CHIRPED PULSE SUPERRADIANT FREE-ELECTRON LASER*

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Abstract

When a short electron bunch traverses an undulator to radiate a wavelength longer than the bunch length, intense superradiance from the electrons can quickly deplete the electron's kinetic energy and lead to generation of an isolated chirped radiation pulse. Here, we develop a theory to describe this chirped pulse radiation in such a superradiant FEL and show the opportunity to generate isolated few-cycle high-power radiation through chirped-pulse compression after the FEL.

INTRODUCTION

High power radiation is useful for applications requiring high energy density. In view of the high-power laser successfully demonstrated by optical chirped pulse amplification, a chirped-pulse FEL followed by a pulse compressor could be an ideal candidate to generate extremely high power radiation in the spectrum no readily accessible by a conventional laser source. It has been suggested previously the use of an energy-chirped electron beam to amplify a frequency-chirped seed laser in an FEL amplifier to obtain temporally compressed high-power radiation at one of the harmonics of the seed laser [1]. It was also suggested the use of an energy-chirped electron pulse to generate self-amplified chirped radiation, which is then filtered to seed a downstream self-amplified-spontaneous-emission (SASE) FEL [2]. Wu *et al.* [3,4] pointed out that manipulating chirps in seed radiation and electron beam to an FEL allows generation of attosecond few-cycle pulses. In this paper we propose a scheme to generate a chirped pulse radiation directly from fast energy depletion of a short electron bunch in an undulator. An external compressor is then used to compress the chirped pulse to achieve few-cycle radiation.

THEORY

From energy conservation, the loss rate of the total electron kinetic energy is equal to the radiation power. To take into account all radiation energy at the expense of the electron kinetic energy, we started from the expression of the coherent synchrotron radiation power of a tightly bunched charge. Given initial electron energy $\gamma = \gamma_0$ at the retarded time $t' = 0$, one can derive the Lorentz factor γ as a function of t' in the relativistic limit $\gamma >> 1$

$$\gamma = \frac{\gamma_0}{1 + t'/\tau_d}, \tag{1}$$

where the pump depletion time τ_d is defined as

$$\tau_d = \frac{W_0}{P_{r,0}}, \tag{2}$$

with W_0 being the initial energy of the electron bunch and $P_{r,0}$ being the initial radiation power of the electrons.

The radiation power is proportional to the square of γ and can thus be expressed as

$$P_{r,N_e}(t') = \frac{P_{r,0}}{(1 + t'/\tau_d)^2}. \tag{3}$$

Furthermore, the wavelength of the dominant radiation mode at t' satisfies

$$\lambda_r = \lambda_u \frac{1 + a_u^2}{2\gamma^2(t')}, \tag{4}$$

where λ_u is the undulator period and $a_u = eB_{\mathrm{rms}}/m_0 c k_{\mathrm{u}}$ is the undulator parameter with $k_{\mathrm{u}} = 2\pi/\lambda_u$, m_0 the electron rest mass, e the electron charge, c the vacuum speed of light, and B_{rms} the rms undulator field. Given a known relationship between the retarded time t' and observation time t, the radiation power in (3) and wavelength in (4) can be expressed in terms of the observation time.

By using Eqs. (3,4), one can write the temporal-dependent radiation field as

$$E(t_n) = \frac{E_0}{(3r_\tau t_n + 1)^{2/3}} \exp[j\phi(t_n)] \tag{5}$$

where E_0 is the maximum or the initial radiation field, $r_\tau = \tau_u/\tau_d$ is the ratio of the electron transit time through the undulator τ_u to the electron energy depletion time τ_d, and $\phi(t_{\mathrm{n}})$ is the radiation phase as a function of time. The time variable $t_{\mathrm{n}} = t/(N_{\mathrm{u}}\lambda_{r0}/c)$ is the observation time normalized to the unperturbed slippage time $N_{\mathrm{u}}\lambda_{r0}/c$ with N_u being the number of undulator periods, and λ_{r0} being the initial radiation wavelength. The time duration $t_{\mathrm{n}} = 1$ is the radiation pulse width or the electron slippage length in the undulator without pump depletion. With pump depletion, the electron slows down when traversing the undulator and the time duration $t_{\mathrm{n}} - 1$ gives the amount of thus increased radiation pulse width. Physically r_τ is a figure indicating the degree of pump depletion in a given undulator. The frequency chirp of the radiation is embedded in the radiation phase

$$\phi(t_n) = 2\pi \frac{N_u}{r_\tau}(3r_\tau t_n + 1)^{1/3} + \phi_0 \tag{6}$$

where ϕ_0 is an arbitrary initial phase.

Figure 1 plots the chirped radiation field as a function of the normalized observation time for $r_\tau = 1$ from a 20-peirod undulator. There are two important features in the plot; the field amplitude reduces and the radiation wavelength increases over time, both due to energy loss of electrons to superradiance in the undulator.

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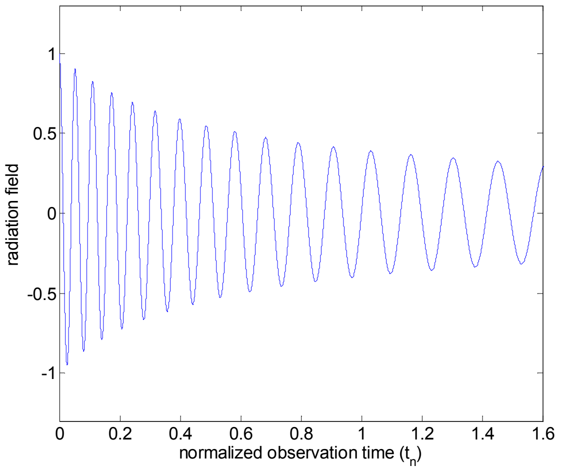

Figure 1: Fequency chirped radiation field from a CPS FEL with $r_\tau = 1$ and $N_u = 20$.

Previously we have assumed a particle-like electron bunch with zero size. To take into account the finite electron bunch length, the radiation power in Eq. (3) is multiplied by the so-called bunch form factor [5], which for a Gaussian electron bunch is given by

$$F(\upsilon_r) = e^{-(2\pi\upsilon_r\sigma_t)^2}, \qquad (7)$$

where υ_r is the radiation frequency and σ_τ is the rms temporal length of the electron bunch. As the electrons propagate down an undulator and radiate, the radiation frequency υ_r decreases due to slowdown of electrons and σ_τ could increase due to debunching. If the multiplication $\upsilon_r\sigma_\tau$ remains approximately a constant in an undulator, the modification needed to be done for the above theory is to divide the pump depletion time τ_d in Eq. (2) by a constant form factor.

THZ AND EUV CPS FEL

In the following we first present a design example for a proof-of-principle experiment at THz frequencies using existing accelerator technologies and one at extreme-ultra-violet (EUV) frequencies driven by a future dielectric laser accelerator (DLA) [6].

For the first design example at THz frequencies, we choose an accelerator system consisting of an S-band photoinjector followed by linear accelerators for electron injection, velocity bunching, and particle acceleration. The accelerator system is expected to generate an electron beam with some nominal parameters: rms electron bunch length = 60 fs, bunch charge = 15 pC, mean electron energy $\gamma = 30$, rms relative energy spread $\Delta\gamma/\gamma = 0.5\%$, positive energy chirp with head-to-tail energy difference $\Delta\gamma = 0.8$, rms normalized emittance = 0.1 mm-mrad, and rms beam radius = 0.5 mm. The initial energy chirp is helpful to maintain a small electron bunch length and energy spread in the undulator. We assume here a 1.5 m long planar undulator with a period of 56 mm and undulator parameter of 1.7, permitting radiation generation at 2.5 THz for a beam with $\gamma = 30$. To confirm maintenance of a short electron bunch in the undulator, we employ the simulation code, General Particle Tracer (GPT) [7], to simulate the propagation of the electron bunch through the undulator. Our simulation result shows that the electron bunch in the undulator undergoes compression first due to the initial positive energy chirp in the beam and is kept in a length of 58±7 fs. The initial energy chirp in the beam also helps to partially compensate the space-charge induced energy spread. To predict the chirped radiation field, we first calculate the bunch form factor with σ_τ = 60 fs and use it to modify the pump depletion time in our theory. At the undulator exit, the calculated degree of pump depletion is $r_\tau = 12\%$. Figure 2 shows the radiation pulse envelope before (blue curve) and after (red curve) pulse compression by using a standard quadratic phase filter [8] with a chirping parameter of 0.0524 in units of the inverse square of the normalized observation time. For comparison, the transform-limited pulse (black dashed line) is also shown in the figure. It is seen that almost all the compressed pulse energy is already in the width of the transform-limited pulse. The full width of the chirped pulse is 12 ps, containing 27 radiation cycles generated from the 27 periods of the undulator. The power of the chirped pulse gradually drops to about 63% of its initial value due to pump depletion. At the end of the undulator the percentage bandwidth of the chirped radiation is broadened to 20.4%. After compression, the peak power of the output radiation pulse is increased nearly 10 times. The half width of the compressed pulse is about 1.3 ps, containing approximately 3 radiation cycles.

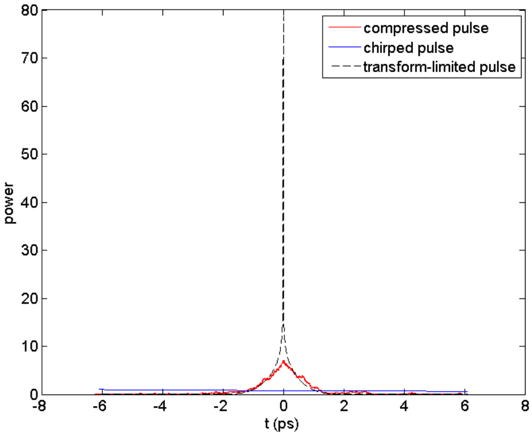

Figure 2: Power envelopes of the radiation pulse before (blue curve) and after (red) compression using an optimized quadratic phase filter for the proposed proof-of-principle CPS FEL at 2.5 THz. After pulse compression, the number of radiation cycles is reduced from 27 to just 3. For comparison, the transform-limited pulse is shown as a black dashed curve.

It is envisaged that a DLA, driven by a laser at ~ 1 μm, would generate electron bunches with a length in the nm range. Give the high repetition rate from the driver laser and the nm-bunched electrons, a CPS FEL could be useful to generate EUV and soft x-ray radiations with both high average and peak powers. As a design example at the EUV wavelengths, we assume a DLA generating an electron bunch with initial energy of $\gamma = 200$ and bunch length of ~1 nm. Debunching of electrons in an undulator is much less concerned for a 100 MeV beam. To integrate with a DLA, a dielectric laser undulator [9] is selected for this design, having a peak undulator field of 3.3 T. The 3.3 T undulator field is consistent with a 1 GV/m laser field near the damage threshold of a dielectric for an incident fs laser pulse [10]. The undulator length is 157.5 mm long, consisting of 150 undulator periods with a period length of 1.05 mm. Therefore, the initial radiation wavelength is 13.5 nm. The bunch charge for injection is 75 *fC*, assuming the electron charge from an accelerator can be scaled by its driving wavelength. The calculated degree of pump depletion for this design is $r_\tau = 5.2\%$.

Figure 3 shows the radiation pulse envelope before (blue curve) and after (red curve) pulse compression by using a quadratic phase filter with a chirping parameter of 0.021 in units of the inverse square of the normalized observation time. The full length of the chirped pulse is 7.3 fs. There are 150 radiation cycles in the pulse, generated from the 150-period undulator. The power of the chirped pulse gradually drops to about 80% of its initial value of 760 W due to pump depletion. At the end of the undulator the percentage bandwidth of the chirped radiation is broadened to nearly 10%. After compression, the peak power is increased by 13 times, reaching a value of 1 kW. The half width of the compressed pulse is about 0.4 fs, containing 9 radiation cycles. The time-bandwidth product of the compressed pulse is ~0.5, which is about the value for a transform-limited Gaussian pulse.

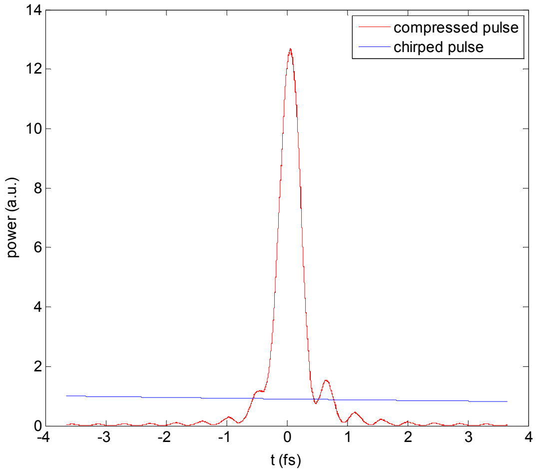

Figure 3: Power envelopes of the proposed EUV CPS FEL radiation pulse before (blue curve) and after (red) compression by a quadratic phase filter. After pulse compression, the peak power of the radiation pulse is increased by about 13 times and the number of the radiation cycles reduces from 150 to 9.

CONCLUSION

A short electron bunch emitting superradiance in an undulator can quickly lose its kinetic energy and generate a chirp-pulse radiation. We have developed a theory to describe the radiation field of such a CPS FEL. The phase of the chirped field strongly depends on how fast the electron energy is depleted in an undulator. In the linear regime where the degree of pump depletion is kept below 10%, we show two design examples of CPS FEL in the THz and EUV spectra that generate few-cycle radiation after pulse compression by using standard quadratic phase filters. In the regime of strong pump depletion, we suggest to use tailor designed chirped mirrors or gratings to compensate the nonlinear chirp in the radiation field to achieve transform-limited pulse compression. A future effort would include numerical simulation of the CPS FEL subject to an arbitrary input beam.

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