at $A = 1/B = h$



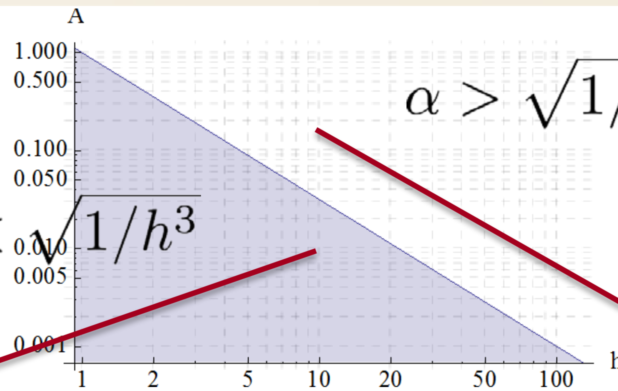
Beam quality determines performance

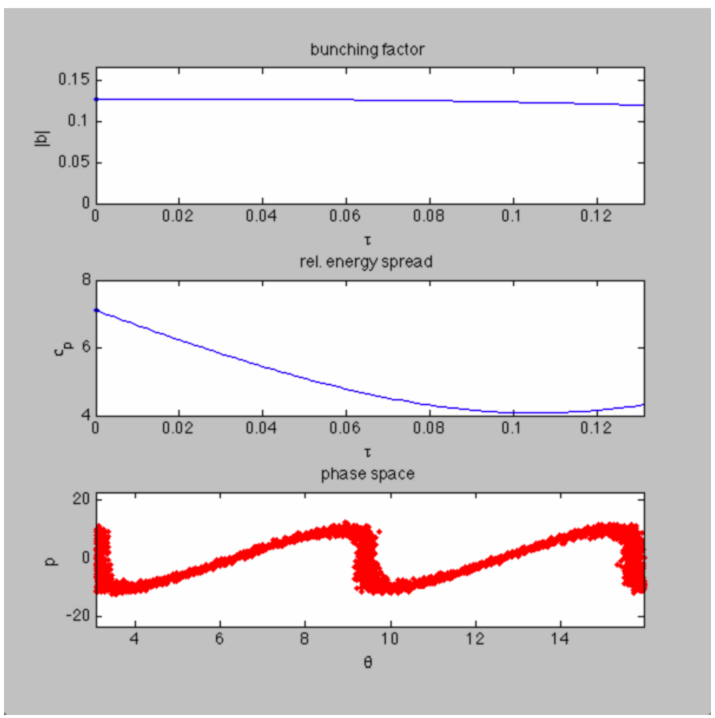
In both cases

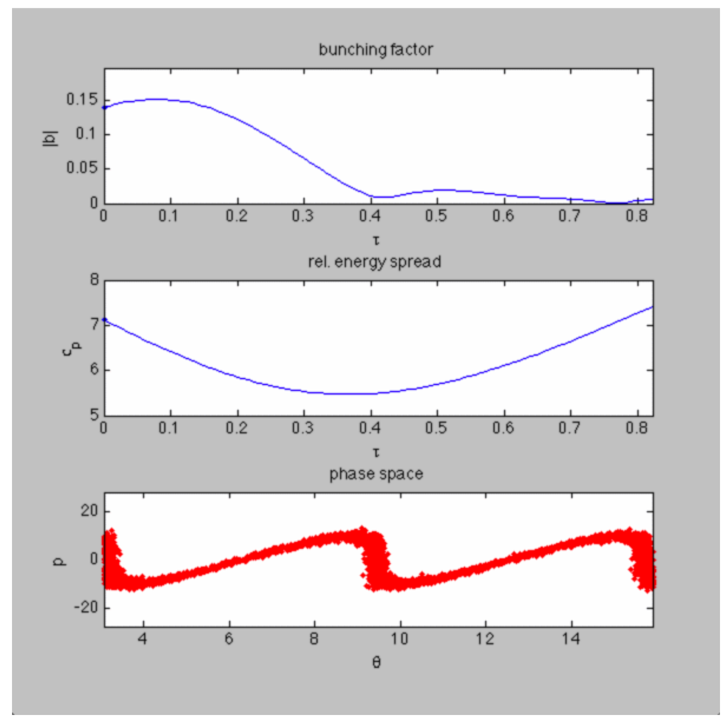
$$\Delta\tau \simeq \alpha h$$

$\alpha > \sqrt{1/h^3}$

$\alpha < \sqrt{1/h^3}$



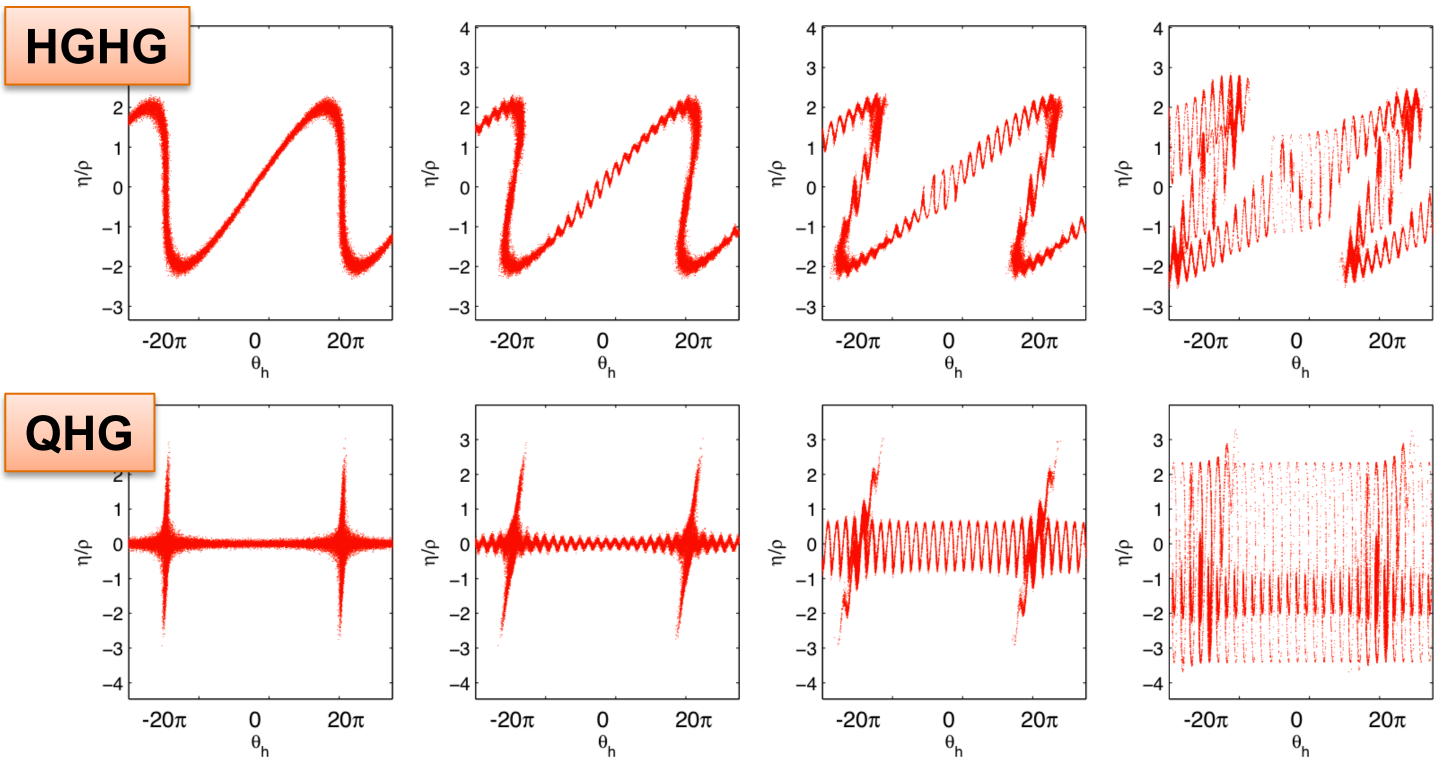




1D FEL performance comparison (h=20)

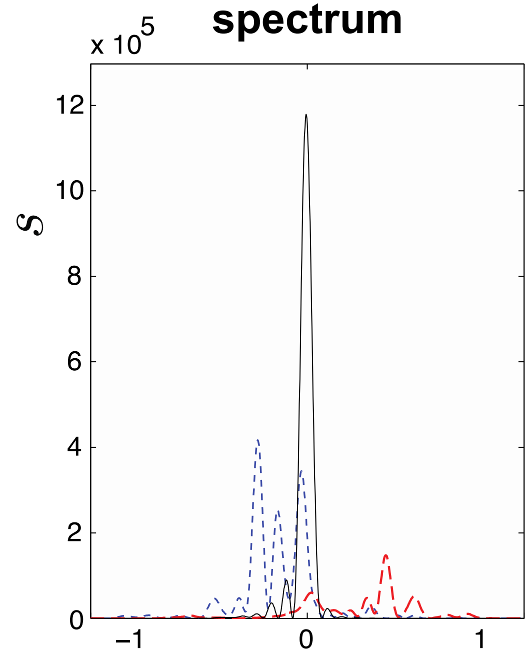
Phase space evolution during lasing at harmonic

- **HGHG beam carries chirped region larger than FEL bandwidth, so frequency competition suppresses gain and destroys temporal coherence**
- **QHG beam has more electrons contribute evenly to lasing**



1D FEL performance comparison (h=20)

Power vs z **Power vs t** **spectrum**



$2k_u\rho z$

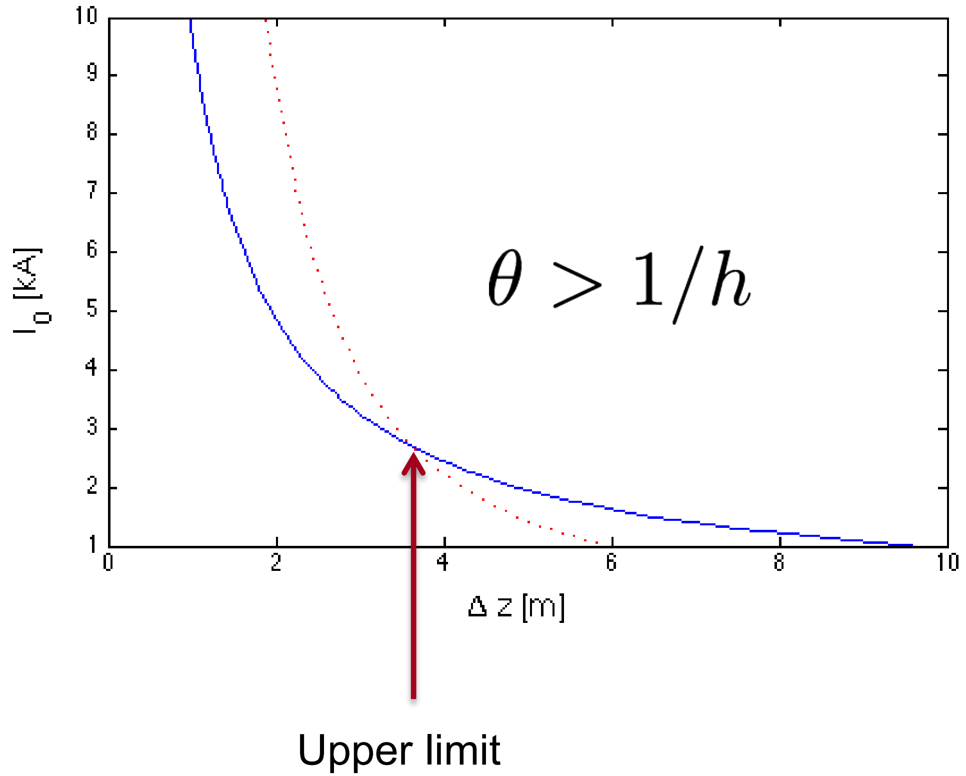
QHG saturates faster, is temporally coherent and has bandwidth <<ρ for long beams

Ex: physical drift length in 1D model

Uniformly filled round beam

- radius r_b=150 μm
- E_0=1 GeV
- $\sigma_\eta=10^{-4}$
- λ=240 nm
- h=20
- (α=0.009)

Required drift length



3D effects

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1D limit assumes the beam is large transversely compared to the laser wavelength in the beam frame:

$$\xi = kr_b/\gamma \gg 1$$

But α cannot be made arbitrarily small without consideration of this limit. In physical units (uniform round beam of radius r_b):

$$\alpha = \xi \sigma_\eta \sqrt{\frac{\gamma I_A}{4 I_0}}$$

Alfven current

Beam current

Modified evolution equations

$$\frac{dp}{d\tau} = \frac{2}{\alpha} \sum_{n=1}^{\infty} \frac{b_n}{n} \sin(n\theta) F_n(R, \xi),$$

$$\frac{d\theta}{d\tau} = \alpha p.$$

Radial dependence of E_z

$$F_n \leq 1$$

$$R = r/r_b$$

Radial LSC field distribution and 3D scaling

Required plasma phase advance

$$\Delta\tau \simeq \alpha A / F_1(0,\xi)$$

Longitudinal motion constraint…

$$\Delta\theta < 1/h$$

…gives plasma advance constraint…

$$\Delta\tau < \sqrt{1/h}$$

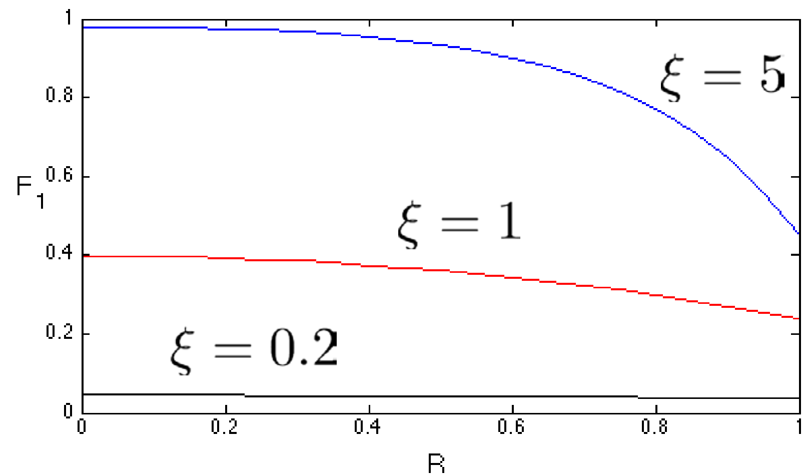
… which defines required beam quality:

$$\alpha < \sqrt{F_1(0,\xi)/h^3}$$

Uniform round beam of radius r_b

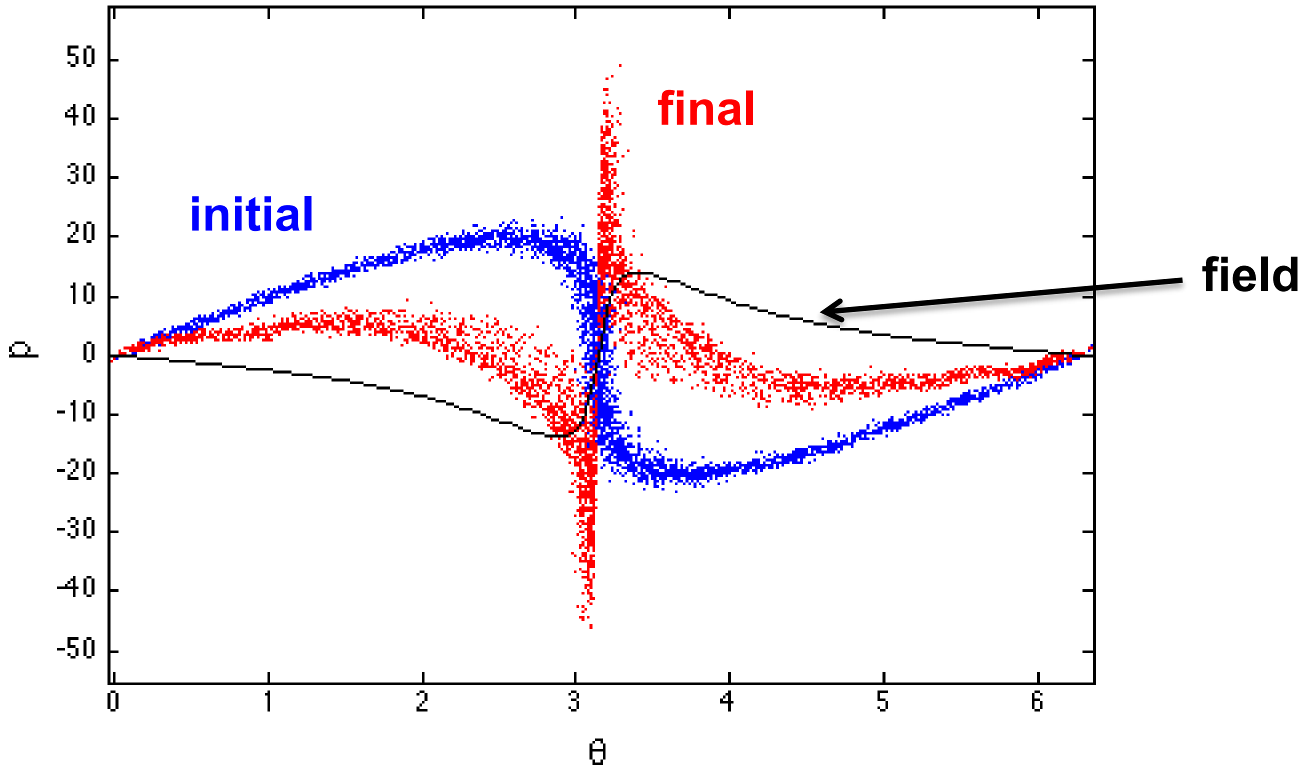
$$F_n(R,\xi) = 1 - n\xi I_0(n\xi R) K_1(n\xi)$$

F vs radius



LSC field amplitude is reduced as ξ<1

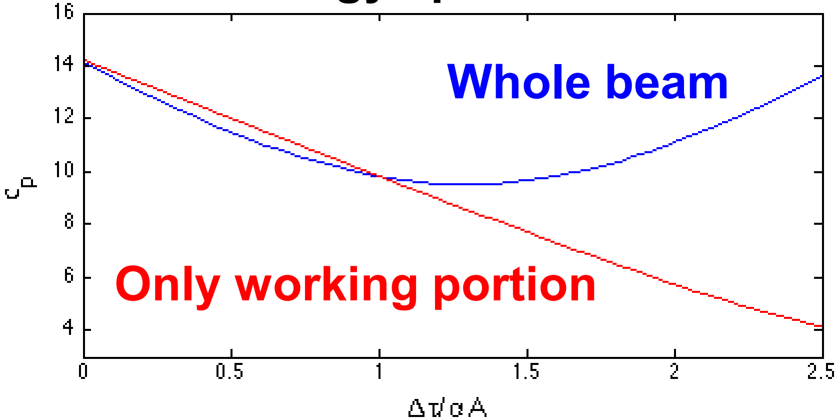
3D effects



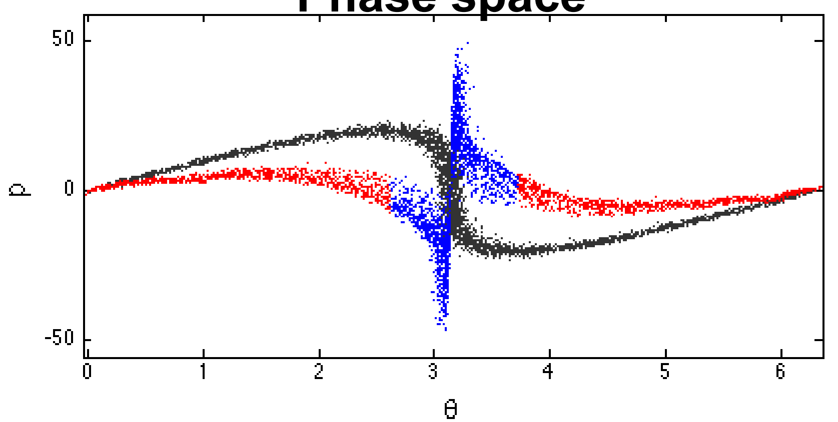
- **3D effects modify the relative amplitudes of the LSC harmonics**
- **LSC field no longer properly mirror the e-beam phase space distribution**

3D fields

rms energy spread vs drift



Phase space



- **Drift length must be increased to more completely reduce energy modulation in "working portion" outside density spike**

Summary

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Potential Advantages

- Access to higher harmonic numbers for HGHG
- Improved performance at modest harmonic numbers through reduction of energy modulation and chirp
- Minimal beam line modifications

Challenges

- Proper tuning of beam and laser modulator/chicane parameters required (no over dispersing)
- Sensitive 3D effects (can worsen phase space)
- Need more detailed realistic simulations to determine practical feasibility on real beams (start to end, emittance, etc)

Thank you!

3D effects