RADIATION AND INTERACTION OF LAYERS IN QUASI-PLANE ELECTRON BUNCHES MOVING IN UNDULATORS

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Abstract

A simple general 1-D theory of coherent spontaneous undulator radiation from dense bunches allows easily taking into account both Coulomb and radiation interaction of the particles in the bunches with arbitrary densities, velocity distributions and energy chirps. The theory can be applied for estimations of THz radiation sources.

RADIATION OF A MOVING PLANE

Generating powerful coherent THz radiation from dense and relatively low-energy electron bunches formed in compact laser-driven photo-injectors [1-8] is a fairly complicated task because of strong Coulomb repulsion of "insufficiently heavy" particles. To overcome it, one can apply energy chirping [6, 8] or/and formation of quasi-plane bunches with increased transverse size [4, 7]. The main properties of radiation and dynamics for such bunches may be found from the simplest 1-D model representing a bunch as a set of moving charged planes. The field of a plane that consists of electrons moving synchronously along arbitrary identical trajectories $\vec{r}(t)$ can be written in the form [3]

$$E_z = -2\pi\sigma s,\ \vec{E}_\tau = 2\pi\sigma\vec{\beta}_\tau(\tilde{t})[1-s\beta_z(\tilde{t})]^{-1},\ \vec{H}_\tau = s\,\hat{z}\times\vec{E}_\tau .$$

Here, $-\sigma$ is a surface charge density, $\vec{\beta}=\vec{v}/c$ is the normalized electron velocity, $s=\mathrm{sgn}[z-z(t)]$, $\tilde{t}$ is the retarded time. Unlike the Lienard-Wiechert field of a point charge, the field of the plane is finite at the charges and determine both the self-action of the charged plane and its radiation. If the particles oscillate in the plane and move with a ultrarelativistic translational velocity β_z, the plane generates coherent spontaneous radiation that is contracted/stretched and increased/decreased by amplitude in the passing/counter $\pm z$ directions, respectively.

The motion of electrons in a linearly polarized undulator field $\vec{H}_u = \vec{y}_0 H\cos(2\pi z/d)$ and own field of the charged plane is described by self-consistent equations

$$\frac{dp_x}{d\zeta} = K\cos\zeta - q\frac{p_x}{p_z},\ \frac{dp_z}{d\zeta} = -K\frac{p_x}{p_z}\cos\zeta - q\frac{p_x^2}{1+p_x^2},\ \frac{d\tau}{d\zeta} = \frac{\gamma}{p_z}.$$

Here, $\zeta = 2\pi z/d$, $\tau = 2\pi ct/d$ are the normalized axial coordinate and current time, $\vec{p}=\gamma\vec{\beta}$, γ are the normalized electron momentum and energy, $K = eH/2\pi mc^2$, $q = e\sigma d/mc^2$ are the undulator and space charge parameters, respectively. For particles with the same energy and zero transverse velocities at the entrance into the undulator field the initial conditions are as follows

$$p_x = 0,\ p_z = \sqrt{\gamma_0^2-1},\ \tau = 0.$$

For electron with ultrarelativistic axial velocity the transverse electric and magnetic radiation self-forces almost completely compensate each another, and because of it the transverse electron momentum differs from its unperturbed value $p_x = K\sin\zeta$ only on terms of the order of small parameter q/γ_0. In the first approximation both the energy and Doppler up-conversion factor for radiating particles averaged by undulator oscillations linearly decrease along the axial coordinate:

$$\bar{\gamma} = \gamma_0(1-\eta),\ \Gamma = (1-\bar{\beta}_z)^{-1} = \Gamma_0(1-2\eta),$$

where

$$\eta = (\gamma_0-\bar{\gamma})/(\gamma_0-1) = \alpha\zeta$$

is the efficiency of radiation, $\alpha = (q/\gamma_0)(1-1/\sqrt{1+K^2})$. The radiation field from the plane presents a signal with duration $\tau_r = 2\pi N\lambda_0/d$ and linear modulation of both amplitude, $E = 1-\alpha\psi$, and frequency, $\omega = \omega_0 E$ (Fig. 1):

$$E_x/2\pi\sigma = (\Gamma_0 K/\gamma_0)E\sin\tilde{\zeta}$$

where $\omega_0 = \Gamma_0(2\pi c/d)$, $\psi = \Gamma_0(\tau-\zeta)$ are the non-perturbed radiation frequency and the phase at the point of observation, $\tilde{\zeta} = \psi(1-\alpha\psi)$ is the retarded coordinate.

The above formulas provide very good coincidence with numerical calculations even for fairly large space charge parameters and undulator lengths. For example, using for the radiating plane the typical parameters of the Israeli THz source (currently under development) [8]: electron energy $E_0 = 5.5$ MeV, surface density $\sigma_0 = 0.1$ nC/mm^2 and *K*=0.47 demonstrates coincidence of analytical and numerical results with high precision; in order to show the differences the density in Fig.1a has been increased up to $3\sigma_0$.

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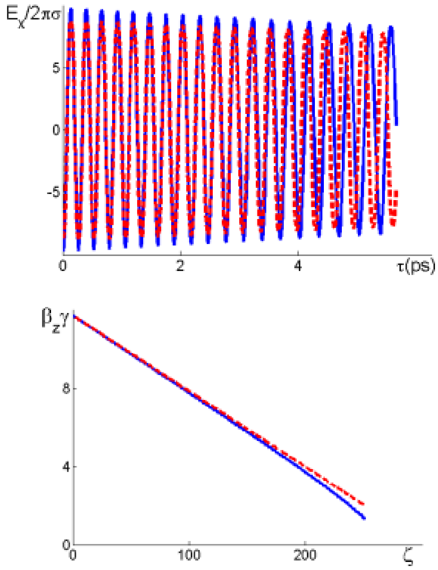

Figure 1: Radiation pulse (a) and electron axial momentum (b) for one plane: analytical (blue) and numerical (red) solutions.

INTERACTING LAYERS

The transverse electron momenta of the particles in a dense moving layer of a finite thickness is also basically determined by the undulator field, but strong axial Coulomb repulsion of sub-layers can quickly increase the axial electron momenta and size of the bunch. In order to mitigate this effect rear particles should be accelerated in a stronger RF field than the front ones during the bunch formation [6, 8] (energy chirp, $\Delta\gamma_0 > 0$). The averaged axial motion of particles in the combined undulator field and fields of all sub-layers is close to the relativistic uniformly accelerated motion. If longitudinal electron masses in sub-layers are close to each other, the particle separation is described by a quasi-nonrelativistic formula. Thus, in the case of two planes

$$\Delta\zeta = \Delta\zeta_0 - \frac{\Delta\gamma_0}{\gamma_0^3}\tau + \frac{(q_1+q_2)\tau^2}{2\gamma_0^3}.$$

For example, the change in separation between two planes with electron energy E_0 and densities $\sigma_0/2$ is 0.1 mm at the length of 20 cm. The separation can be significantly decreased and then returns back to the initial value at the energy chirp $\Delta\gamma_0 \sim 1$.

Coulomb and radiation forces change the kinetic and potential energy of the particles. The radiation flux is equal to the change in the surface density of the total energy,

$$w = q_1(\gamma_1 - 1) + q_2(\gamma_2 - 1) - q_1 q_2(\zeta_1 - \zeta_2)\,,$$

and for two planes one has the approximate equation

$$\frac{dw}{d\tau} = -q_1^2\frac{p_{1x}^2}{1+p_{1x}^2} - 2q_1q_2\frac{p_{1x}}{\gamma_1}\left(\frac{\gamma_2 p_{2x}}{1+p_{2z}^2}\right)_{\tilde\tau_2} - q_2^2\frac{p_{2x}^2}{1+p_{2x}^2}.$$

Here, $\tilde\tau_2$ is the retarded time corresponding to the radiation of the second plane which acts on the first one at the moment τ. If the separation between the planes is much smaller than the characteristic radiation wavelength, the planes radiate coherently. For significantly changing separations, regions of constructive and destructive interference quickly replace one another.

The front plane is decelerated by the self-radiation and accelerated by the Coulomb and radiation forces from the back plane. If the separation between them changes quickly, the latter force can be neglected. Then the self-radiation force for the first plane can compensate its Coulomb repulsion by the second plane and stabilize the axial velocity of the first plane if $q_2 \approx (1 - 1/\sqrt{1+K^2})q_1$ (Fig. 2a). In this case, the radiation of the first plane can be powerful and narrowband while the radiation of the decelerated second plane provides a low-frequency part of the spectrum (Fig. 2b).

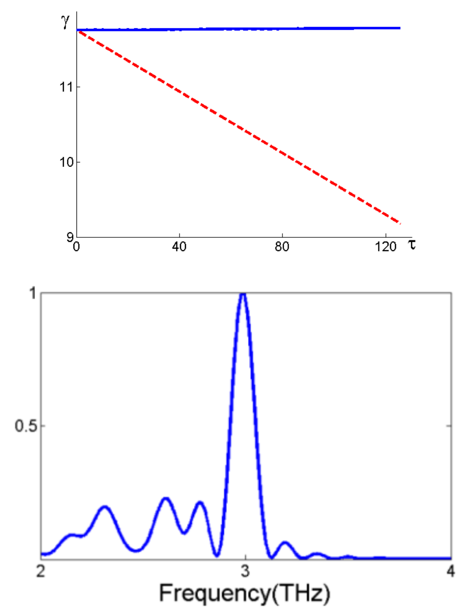

Figure 2: The Lorentz-factors and radiation spectrum for two planes with space charge parameters q_1 (blue) and q_2 (red). Coulomb acceleration of the front plane in the field of the rear one is compensated by the radiation deceleration.

In order to simulate radiation and dynamics of a quasi-plane electron bunch it is needed to provide a sufficient number of the electron sub-layers (moving planes) in each axial area with the scale of the order of the radiation wavelength at any time. If the initial thickness of the layer is much smaller than the wavelength and its changes during the motion in the undulator field are small, the resulting radiation is certainly close to that from one plane. However, at the layer surface density of the order

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of σ_0 and particle energy of the order of E_0 without energy chirping the Coulomb repulsion increases the layer thickness many times (even for short interaction length). At the initial layer thickness 0.1 mm and fairly large optimum chirp $\Delta\gamma_0 \approx 2.7$ the minimum thickness is very small (Fig. 3).

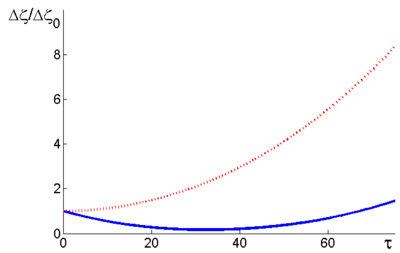

Figure 3: Change in the normalized pulse length of a moving layer with the surface density σ_0 and initial duration 0.3 ps (uniform initial charge distribution; red - without energy chirp, blue - $\Delta\gamma_0 = 2.7$).

Then the thickness increases up to the initial value at 11 undulator periods when the radiation is saturated with efficiency close to 1% (Fig. 4a). The corresponding radiation energy, pulse duration and power flux are 0.5 mJ/cm^2, 3.5 ps and 140 MW/cm^2, respectively. This radiation is distributed over a fairly broadband frequency spectrum with the central frequency 3.7 THz and width of the order of 30% (Fig. 4b).

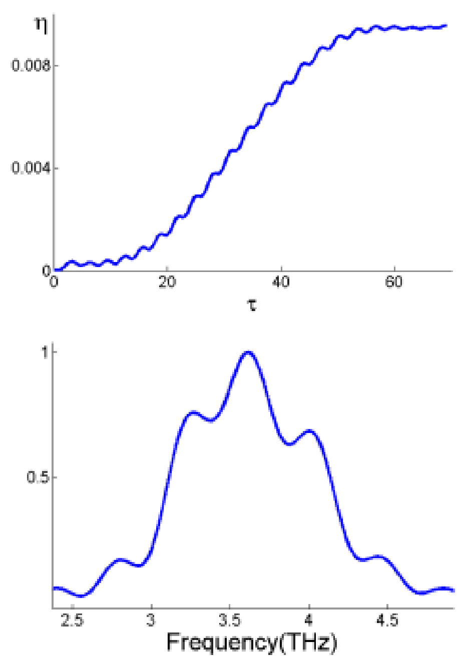

Figure 4: Current efficiency and radiation spectrum for a moving layer with initial duration 0.3 ps.

CONCLUSION

The developed 1-D approach for coherent spontaneous undulator radiation based on using the field of a moving charged plane is much simpler than the existing exact 3-D methods which take into account finite transverse sizes of the bunches, transverse non-homogeneity of the undulator fields and influence of electrodynamic systems (waveguides) on the radiation. Due to compensation of transverse radiation forces there exists a small parameter of the problem even in the case of a very large charge density. Correspondingly, the field of a dense bunch weakly changes transverse undulator particle oscillations, and the particle dynamics can be easily studied using simple averaging of equations by undulator oscillations. The same method can be also used for essential simplification of equations in a 3-D approach. Though the 1-D approach provides only estimations and obviously overestimates the influence of the Coulomb particle repulsion, it gives a clear physical picture and demonstrates the problems associated with maintaining coherence and narrowband spectrum of the radiation for large charges and relatively low electron energy.

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