

X-RAY SMITH-PURCELL RADIATION FROM A BEAM SKIMMING A GRATING SURFACE

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Abstract

X-Ray Smith-Purcell radiation, i.e. the radiation from a beam of charged particles moving above a periodical target parallel to its surface, is considered for the case when a part of the beam crosses the target. The radiation arising is the superposition of Smith-Purcell radiation and transition radiation (TR) from the grating. The analytical expression for spectral-angular distribution of radiation is obtained. It is shown that characteristics of radiation in this case differ considerably from the characteristics of radiation for the bunch with uniform distribution. The incoherent form-factor of bunch with Gaussian distribution of particles has been obtained; it is proved to provide a considerable increase of the radiation intensity in conditions when bunch skims the grating.

INTRODUCTION

Smith-Purcell radiation as a base of Free Electron Lasers is actively studied experimentally and theoretically in recent years [1]-[3]. Usually the beam is supposed to move at some distance above the target surface. In practice this distance is chosen to be minimal in order to broaden the spectrum of radiation to high frequencies, therefore the beam passes very close to the target surface. Along with that, experimental data contains the information about grating heating, which is apparently caused by interaction of the beam with the grating. For example, authors of article [4] suppose that the beam skims the grating surface. There has been no theory of Smith-Purcell radiation for such conditions yet. We give the analytical description of X-Ray radiation arising when the beam of charged particles moves above the periodical target and a part of the beam crosses the target. The radiation arising is the superposition of Smith-Purcell radiation and transition radiation (TR) from the grating. This radiation determines the process of beam bunching and, consequently, gains of radiation.

Talking of SPR in this article we shall mean that this is the special case of DR – Diffraction radiation for periodical target, i.e. grating. So, sometimes we shall mention DR, meaning that lion share of the bunch goes above the target surface, and sometimes mention TR, when considerable part of the bunch intersects the edges of the target.

FIELD OF RADIATION

We consider Smith-Purcell radiation from the bunch of N particles. The grating consists of N_{st} strips with

dielectric permittivity $\varepsilon(\omega)$ and vacuum between the strips. The width of a strip is a , the grating period is d . We assume that each particle has the charge e and moves uniformly with the constant velocity $\mathbf{v} = (v_x, v_y, 0)$, α is the angle between the beam velocity and axis x , see Fig.1. The center of the bunch is at a distance h above the grating surface ($h > 0$) or under the surface ($h < 0$). The radius-vector of m -th particle is $\mathbf{r}_m = (x_m, y_m, z_m)$. We would like to emphasize that these coordinates can be negative. To find the field of radiation we use the method of polarization, described for single-particle radiation in more detail in monograph [5] and developed for the radiation from the bunch in [6], [7].

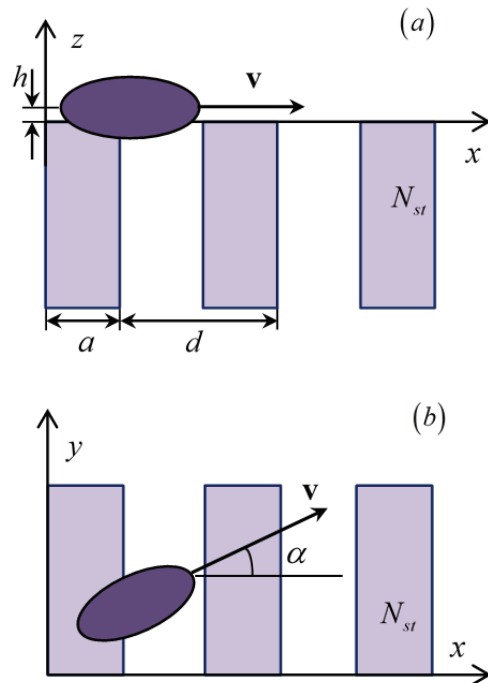


Figure 1: Bunch skimming a grating surface generates radiation (a) side view; (b) top view.

In X-Ray frequency range the dielectric permittivity has the form:

$$\varepsilon = 1 + \chi' + i\chi'', \quad (1)$$

where $\chi' = -\omega_p^2/\omega^2$, $\omega \gg \omega_p$, ω_p is the plasma frequency. Now we consider non-absorbing medium i.e. $\chi'' \ll |\chi'|$.

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The Coulomb field of each particle polarizes the target and due to it the radiation is generated. The polarization current density has the form:

$$\mathbf{j}(\mathbf{r}, \omega) = \frac{\omega}{4\pi i} (\varepsilon(\omega) - 1) \mathbf{E}_0(\mathbf{r}, \omega). \quad (2)$$

Here $\mathbf{E}_0(\mathbf{r}, \omega)$ is the Coulomb field of the bunch. Its Fourier image can be written as:

$$\mathbf{E}_0(\mathbf{q}, \omega) = - \sum_{m=1}^N \frac{ie}{2\pi^2} \frac{\mathbf{q} - \mathbf{v}\omega/c^2}{q^2 - \omega^2/c^2} e^{-i\mathbf{q}\mathbf{r}_m} \delta(\omega - \mathbf{q}\mathbf{v}). \quad (3)$$

The field of radiation is defined by polarization current density [1]:

$$\mathbf{E}(\mathbf{r}, \omega) = \frac{i\omega}{c^2} \frac{e^{ikr}}{r} \left[\mathbf{n}' \left[\mathbf{n}' \int_V d^3r' e^{-ikr'} \mathbf{j}(\mathbf{r}', \omega) \right] \right]. \quad (4)$$

Here $\mathbf{k}' = \mathbf{n}' \sqrt{\varepsilon(\omega)} \omega/c$ and the prime means the value in the medium. In Eq. (4) it is integrated over the area of generation of the radiation, i.e. over the volume of the target V .

It should be noticed that if the problem contains the interference and coherence phenomena then it is needed to take into account the law of refraction even in high-frequency region [6]. In this paper we allow it for the upper edge:

$$\mathbf{n}' = \varepsilon(\omega)^{\frac{1}{2}} (n_x, n_y, \sqrt{\varepsilon(\omega) - 1 + n_z^2}), \quad (5)$$

$\mathbf{n} = (n_x, n_y, n_z)$ - the unit vector of the wave-vector in vacuum: $\mathbf{k} = \mathbf{n}\omega/c$. It is easy to see that the minimal value of n_z exists:

$$n_z^{\min} = \sqrt{1 - \varepsilon(\omega)}, \quad (6)$$

for which the waves incident to the target edge are ordinary plane waves. For $n_z < n_z^{\min}$ the incident waves are evanescent. In this paper we shall consider only case of comparatively big angles $n_z > n_z^{\min}$, because the case of radiation at grazing incidence needs more the case, so, we have the limit for defined by Eq. (6).

Thus, after integrating in Eq. (4) using Eqs. (2), (3) and taking into account the law of refraction Eq. (5) one can obtain the field of Smith-Purcell radiation from the bunch skimming a grating surface:

$$\mathbf{E}(\mathbf{r}, \omega) = -ie \frac{\varepsilon(\omega) - 1}{2\pi v_x} \frac{\omega^2}{c^2} \frac{e^{ikr}}{r} e^{i\frac{\varphi a}{2}} e^{i(N_{st}-1)\frac{\varphi d}{2}} F_1 F_{st} \times \quad (7)$$

$$\times \sum_{m=1}^N e^{-i\xi x_m} e^{-ik_y y_m} e^{-ik'_z z_m} [\mathbf{n}' [\mathbf{n}' \mathbf{L}_m]].$$

$$\text{Here } F_1 = \frac{\sin(\varphi a/2)}{\varphi}, F_{st} = \frac{\sin(N_{st}\varphi d/2)}{\sin(\varphi d/2)}, \xi = \frac{\omega - k_y v_y}{v_x}$$

$$\mathbf{A} = \xi \mathbf{e}_x + k_y \mathbf{e}_y - \mathbf{e}_x v_x \omega/c^2 - v_y \mathbf{e}_y \omega/c^2, \quad \varphi = \xi - k_x,$$

$$\mathbf{L}_m = \frac{\mathbf{A}\rho^{-1} - i\mathbf{e}_z}{\rho - ik'_z} + \frac{\mathbf{A}\rho^{-1} \text{sgn}(z_m) - i\mathbf{e}_z}{\rho - ik'_z \text{sgn}(z_m)} \left[e^{-z_m \rho \text{sgn}(z_m) + ik'_z z_m} - 1 \right],$$

$$k'_z = \varepsilon(\omega)^{\frac{1}{2}} \frac{\omega}{c} \sqrt{\varepsilon(\omega) - 1 + n_z^2}, \quad \rho^2 = \xi^2 + k_y^2 - \omega^2/c^2.$$

INCOHERENT AND COHERENT FORM-FACTORS

Knowing the field Eq. (7) one can obtain the spectral-angular distribution of radiation:

$$\frac{dW(\mathbf{n}, \omega)}{d\Omega d\omega} = \left\langle cr^2 |\mathbf{E}(\mathbf{r}, \omega)|^2 \right\rangle. \quad (8)$$

Angle brackets $\langle \dots \rangle$ mean the averaging over the locations of all the particles. Eq. (8) can be written as

$$\frac{dW(\mathbf{n}, \omega)}{d\Omega d\hbar\omega} = \frac{1}{137} \left(\frac{\varepsilon(\omega) - 1}{2\pi\beta_x} \right)^2 \frac{\omega^2}{c^2} F_1^2 F_{st}^2 G, \quad (9)$$

where

$$G = \left\langle \frac{\omega^2}{c^2} \left| \sum_{m=1}^N e^{-i\xi x_m} e^{-ik_y y_m} e^{-ik'_z z_m} [\mathbf{n}' [\mathbf{n}' \mathbf{L}_m]] \right|^2 \right\rangle. \quad (10)$$

The factor G determines the radiation from the bunch, that is why we will call it form-factor; however, it differs from what is usually called form-factor (see Eq. (19) or monograph [5] in more detail). The reason to use this new denomination is that in our case it is impossible to separate from G the multiplier usually contained in the spectral-angular distribution of the single particle. In the limiting cases of "pure" DR and TR our form-factor G (Eq. (10)) turns into the production of usual form-factor and the part of single-electron density of radiation.

Factor F_{st}^2 defines the dispersion relation of Smith-Purcell effect in Diffraction radiation (which is usually called mere Smith-Purcell radiation, SPR): it has sharp maxima at

$$\varphi d = 2\pi n_{SP}, \quad n_{SP} = 1, 2, \dots, \quad (11)$$

where

$$\varphi = (\omega - \mathbf{k}\mathbf{v})/v_x. \quad (12)$$

Using properties of sum and averaging in Eq. (10) one can obtain the form factor as the sum of coherent and incoherent form-factors (see also [6]-[10]):

$$G = NG_{inc} + N(N-1)G_{coh}. \quad (13)$$

Let us consider the bunch of non-interacting particles with Gaussian distribution. In this case the distribution function is

$$f = \frac{e^{-\frac{(x_m \cos \alpha + y_m \sin \alpha)^2}{\sigma_x^2}} e^{-\frac{(-x_m \sin \alpha + y_m \cos \alpha)^2}{\sigma_y^2}} e^{-\frac{(z_m - h)^2}{\sigma_z^2}}}{\pi^{3/2} \sigma_x \sigma_y \sigma_z}, \quad (14)$$

where h is z -position of the bunch center, which can be both positive and negative. After some calculations it is possible to find G_{coh} and G_{inc} from Eq. (10):

$$\begin{aligned} G_{inc} = & [C_1 - \text{Re} C_2] [1 - \Phi(h/\sigma_z)] + \\ & + \frac{C_1}{2} e^{2h\rho} e^{\rho^2 \sigma_z^2} [1 - \Phi(h/\sigma_z + \rho\sigma_z)] + \\ & + \frac{C_1}{2} e^{-2h\rho} e^{\rho^2 \sigma_z^2} [1 + \Phi(h/\sigma_z - \rho\sigma_z)] - \\ & - \text{Re} \left(e^{h(\rho - ik'_z)} e^{\frac{\sigma_z^2 (\rho - ik'_z)^2}{4}} (C_2 - C_1) \right) + \\ & \text{Re} \left(e^{h(\rho - ik'_z)} e^{\frac{\sigma_z^2 (\rho - ik'_z)^2}{4}} (C_2 - C_1) \Phi \left(\frac{h}{\sigma_z} + \sigma_z \frac{\rho - ik'_z}{2} \right) \right) \end{aligned} \quad (15)$$

$$C_1 = \frac{\omega^2 \mathbf{A}^2 \rho^{-2} + 1 - (\mathbf{A}\mathbf{n}')^2 \rho^{-2} - (n'_z)^2}{c^2 \rho^2 + k'_z{}^2},$$

$$C_2 = -\frac{\omega^2 \mathbf{A}^2 \rho^{-2} + 1 - (\mathbf{A}\mathbf{n}')^2 \rho^{-2} - (n'_z)^2 + 2i(\mathbf{A}\mathbf{n}')\mathbf{n}'_z \rho^{-1}}{c^2 \rho^2 + k'_z{}^2 - 2i\rho k'_z},$$

and

$$\begin{aligned} G_{coh} = & \frac{1}{4} e^{-l} \left| e^{-h\rho} e^{\frac{\sigma_z^2 \rho^2}{4}} \left[1 + \Phi \left(\frac{h}{\sigma_z} - \frac{\rho\sigma_z}{2} \right) \right] \mathbf{C}_3^* - \right. \\ & \left. - e^{h\rho} e^{\frac{\sigma_z^2 \rho^2}{4}} \left[1 - \Phi \left(\frac{h}{\sigma_z} + \frac{\rho\sigma_z}{2} \right) \right] \mathbf{C}_3 + \right. \\ & \left. + e^{-\frac{\sigma_z^2 k'_z{}^2}{4}} e^{-ik'_z h} \left[1 - \Phi \left(\frac{h}{\sigma_z} - \frac{ik'_z \sigma_z}{2} \right) \right] \left[\mathbf{C}_3 + \mathbf{C}_3^* \right] \right|^2, \end{aligned} \quad (16)$$

where $\Phi(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt$ - is the Laplace function, * - means conjugate complex value;

$$\mathbf{C}_3 = \mathbf{n}' \frac{\omega (\mathbf{A}\mathbf{n}') \rho^{-1} + in'_z}{c \rho + ik'_z} - \frac{\omega \mathbf{A} \rho^{-1} + i\mathbf{e}_z}{c \rho + ik'_z},$$

$$I = \frac{(sk_y - \sin \alpha \cos \alpha [\sigma_y^2 - \sigma_x^2] \xi)^2 + \sigma_x^2 \sigma_y^2 \xi^2}{2s},$$

$$s = \sigma_y^2 \cos^2 \alpha + \sigma_x^2 \sin^2 \alpha.$$

We would like to draw reader's attention to the exponents in Eqs. (15) and (16) which are proportional to

the square of the radiation frequency (which is contained in ρ , for example). It may seem that these exponents increase indefinitely with $\omega \rightarrow \infty$. However, it is known that for $x \rightarrow \infty$ the asymptotic form of Laplace function is

$$\Phi(x \rightarrow \infty) \approx 1 - \frac{e^{-x^2}}{x\sqrt{\pi}}. \quad (17)$$

Thus, G_{coh} and G_{inc} decrease with growing of frequency (one can see it above in Fig. 2). Also, considering the limiting cases $\omega \rightarrow \infty$, $h > 0$ (Diffraction radiation) and $\omega \rightarrow \infty$, $h \rightarrow -\infty$ (Transition radiation) it is useful to keep in mind that $\Phi(-x) = -\Phi(x)$. The results obtained (Eq. (9) with Eqs. (13), (15) and (16)) for radiation of a single charged particle turn into results of article [11] in case of $h > 0$ (DR), and into results of [12] in case of $h < 0$ (TR).

ANALYSIS

To plot the figures we put:

$$\begin{aligned} n_x &= \sin \theta \cos \phi, \\ n_y &= \cos \theta, \\ n_z &= \sin \theta \sin \phi. \end{aligned} \quad (18)$$

In all figures with the exception of Figs. 2 and 4 the length of the bunch is too big to observe the coherent radiation and this is the incoherent radiation that makes the main contribution.

Figure 2 demonstrates the coherent and incoherent form-factors for Gaussian distribution of the particles in the bunch in dependence on the wavelength of radiation. It is plotted for parameters of SLAC (FACET, $E = 20 \text{ GeV}$). To compare this two function we have to choose very short bunches ($\sigma_x = 10 \text{ nm}$). Had we not use that order of the bunch length, the coherent form-factor would have been too small to be noticeable. This functional behaviour is different for the bunch with uniform distribution of the particles [6]. Moreover, as opposed to the uniform distribution of the particles in the bunch, the incoherent form-factor dominates starting from $\lambda \approx \sigma_x$ and less, see Fig. 2. Nevertheless, both for Gaussian and uniform distributions the incoherent part of form-factor should be taken into account (see comparison between incoherent form-factors for Gaussian and uniform distributions in Fig. 3). Figure 3 is plotted until value of wavelength corresponding to $\omega = 3\omega_p$.

In Fig.4 the dependence of coherent and incoherent factors on α is demonstrated. It is plotted for value of α satisfying to the Eq. (6). The values of viewing angles satisfy the third maximum from Smith-Purcell dispersion relation ($n_{sp} = 3$). One can see that the more the angle between the beam velocity and the rulings direction is, the

more intensive radiation can be obtained. We would like to stress that in case $\alpha \neq 0$ the radiation is distributed over the cone surface [13] with cone angle $\theta = \arccos(\beta^{-1} \sin \alpha)$. The features of this case demand the separate consideration. For clarity, all figures hereafter will be plotted at $\alpha = 0$, excluding Fig. 4.

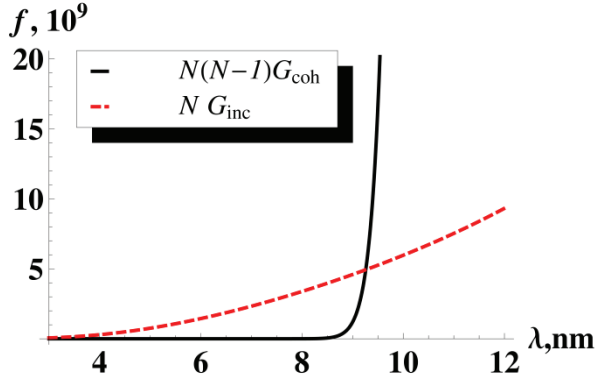


Figure 2: The coherent (solid black) and incoherent (dashed red) form-factors for Gaussian distribution of the particles in the bunch. Here $\hbar\omega_p = 26.3 eV$ (beryllium), $\gamma = 4 \cdot 10^4$ (energy of FACET, SLAC), $N = 10^{10}$, $\alpha = 0$, $\phi = \pi/6$, $h = 60 \mu m$, $\sigma_x = 10 nm$, $\sigma_y = \sigma_z = 5 \mu m$.

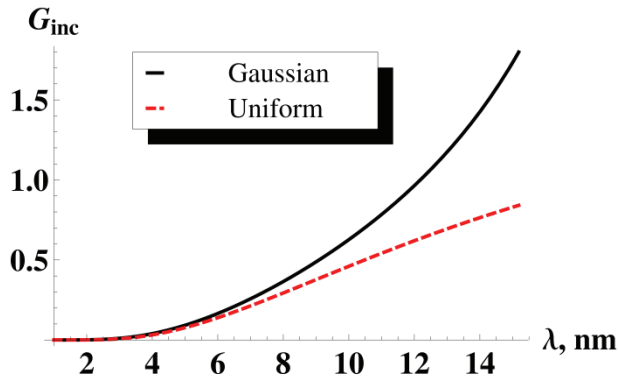


Figure 3: The incoherent form-factor for uniform (dashed red, results from [6]) and Gaussian (solid black) distribution. Here $\hbar\omega_p = 26.3 eV$ (beryllium), $\gamma = 4 \cdot 10^4$ (energy of FACET, SLAC), $N = 10^{10}$, $\alpha = 0$, $\phi = \pi/6$, $h = 60 \mu m$, $\sigma_x = 20 \mu m$, $\sigma_y = \sigma_z = 15 \mu m$.

The spectral-angular distribution of Smith-Purcell radiation for different values of impact-parameter is shown in Fig. 5. Our results permit to see the transition of diffraction radiation (DR) into TR and, moreover, they describe the case when one half of the bunch is above the target surface, while the other is under it (see the black solid curve in Fig.5).

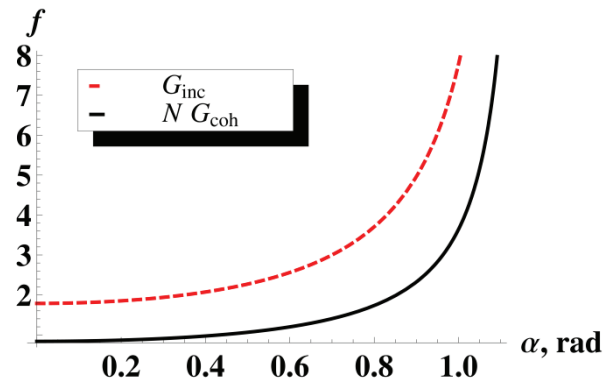


Figure 4: The coherent (solid black) and incoherent (dashed red) form-factors for Gaussian distribution of the particles in the bunch depending on α . Here $\hbar\omega_p = 26.3 eV$ (beryllium), $\lambda = 9.1 nm$, $\gamma = 4 \cdot 10^4$ (energy of FACET, SLAC), $N = 10^{10}$, $h = 60 \mu m$, $\sigma_x = 10 nm$, $\sigma_y = \sigma_z = 5 \mu m$, $\theta = \arccos(\beta^{-1} \sin \alpha)$,

$$n_{SP} = 3, \phi = \arccos \left[\frac{1 \cos \alpha - 2\pi c n_{SP} / (d\omega\beta)}{\sin \theta} \right].$$

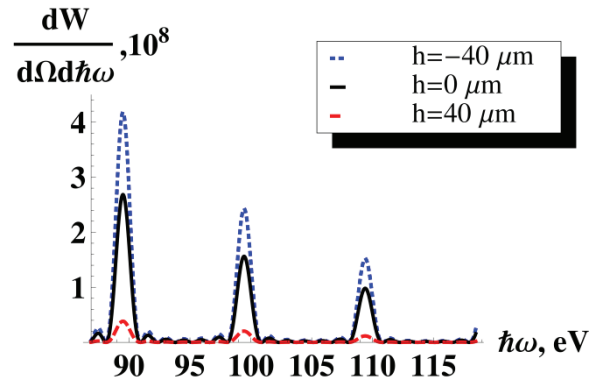


Figure 5: The spectral-angular distribution of radiation from the bunch when its center is above the target surface (dashed red), under it (dotted blue), on the surface (solid black). Here $N_{st} = 7$, $d = 0.9 \mu m$, $a = d/2$ and other parameters as in Fig. 3.

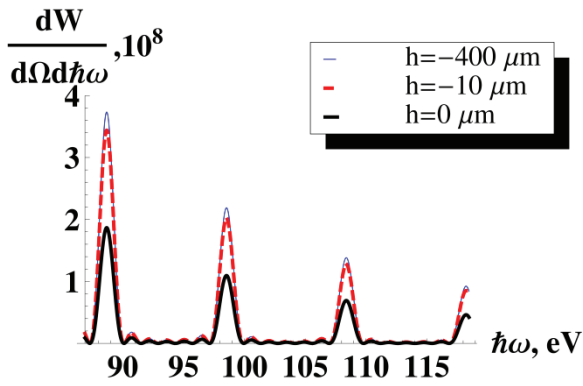


Figure 6: The spectral-angular distribution of radiation from the bunch when its center is under the target surface (dashed red and dotted blue) and on the surface (solid black). Here $\hbar\omega_p = 26.3 eV$ (beryllium), $\gamma = 20$ (energy of LUCX (KEK, Japan)), $N_{st} = 7$, $N = 10^{10}$, $\alpha = 0$, $\phi = \pi/6$, $\sigma_x = 30 \mu m$, $\sigma_y = \sigma_z = 10 \mu m$, $d = 0.9 \mu m$, $a = d/2$.

The spectral-angular distribution of the radiation from the bunch moved under the target surface for comparatively low energy of electrons is shown in Fig. 6. With growing of the distance between the bunch center and the target surface the differences in intensity of radiation becomes insignificant and goes to the distribution of TR from the infinity strips.

Usually, the spectral-angular distribution of radiation from the bunch is written in the following form:

$$\frac{dW(\mathbf{n}, \omega)}{d\Omega d\hbar\omega} = \frac{dW_1(\mathbf{n}, \omega)}{d\Omega d\hbar\omega} [N + N(N-1)G'_{coh}], \quad (19)$$

where $dW_1(\mathbf{n}, \omega)/d\Omega d\hbar\omega$ is the spectral-angular distribution of radiation from a single particle; the designation G'_{coh} with prime is used in order to indicate that it does not coincide with the coherent form-factor G_{coh} from our expression (see Eqs. (13) and (16)).

We would like to stress, that form of Eq. (19) is, generally speaking, not correct. There are two reasons for this. First is that in Eq. (19) the incoherent form-factor has not been taken into account. Second, using Gaussian distribution we always deal with the polarization radiation, which is the mixture of Diffraction radiation and Transition radiation – there are the “tails” of distribution that are on the other side of the upper edge of the target. Therefore, we can tell about TR or DR only in limiting cases, when bunch is distant enough from the upper edge of the target.

In Fig.7 we plot the correct distribution (solid black curve) and distribution from Eq. (19). As one can see, the difference can be considerable.

The spectral-angular distribution depending on impact-parameter is shown in Fig. 8. The angle ϕ corresponds to the third peak of SPR. Black solid curve corresponds to

our results, red dashed - to the results obtained by usual way (see Eq. (19)). One can see that for impact-parameter $h \geq \sigma_z$ they are very close to each other, but for negative h situation changes.

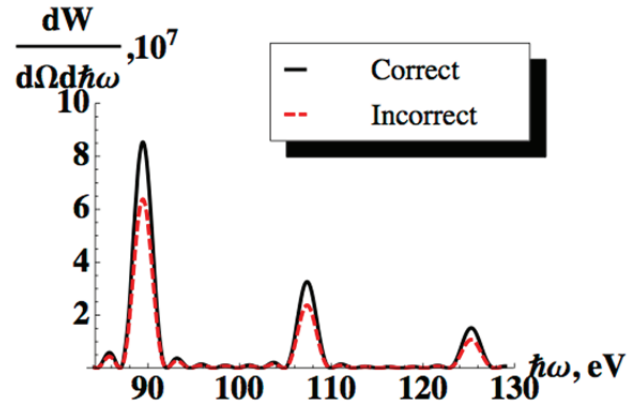


Figure 7: The spectral-angular distribution of incoherent radiation for correct (black solid, G_{inc} from Eq. (15)) and incorrect (red dashed, $G_{inc} = 1$) calculation of the form-factor. Here $N_{st} = 7$, $d = 0.5 \mu m$, $a = d/2$, $\omega = 3\omega_p$, $h = 17 \mu m$ and other parameters as in Fig. 3.

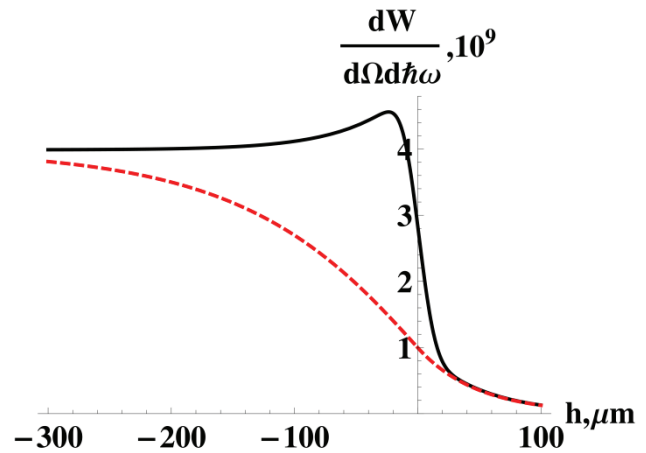


Figure 8: The spectral-angular distribution depending on impact-parameter: DR for $h > 0$, TR for $h < 0$. Black solid curve corresponds to our results, red dashed - to the usually considered results (see Eq. (19)). Here $N_{st} = 7$, $d = 0.5 \mu m$, $a = d/2$, $\phi = 24.6^\circ$ ($n_{SP} = 3$) and other parameters as in Fig. 3. The curves are plotted for $\omega = 3\omega_p$, i.e. $\lambda \ll \sigma_x = 20 \mu m$, which corresponds to incoherent radiation.

CONCLUDING REMARKS

The existence of incoherent form-factor was denoted more than a decade ago [9], [10], but for the first time it has been obtained in analytical form and analyzed only recently [6] for uniform distribution of the electrons in bunch.

In this work we obtain the incoherent form-factor for Gaussian distribution, along with the coherent one. It is proved, that for the case considered (particles of the bunch go both through the target and above it), spectral-angular distribution of radiation has the features both resonant DR and resonant TR. Therefore, the resulting intensity of radiation coincides with habitual Eq. (19) in case of incoherent X-ray TR only when $|h| > \gamma\beta\lambda/4\pi$, and in case of DR only when $h > \min\{\sigma_z, \gamma\beta\lambda/4\pi\}$ - see Fig. 8. It is interesting to see, that Fig. 8 demonstrates the strong effect of the edge, and along with that shows the peak of transition radiation caused by incoherent form-factor.

The theory developed is valid for arbitrary energies of the particles, for Gaussian distribution and for UV and X-ray domain of photon energies ($\omega \gg \omega_p$, which in practice stands for $\omega > 3\omega_p$).

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