QUASI-OPTICAL THEORY OF TERAHERTZ SUPERRADIANCE FROM AN EXTENDED ELECTRON BUNCH*

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Abstract

We consider the superradiance of an extended relativistic electron bunch moving over a periodically corrugated surface for the generation of multi-megawatt terahertz pulses. To study the above process we have developed a three-dimensional, self-consistent, quasi-optical theory of Cherenkov stimulated emission which includes a description of the formation of evanescent wave over a corrugated surface and its excitation by RF current induced in the electron bunch.

INTRODUCTION

In recent years, significant progress was gained in the generation of electromagnetic pulses in the centimeter and millimeter wavelength ranges based on the Cherenkov superradiance (SR) of high-current subnanosecond electron bunches with particle energies of 300–400 keV [1–5] propagating in periodically corrugated single-mode waveguides. In these frequency ranges SR pulses of subnanosecond duration with record-breaking gigawatt peak power were obtained by means of compact high-current accelerators. Typical duration of bunches employed in these experiments was, on the one hand, large compared to the wavelength and, on the other hand, it was limited by the so-called coherence length within which coherent emission of a single electromagnetic pulse from the entire bunch volume is possible due to the slippage of radiation relative to the particles. This SR emission includes electron self-bunching, and the peak power of SR pulse is approximately proportional to the square of the total number of particles in the electron pulse.

A natural continuation of this research is a development of Cherenkov SR sources operating at shorter wavelength values, including the terahertz frequency range. Technological advancements in the fabrication of spatially periodic microstructures also encourage such studies. In order to generate powerful single pulses in the THz range, it is necessary to reduce the duration of the electron bunches to several tens of picoseconds with corresponding increase in their densities. In turn, to obtain stable transverse focusing of dense electron bunches, it would be necessary to increase the particle energy up to several MeV. This increase is also a positive factor in view of improving the impedance of coupling to a surface wave. It should be recalled that for the Cherenkov radiation mechanism the spatial scale of the transverse field decay $L_\perp \sim \lambda\gamma/2\pi$ (where $\gamma = \left(1-\beta_0^2\right)^{-1/2}$ is the relativistic Lorentz factor) increases with the particle energy due to a decrease in the requirements for the waves deceleration. High brightness electron bunches generated by photoinjectors [6, 7] can satisfy the above conditions.

We should emphasize that the methods used for the theoretical description of stimulated Cherenkov radiation from relativistic electron beams in the short wavelength range must differ significantly from the approach developed previously for the microwave range. The existing theory of relativistic Cherenkov radiation sources operating in the regimes of both long-pulse (quasi-stationary) [8-10] and short-pulse (SR) [1-3] generation was based on an assumption that the transverse size of the microwave system is comparable to the radiation wavelength. Under these conditions, the Cherenkov radiation was described using a formalism according to which the electron beam interacted with a spatial harmonic of the volume waveguide mode possessing a fixed transverse structure.

For wavelengths shorter than one millimeter, the conditions of ensuring the electron beam transport and reducing Ohmic losses imply the use of oversized (or open) slow-wave systems. Accordingly, it is necessary to take into account the diffraction effects and to use a quasi-optical approach for the description of Cherenkov radiation from relativistic electron beams moving over periodically corrugated surfaces. In the case of a quasi-stationary electron beam such an approach was developed in [11] for consideration of surface-wave oscillators. In this letter we demonstrate that a similar method can be effectively used for analysis of stimulated emission of extended electron bunches moving above a corrugated surface. The validity of our consideration is confirmed by direct particle-in-cell (PIC) simulations based on CST STUDIO 3D code.

BASIC MODEL

We consider a three dimensional (3D) model of the Cherenkov SR from an extended electron bunch that moves rectilinearly at a velocity of $v_0 = \beta_0 c$ along guiding magnetic field $\vec{H}_0 = H_0\vec{z}_0$ over a plane with a shallow periodic sinusoidal corrugation

$$b(z) = b_1 \cos\left(\bar{h}z\right), \qquad (1)$$

where $b_1 << d$ is the corrugation amplitude, d is the period, and $\bar{h} = 2\pi/d$. We assume that an electron bunch has finite dimensions $l^e_{x,y,z}$ in three space coordinates (Fig. 1a).

Radiation field near the corrugated surface can be presented as a sum of two counter-propagating TM-

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polarized wave beams with the following components of magnetic field

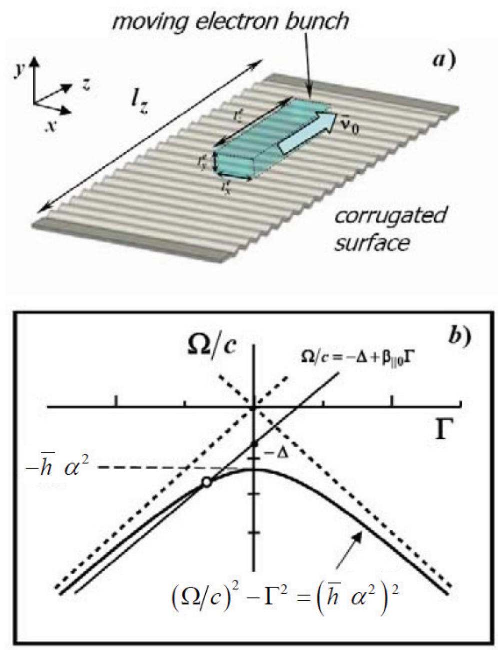

Figure 1: **(a)** Principal scheme of generation of SR pulses by an extended electron bunch moving over a periodically corrugated surface. **(b)** The dispersion diagram of the normal surface wave and resonance point in the case of a relativistic rectilinear electron beam.

$$H_x = \mathrm{Re}\left[A_+(z,x,y,t)e^{i(\omega t-kz)} + A_-(z,x,y,t)e^{i(\omega t+kz)}\right], \quad (2)$$

$k=\omega/c$. Correspondingly, the electric field satisfies the Maxwell equation $\vec{E}=-i/k\, rot\vec{H}$ and possesses components:

$$E_y = -\mathrm{Re}\left[A_+e^{i(\omega t-kz)} - A_-e^{i(\omega t+kz)}\right],$$
$$E_z = -\mathrm{Re}\frac{i}{k}\left[\frac{\partial A_+}{\partial y}e^{i(\omega t-kz)} + \frac{\partial A_-}{\partial y}e^{i(\omega t+kz)}\right]. \quad (3)$$

Under the Bragg resonance condition $\bar{h}\approx 2k$ coupling and mutual scattering of the counter-propagating wavebeams (2) takes place. To describe the waves coupling on the periodic structure located at $y=0$ we use the surface magnetic currents concept [12,13]. In the planar geometry under consideration, the surface current can be written as

$$j_x^m = -\frac{c}{4\pi}E_z = -\frac{c}{4\pi}\left(\frac{\partial\left(b(z)E_y\right)}{\partial z} + i\omega H_x\frac{b(z)}{c}\right). \quad (4)$$

In general form, field excitation by the surface magnetic j_x^m and volume electric j_z^e (see below) currents is described by the wave equation:

$$\Delta H_x - \frac{1}{c^2}\frac{\partial^2 H_x}{\partial t^2} = \frac{4\pi}{c^2}\frac{\partial j_x^m}{\partial t}\delta(y) - \frac{4\pi}{c}\frac{\partial j_z^e}{\partial y}. \quad (5)$$

Here, $\delta(y)$ is the delta function. In (2) and (3) we chose carrier frequency equal to the Bragg frequency $\bar{\omega}=\bar{h}c/2$. Taking into account the Bragg resonance condition after substituting the fields (2), (3) and the magnetic currents (4) into Eq.(5), we obtain a system of coupled parabolic equations (here we take $j_z^e=0$)

$$\pm\frac{\partial A_\pm}{\partial z} + \frac{\partial A_\pm}{c\partial t} + \frac{i}{\bar{h}}\frac{\partial^2 A_\pm}{\partial x^2} + \frac{i}{\bar{h}}\frac{\partial^2 A_\pm}{\partial y^2} = i\alpha A_\mp\delta(y), \quad (6)$$

where $\alpha=\bar{h}b_1/4$ is the coupling parameter.

To demonstrate that Eqs.(6) describe formation of surface waves, let us derive a dispersion relation for normal wave propagating near the infinite corrugated surface. For simplicity we consider here a 2D model, neglecting the radiation diffraction in the *x* direction. We should present the solution of the above equations in the region $y>0$ as $A_\pm \sim \exp i(\Omega t-\Gamma z-g_\pm y)$, where $g_\pm = i\sqrt{-\bar{h}(\Omega/c\mp\Gamma)}$ are the transverse wave numbers. Taking into account the boundary conditions on the corrugated surface $\left(\partial A_\pm/\partial y-\alpha\bar{h}A_\mp\right)\Big|_{y=0}=0$ which follow from Eqs. (6) we get the dispersion equation:

$$g_+g_- = -\bar{h}^2\alpha^2 \quad \text{or} \quad \frac{\Omega^2}{c^2}-\Gamma^2=\bar{h}^2\alpha^4. \quad (7)$$

As seen from Fig. 1b, a dispersion curve of the normal wave is located below the light cone and corresponds to an evanescent slow-wave with exponential decay of amplitude along the vertical *y* coordinate. For ultra-relativistic electrons and shallow corrugation the typical crossing point of the dispersion curve of the normal surface wave and the electron beam line $\Omega/c=-\Delta+\beta_0\Gamma$ belongs to the slope where the group velocity of the surface wave is co-directed with the electron longitudinal velocity.

Under the conditions of Cherenkov interaction, the microbunching of electrons takes place under the action of the longitudinal electric field component E_z. This process is described by the equations

$$\left(\frac{\partial}{\partial Z}+\frac{1}{\beta_0}\frac{\partial}{\partial\tau}\right)^2\theta = \mathrm{Re}\left(\frac{\partial\hat{A}_+}{\partial Y}e^{i\theta}\right) \quad (8)$$

$$\theta|_{Z=0}=\theta_0+r\cos\theta_0,\ \theta_0\in[0,2\pi),\ r<<1,$$
$$\left(\frac{\partial}{\partial Z}+\frac{1}{\beta_0}\frac{\partial}{\partial\tau}\right)\theta\Big|_{Z=0}=\hat{\Delta}, \quad (9)$$

where $\theta=\bar{\omega}(t-z/c)$ is the phase of electrons relative to the co-propagating partial wave A_+, $\hat{\Delta}=2\Delta/\bar{h}G$. The microbunching induces a high frequency electron current that can be presented in the form:

$$j_z^\omega = -\frac{Qv_0}{V}f(z-v_0t,x,y)J, \quad (10)$$

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where $J(z,x,y) = \frac{1}{\pi}\int_0^{2\pi} \exp(-i\theta)d\theta_0$, Q is the bunch total charge, V is its volume, and the function $f(z,x,y)$ defines the unperturbed bunch profile.

Substituting (10) in Eq.(5) we get equations that describe the excitation of a surface wave by the electron bunch:

$$\frac{\partial \hat{A}_+}{\partial Z} + \frac{\partial \hat{A}_+}{\partial \tau} + i\frac{\partial^2 \hat{A}_+}{\partial X^2} + i\frac{\partial^2 \hat{A}_+}{\partial Y^2} + \sigma\delta(Y)\hat{A}_+$$
$$= i\hat{\alpha}\delta(Y)\hat{A}_- - \frac{\partial}{\partial Y}\left(JF(Z-\beta_0\tau, X, Y)\right), \qquad (11)$$
$$-\frac{\partial \hat{A}_-}{\partial Z} + \frac{\partial \hat{A}_-}{\partial \tau} + i\frac{\partial^2 \hat{A}_-}{\partial X^2} + i\frac{\partial^2 \hat{A}_-}{\partial Y^2} + \sigma\delta(Y)\hat{A}_- = i\hat{\alpha}\delta(Y)\hat{A}_+.$$

Self-consistent system (8), (11) was written using the following normalization:

$Z = G\bar{\omega}z/c$, $X = \sqrt{2G}\bar{\omega}x/c$, $Y = \sqrt{2G}\bar{\omega}y/c$, $\tau = G\bar{\omega}t$,

$\hat{A}_\pm = \sqrt{2}eA_\pm \Big/ \left(mc\bar{\omega}\gamma_0^3 G^{3/2}\right)$, $\hat{\alpha} = \sqrt{2/G}\alpha$,

$G = \left(\frac{e}{mc^3}\frac{\lambda^2}{\pi\gamma_0^3}\frac{Qv_0}{V}\right)^{1/2}$ is the gain parameter. Below we assume for simplicity that the electron density has uniform distribution over all three space coordinates. It should be noted that in Eqs.(11) the Ohmic losses are also taken into account through being characterized by the parameter $\sigma = k\varepsilon\sqrt{2/G}$, where ε is the skin depth.

In the simulations we have used the following boundary conditions. In the longitudinal direction we assume that the corrugation has a finite length $L = G\bar{\omega}l/c$ and electromagnetic energy fluxes from outside are absent: $\hat{A}_+\big|_{Z=0} = 0$, $\hat{A}_-\big|_{Z=L} = 0$. In the vertical y direction we take into account that in practice the electron bunch should be transported in the vacuum channel formed by the planar waveguide. In this case Eqs.(11) should be supplemented by an additional boundary condition at the second regular plate of the waveguide: $\partial\hat{A}_\pm/\partial Y\big|_{Y=B} = 0$, where $B = \sqrt{2G}\bar{\omega}b/c$ is the normalized gap between the plates. If this gap is sufficiently large ($B >> 1$) the position of the second plate does not affect the characteristics of the electron-wave interaction due to the exponential decay of the surface wave in the y direction. Nevertheless it allows us to present the solution of Eqs.(11) as a sum of the modes of the regular planar waveguide $\hat{A}_\pm = \sum_{n=0}^{\infty} \hat{A}_\pm^n(Z,X,\tau)\cos(n\pi Y/B)$ for the numerical simulation. In the horizontal x direction we used the artificial cyclical boundary conditions which were imposed at a large distance from the bunch.

SIMULATION RESULTS

Simulations of the Cherenkov SR were performed in the terahertz range for an electron bunch with a length of l_z^e=1.2 cm, transverse dimensions of $l_x^e = 0.45$ mm and $l_y^e = 0.3$ mm, particle energies of 1.4 MeV and a total bunch charge of 2.2 nC. These parameters can be obtained for the electron bunches generated by photoinjectors [6,7]. The bunch was propagating over a plane with a corrugated region of length 13 cm, a corrugation period of 0.15 mm and a corrugation amplitude of 22 µm. For a copper surface the skin depth in the terahertz range is $\varepsilon \approx 0.07\ \mu m$. These physical parameters correspond to the normalized quantities $G = 3.5\cdot 10^{-3}$, $\hat{\alpha} = 5.5$, $\Delta = 10.5$, $L = 9.7$, $L_x^e = 0.8$, $L_y^e = 0.53$, $L_z^e = 0.9$, $\sigma = 0.035$. We used the initial small electron density fluctuations governed by the parameter $r << 1$ as initial conditions.

The results of these simulations showed that the main fraction of the radiation is emitted in the form of a short SR pulse in the positive direction of the axis z, i.e., in the direction of propagation of the electron bunch. The temporal dependence of the total radiation power $P_+ = G^2\frac{\gamma_0^2}{32\pi}\frac{m^2c^5}{e^2}\int_0^\infty\int_0^\infty\left|\hat{A}_+(Z=L)\right|^2 dXdY$ is shown in Fig. 2. The process of pulse formation is illustrated by Fig. 3, where spatial structures of the partial waves are presented. One can see that the amplitudes of both partial waves exponentially decay in the direction of the y axis with increasing distance from the corrugated surface. In Fig. 4, spatial profiles of the electric field on the cross-

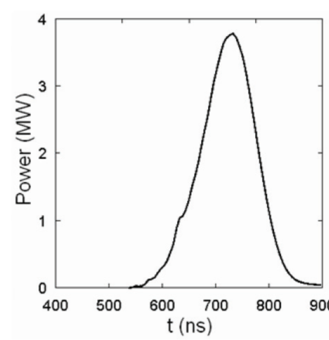

Figure 2: SR pulse temporal profile.

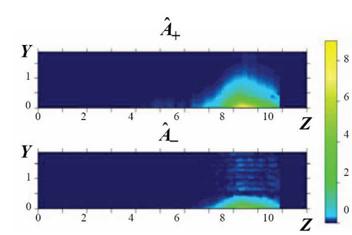

Figure 3: Spatial structures of the partial waves $\hat{A}_\pm(Y,Z)$ that corresponds to formation of normal surface wave ($X = 0$, $\tau = 10$).

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section $Y = 0.6$ are shown in consecutive moments of time. As follows from the dispersion diagrams Fig. 1b the electron velocity in the resonance point exceeds the group velocity of the normal wave. It leads to formation of wake fields behind the electron bunch that is clearly observed in Fig. 4. The peak power of the SR pulse amounted to 3.5 MW at a pulse duration of ~100 ps.

Results obtained in the framework of a quasi-optical model were confirmed by direct CST STUDIO PIC simulations. Figure 5a,b shows the SR pulse and its spectrum with a central frequency of 0.8 THz obtained for

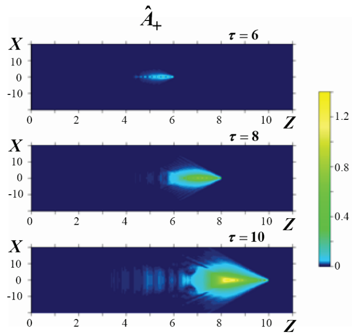

Figure 4: Formation of wake field after electron bunch. Profiles of electric field $\hat{A}_+(X,Z)$ on the surface $Y = 0.7$ in consecutive points in time.

the physical parameters of the periodic system and electron bunch indicated above. The total estimated radiation power was about 3.7 MW. The fields had the structures corresponding to a surface evanescent wave located near the corrugated surface. Figure 5c demonstrates the formation of the wake-field.

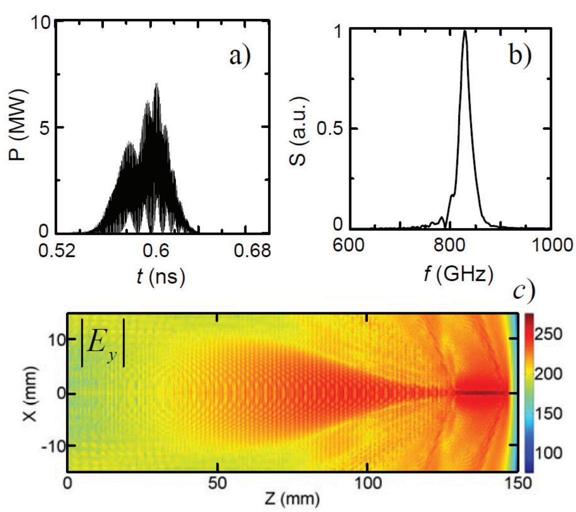

Figure 5: Results of 3D direct PIC simulations of Cherenkov SR in terahertz range for an electron bunch formed by a photoinjector gun. **(a)** – SR pulse, **(b)** – radiation spectrum, **(c)** – formation of wake field.

CONCLUSION

Thus, the results of our analysis show the possibility of generating single high-power terahertz pulses using the phenomenon of Cherenkov superradiance of extended electron bunches that move in free space over a corrugated surface. We believe that it is expedient to study the possibility of moving to still shorter wavelengths, which can be achieved by decreasing the period of the diffraction gratings and increasing the density and energy of the particles in the electron bunches.

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