USING LORENTZ TRANSFORMATIONS FOR SIMULATIONS OF WIGGLER SUPERRADIANCE FROM THE PICOSECOND ELECTRON BUNCHES*

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Abstract

In this paper we present a theoretical analysis of superradiance (SR) from picosecond electron bunches wiggling in periodical undulator field based both on the method of averaged ponderomotive force and on a direct numerical PIC (particle-in-cell) simulation. Within both approaches the analysis takes place in the reference frame co-moving with electrons which allows simplifying the procedure of simulation significantly due to the fact that all the spatial scales including the radiation wavelength, the length of the beam and the length of the pump field pacet into which the undulator field is transformed are of the same order. We show that in the reference frame the SR effect can be interpreted as a formation of the distributed Bragg mirror in the bulk of the electron beam which is effectively reflecting (scattering) the pump wave. A possibility of generation of multimegawatt pulses in terahertz and far infrared wave ranges is demonstrated.

INTRODUCTION

Recently, a significant progress has been achieved in generation of ultrashort electromagnetic pulses in centimeter and millimeter waveband based on superradiance (SR) of high-current electron bunches [1]. Generated pulses are characterized by record-breaking (gigawatt) peak powers. As it was shown both theoretically and experimentally at particle energies of ~300 keV, currents ~1 kA and the bunch durations ~ 1 ns the most effective mechanism of SR pulses generation is the Cherenkov one, realized in a periodic slow-wave structure.

The advancement of SR sources further into short wave ranges can be obtained by using the emission of electron bunches moving in the undulator field. In this case the particles energy should be increased up to 4-5 MeV and the bunch duration should be about several picoseconds. The bunches formed by photo-injection guns possess the necessary characteristics [2]. In this paper under assumption that the electron bunch propagates in a planar waveguide a theoretical analysis and KARAT PIC code simulations of above process were performed. In both approaches we analysis SR effects in the co-moving with electrons reference frame K' [3]. In particular, this allows to simplify the numerical simulation procedure significantly, because at relativistic factor values $\gamma \sim 10$ the length of undulator of about several meters according to Lorentz transformations turns into several tens of centimeters, whereas the length of picosecond electron bunch in the rest reference frame stretches up to several centimeters (see Fig. 1). Besides the radiated wavelength transforms from the submillimeter to millimeter range. Proportionality of all scales, including the transverse size of the bunch, allows us to simulate the processes without using significant computational resources, finding then the parameters of radiated pulses in the lab frame using again the relativistic transformations.

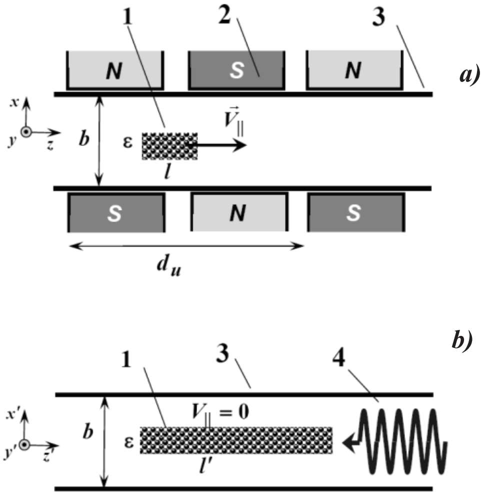

Figure 1: Scheme of generation of superradiance pulse (a) in the lab reference frame and (b) in the co-moving reference frame. (1) - electron beam, (2) - undulator, (3) - planar waveguide, (4) - electromagnetic TE wave, into which the undulator field transforms.

SIMPLIFIED MODEL

We consider here a two-dimensional model assuming that an electron bunch with a length of l moves in a planar waveguide with the gap between plates b_0 (See Fig. 1). Electrons oscillate in a planar undulator field with vector potential:

$$\vec{A}(z,t) = \mathrm{Re}\left(\vec{y}_0 A_u \exp\left[ih_u z\right]\right), \qquad (1)$$

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where $d = 2\pi/h_u$ is the undulator period, A_u is an amplitude. Further analysis will be conducted in the co-moving reference frame where the undulator field transforms into propagating pump wave

$$\vec{A}'(z',t') = \mathrm{Re}\left(\vec{y}_0 A'_u \exp\left[i\omega' t + ih' z\right]\right), \qquad (2)$$

where $\omega' = \gamma V_{||} h_u$, $h' = \gamma h_u$. In K' the electron bunch (layer) of finite length l' can be divided into a set of macroparticles differing by initial location z'_0.

Radiated field in the planar waveguide with a gap of b between the plates can be presented as a sum of TE modes with a frequency determined by the electron oscillations frequency and with different longitudinal wavenumbers $h'_n = \sqrt{k'^2 - g_n^2}$, $g_n = (2n-1)\pi/b$:

$$\vec{\mathbf{A}}_{\mathbf{s}} = \mathrm{Re}\left[\vec{y}_0 \sum_n A_n(z') \cos(g_n x') e^{i\omega' t' \pm i h'_n z'}\right] \qquad (3)$$

Mode amplitudes excited by one oscillating macro-particle in the instant of time when its coordinate is $z'(z'_0)$ can be found from Helmholtz equation:

$$\frac{d^2 A_n}{dz'^2} + h'^2_n A_n = -4\pi\alpha_u \theta_n e^{i h'_n z'(z'_0)} \delta\left(z' - z'(z'_0)\right), \qquad (4)$$

where $\theta_n = (2/b) \int\limits_{-\varepsilon/2}^{\varepsilon/2} \cos(g_n x') dx'$. Solution of (4) can be presented as a composition of two components propagating in $\pm z'$ directions:

$$A_n(x',z') = \frac{2\pi}{i h'_n} \alpha_u \sigma \theta_n e^{i h' z'(z'_0) - i h'_n |z' - z'(z'_0)|} \qquad (5)$$

Fields (5) acting together with (1) lead to the emergence of the averaged ponderomotive force $F_{pond} = -\frac{e^2}{2mc^2} \nabla \left\langle A^2 \right\rangle$ acting from one of the macroparticles on the other one. This force can be presented in the form:

$$\begin{gathered} F_{pond}(x',z') = \\ = \pi e \sigma \alpha_u^2 \sum_n \frac{k'}{h'_n} \theta_n f_n \left[k'\left(z' - z'(z'_0)\right)\right] \cos(g_n x'), \\ f_n(\zeta) = \mu_n^+ \cos(\mu_n^+ \zeta),\ \zeta > 0, \\ f_n(\zeta) = \mu_n^- \cos(\mu_n^- \zeta),\ \zeta < 0, \end{gathered} \qquad (6)$$

where $\mu_n^\pm = (h' \pm h'_n)/k'$, $k = \omega'/c$ This force is sign-variable and attracting near the particle. It is depicted in Fig. 2 for different values of the gap between the plates. We also take into account the Coulomb forces which can be written as:

$$F_C = -4e\sigma f_C(z),$$

$$f_C(z) = \frac{z}{\varepsilon}\left[R\left(\frac{\varepsilon}{z}\right) + \sum_{m=1}^{\infty} (-1)^m \left(R\left(\frac{mb - \varepsilon}{z}\right) + R\left(\frac{mb + \varepsilon}{z}\right) - 2R\left(\frac{mb}{z}\right)\right)\right]$$

Thus the dynamics of particles can be described by the following equations:

$$\begin{gathered} \frac{\partial Z(Z_0,\tau)}{\partial \tau} = P(Z_0,\tau), \\ \frac{\partial P(Z_0,\tau)}{\partial \tau} = \hat{F}_{pond} + \hat{F}_C, \\ \hat{F}_{pond} = \\ = -\sum_n \frac{\omega'^2_p}{4\omega' c h'_n} \theta_n^2 \frac{b}{2\varepsilon} \alpha_u^2 \int\limits_0^L \chi(\tilde{Z}_0) f_n\left(Z(Z_0,\tau) - Z(\tilde{Z}_0,\tau)\right) d\tilde{Z}_0, \\ \hat{F}_C = \frac{\omega'^2_p}{\pi \omega'^2} \int\limits_0^L \chi(\tilde{Z}_0) f_C\left(\frac{Z(Z_0,\tau) - Z(\tilde{Z}_0,\tau)}{k'}\right) d\tilde{Z}_0. \end{gathered} \qquad (7)$$

where $Z = \omega' z'/c$, $L = \omega' l' z/c$, $\tilde{p} = p'/mc$, $\tau = \omega' t'$, $\omega_p = \sqrt{4\pi e^2 \rho'/m}$, $\alpha_i = e A'_u / mc^2$. At initial moment all of the electrons are uniformly distributed over the longitudinal coordinate in the interval $[0, B]$. Normalized amplitudes of the component of scattered field which is radiated in the direction which is opposite to direction of pump wave are given by

$$\left|\alpha_s^+\right| = \frac{i\omega_p^2 \alpha_i}{2\omega^2} \left|\int\limits_0^B \exp\left(2iZ(\tilde{Z}_0)\right) d\tilde{Z}_0\right|.$$

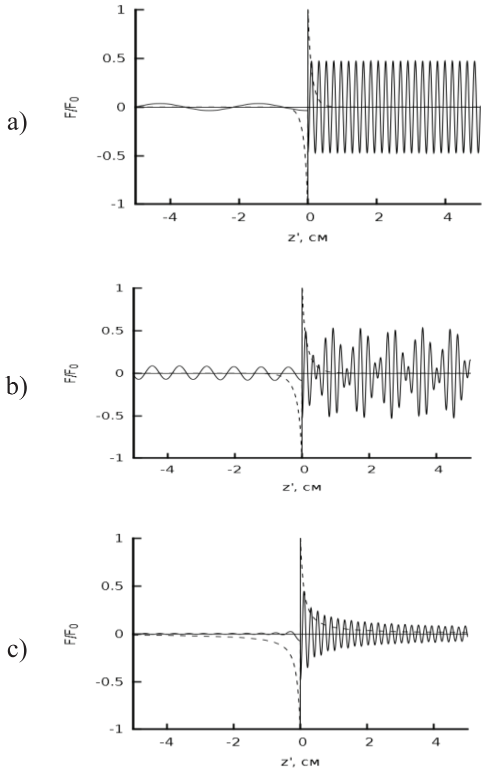

Figure 2: Dependence of the ponderomotive (solid line) and Coulomb (dotted line) forces on the longitudinal coordinate for a waveguide with one propagating mode (a, b=0.4 cm), with two propagating modes (b, b =0.7 cm) and in the free space (c, b=60 cm).

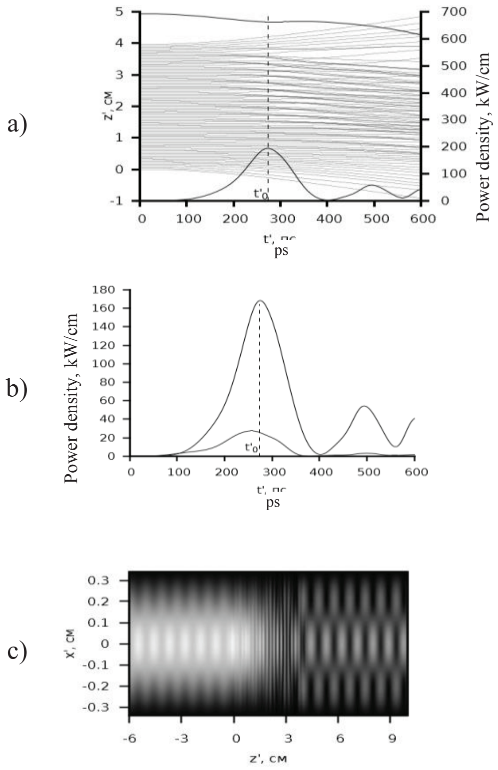

Figure 3: Simulation of the undulator SR process based on Eqs. (7): (a) bunching of electrons and time dependence of integral powers radiated forward and backward, (b) temporal dependencies of mode intensities with different transverse indices, (c) spatial distribution of a field at the instant of time corresponding to the peak amplitude. $L = 63$, $\alpha_u = 0.7$, $\varepsilon/b = 0.29$, $\omega'_p/\omega' = 0.024$

RESULTS OF SIMULATION WITHIN THE AVERAGED MODEL

Simulations of Eqs. (7) show that due to attracting feature of pondermotive force (6) the bunching of particles developed (Fig. 3a). Correspondingly at some instant of time (Fig. 3b) coherent summation of fields radiated by most of particles takes place and the generated field constitutes a short SR pulse. Initial level of signals corresponds to the level of radiation of the bunch without modulation and in fact is determined by the front edges of electron pulse. This radiation in the case under consideration is a seed initiating the further development of SR process.

We simulated the radiation of the electron bunch with particles energy of 5 MeV, with a length of ~4 mm, charge of 5 nC/cm (linear current density of 375 A/cm) and the transverse size $\varepsilon \sim 2$ mm oscillating in the field of undulator with period of 4 cm and field amplitude of 0.19 T. The radiation was assumed to take place in a planar waveguide with the gap between plates of 7 mm. In the rest frame K' the length of the bunch was 4 cm. The undulator field transforms into a wave with a wavelength of 4 mm and the power density of 5.6 MW/cm.

As follows from Fig.3b the peak radiation power density is 0.17 MW/cm and the pulse duration of 120 ps. By means of Lorentz transformations we obtain the power density of the radiation of the forward propagating wave in the lab frame of 70 MW/cm. Radiation frequency is about 1.5 THz and the pulse duration decreases down to 6 ps. Note that in the rest frame the pump pulse was about 20 wavelength. In the lab frame it corresponds to 20 period undulator with a total length of about 0.8 m.

RESULTS OF PIC SIMULATIONS IN CO-MOVING REFERENCE FRAME

The analysis of SR process within the model that employed the averaged ponderomotive force approach does not account for a number of important factors, including transverse inhomogeneity of the undulator field along the transverse coordinate. Another significant simplification is the assumption of relatively small undulator field leading to weak relativism of the electrons movement in co-moving reference frame. Thus, our semi-analytical consideration was supplemented by a direct PIC simulation in the rest frame, where, as we stated in the Introduction, different dimensions of the system are of the same scale.

We simulated the system dynamics using the KARAT PIC-code [4]. In the co-moving frame we simulated the situation when a pump pulse incidents on a stationary plasma bunch. Initial geometry of the interaction space is presented in Fig. 4a. Note that transverse stability pf the beam is provided by transverse inhomogeneity of the unudulator field which is illustrated by Fig. 4b where also the spatial modulation of the beam width with a period inversely proportional to the amplitude of the pump wave.

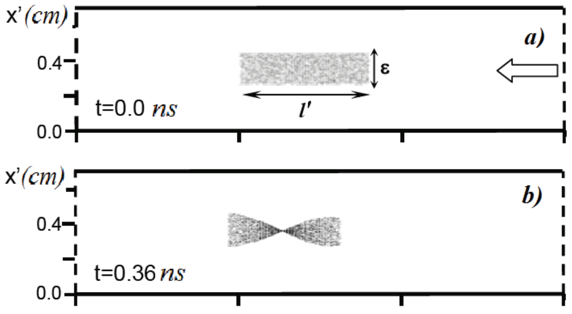

Figure 4: (a) Interaction space geometry and the initial position of the electron bunch. Arrow is the direction of the pump pulse propagation. (b) Electron bunch focusing under the transversely inhomogeneous pump field.

Simulation parameters correspond to those used in previous Section, although we took the larger undulator parameter $\alpha_u = 0.9$. Fig. 5a shows the SR pulse having in the rest frame power density 90 kW/cm and duration of 100 ps. Radiation spectrum is presented in Fig. 5b with a central frequency of 48 GHz. The frequency is downshifted with respect to the pump frequency. This effect emerges due to the decrease of the average translational velocity with the rise of bounce oscillations.

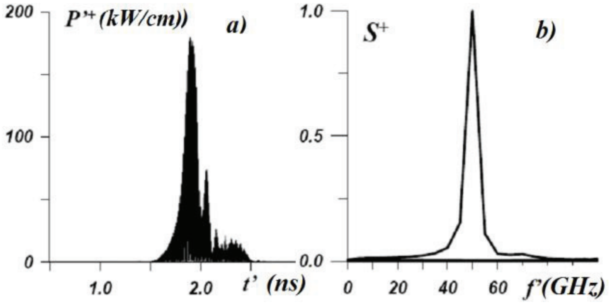

Figure 5**:** Results of PIC simulation of the undulator SR in co-moving reference frame: (a) pulse of superradiance propagating in the positive direction of the z axis, (b) it's spectrum.

After transformation into the lab frame the center frequency of the forward-radiated pulse increases up to 0.9 THz. It's power increases $4\gamma^2=400$ times reaching 36 MW/cm and the pulse duration decreases $\gamma=10$ times down to 10 ps. It is interesting to note that the time of development of the process in the rest frame is about 200-300, still the bunch moving with a velocity close to the speed of light passes the distance of about 1 m. Thus the simulation conducted is an example of unconventional use of relativity theory in applied problems.

CONCLUSION

Thus the analysis undertaken demonstrates the possibility of using the undulator SR for generation of powerful (multi-MW) pulses in THz wave range based on bunches formed by photo-injecting guns. Obviously, increasing the particles energy leads to the radiation frequency increase alongside with peak power increase. One of the important results of the work is the development of approaches giving the general physical picture and allowing detailed simulation.

Note also that instead of the undulator field the wiggling of electrons can be provided by an electromagnetic wave in RF or optical range., In the electrons' rest frame the processes of stimulated scattering are close to the processes of undulator radiation. Correspondingly, the models developed here can be applied to the scattering problem.

REFERENCES

[1] S.D.Korovin, et al., "Generation of Cherenkov superradiance pulses with a peak power exceeding the power of driving short electron beam", Phys. Rev. E. V. 74, P. 016501-016509 (2006).
[2] Kurennoy S.S., et al., "Development of photoinjector RF cavity for high-power CW FEL", Nucl. Instr. & Meth. in Phys. Res. A. V.528, Iss 1-2, P. 392-396. (2004).
[3] N.S.Ginzburg, A.S.Sergeev "Superradiance of an electron swarm in a periodic magnetic field", JETP Lett. V.54, P.446. (1991).
[4] V.P.Tarakanov, "User's Manual for Code KARAT", Springfield: BRA (1992).