AN ANALYSIS OF OPTIMUM OUT-COUPLING FRACTION FOR MAXIMUM OUTPUT POWER IN OSCILLATOR FEL

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Abstract

The effect of the out-coupling fraction on the output power in oscillator FEL is analyzed. The formulas of the optimum out-coupling fraction and the corresponding maximum output power are given. They are dependent on the initial small signal gain and the passive loss rate of the light in the optical cavity. The initial comparison show that the result given by the formula agree well with the results in references.

INTRODUCTION

The basic working modes of free-electron lasers (FELs) include the amplifier, the oscillator and the self-amplified spontaneous emission (SASE). The oscillator FEL work in the low gain regime with the multi-amplifying. For oscillator FEL，one main goal of the design and optimization is to achieve the maximum output power. An important work is to optimize the out-coupling fraction of the light. It has been discussed by several authors [1-3], in this paper we analysis the optimization of the out-coupling fraction for maximum output power, here we don't consider what specific way of the out-coupling is used.

ANALYSIS

By expanding FEL equations and taking some approximation, the optical field gain at the pass n can be given as [4]

$$g_n \approx \frac{e^{g_{ss}}-1}{1+e^{g_{ss}}P_{n-1}/P_c}, \quad (1)$$

where P_{n-1} is the optical power at the pass n-1,the power of the optical pulse at the undulator exit during the nth passage, g_{ss} is initial small signal gain:

$$g_{ss} = -(2k_u\rho L)^3 \left\langle \frac{\partial}{\partial x}\frac{\sin^2(x/2)}{(x/2)^2} \right\rangle_{\phi_0'}, \quad (2)$$

where ρ is FEL parameter, k_u and L is the wave vector and length of the undulator, $x=\phi_0'L$, the angular bracket represents the average over the electron's initial phase velocities (i.e. the tuning parameter) ϕ_0'; and

$$P_c = \frac{g_{ss}}{\beta}\rho P_e. \quad (3)$$

P_e is electron beam power and [4, 5]

$$\beta = (2k_u\rho L)^7 \left\langle \frac{1}{x^7}\{x(6-x^2)\sin x + 4(1-x^2)\cos x + \frac{3}{2}x\sin 2x + \frac{1}{2}(\frac{5}{2}-x^2)\cos 2x - \frac{21}{4}\}\right\rangle_{\phi_0'}. \quad (4)$$

When the power equal to P_c, i.e. $P_{n-1}=P_c$, from Eq. 1 it has

$$g_n = \frac{e^{g_{ss}}-1}{e^{g_{ss}}+1} = \mathrm{cth}(\frac{g_{ss}}{2}) \approx g_{ss}/2.$$

Therefore P_c is the value of the intensity halving the small signal gain of the device. The profiles of the angular bracket parts in the expressions of g_{ss} (Eq. 2) and β (Eq. 4) are given in Fig. 1, from which we have $g_{ss}/\beta \sim 100/(2k_u\rho L)^4$. Therefore we can estimate the value of P_c : $P_c \sim 100(\sqrt{3}L_g/L)^4\rho P_e$, for low gain regime L<3L_g, thus we have $P_c >\sim 10\rho P_e$. Notice the saturation power $P_s > P_c$ (Eq. 7), we can know that the saturation power of oscillator FEL is larger than that of SASE FEL(~ρP_e).

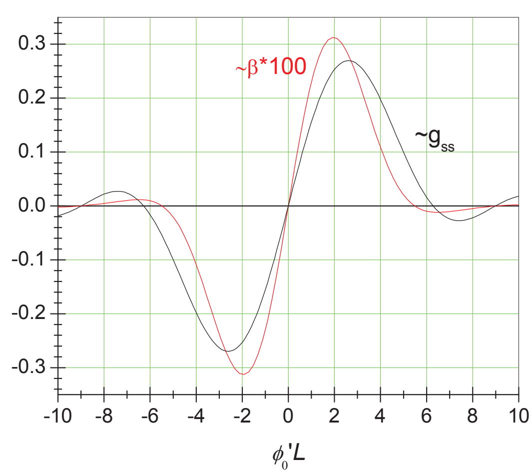

Figure 1: the angular bracket parts of g_{ss} and β in Eq. 2 and 4.

The intra-cavity power at pass n is

$$P_n = P_{n-1}(1+g_n)(1-\alpha) = P_0\prod_{i=1}^{n}(1+g_i)(1-\alpha), \quad (5)$$

where α is the total loss ratio in optical cavity including the output coupling fraction and the passive loss. P_0 is the initial emission power, i.e. the spontaneous emission power [6]:

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$$P_0 = (2k_u \rho L)^2 \frac{1}{N_{e,s}} \rho P_e \cdot \quad (6)$$

N_{es} is the number of electrons in the slippage distance.

As the optical field intensity increase, the gain decrease, when the net gain equal to zero, it has $g_n = \alpha/(1-\alpha)$, the system reaches equilibrium, namely the optical field reaches saturation. From Eq. 1 we obtain the saturation power in cavity

$$P_s = \frac{1-\alpha-e^{-g_{ss}}}{\alpha} P_c \sim \frac{g_{ss}-\alpha}{\alpha} P_c \, . \quad (7)$$

The saturation powers given by Eq. 7 agree well with the one-dimensional numerical results [1].

With Eq. 7 the gain at the pass n (Eq. 1) can also be written as

$$g_n = \frac{(e^{g_{ss}}-1)\alpha}{\alpha + [e^{g_{ss}}(1-\alpha)-1]P_{n-1}/P_s} \, . \quad (8)$$

The time required to reach saturation can be estimated as

$$T_e \geq \frac{4L_c \ln(P_s/P_0)}{c(g_{ss}-\alpha-g_{ss}\alpha)} \, , \quad (9)$$

where L_c is the length of optical cavity, We denote the output coupling fraction and the passive loss of the cavity as α_{oc} and α_{lo}, respectively, and from $1-\alpha = (1-\alpha_{lo})(1-\alpha_{oc})$, we have $\alpha = \alpha_{lo} + \alpha_{oc} - \alpha_{oc}\alpha_{lo}$. Then the output power is $P_{out} = P_s(1-\alpha_l)\alpha_{oc}$. For a given intra-cavity optical power of oscillator FEL, the larger out-coupling fraction gives a larger fraction of the out-coupled power. But the larger out-coupling fraction means the smaller net gain, which leads to the optical field saturated earlier, i.e. the lower intra-cavity power. Therefore there exists an optimum output coupling fraction for maximum out-coupled power (Fig. 2).

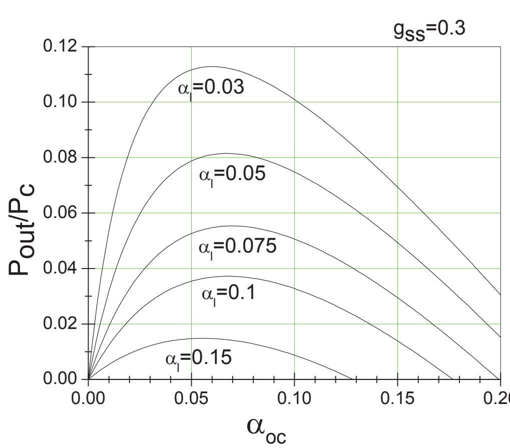

Figure 2: the output power with output coupling fraction for different cavity loss, the initial gain is 0.3.

Using Eq. 7 we differentiate the output power P_{out} with respect to α_{OC}, and set the derivative equal to zero, then we get the optimum out-coupling fraction

$$\begin{aligned} \alpha_{oc,m} &= (\alpha-\alpha_{lo})/(1-\alpha_{lo}) \\ &= (\sqrt{(1-e^{-g_{ss}})\alpha_{lo}}-\alpha_{lo})/(1-\alpha_{lo}) \end{aligned} \, . \quad (10)$$

We can see that the optimal output coupling is determined by the passive losses and the initial small signal gain (Fig. 3).

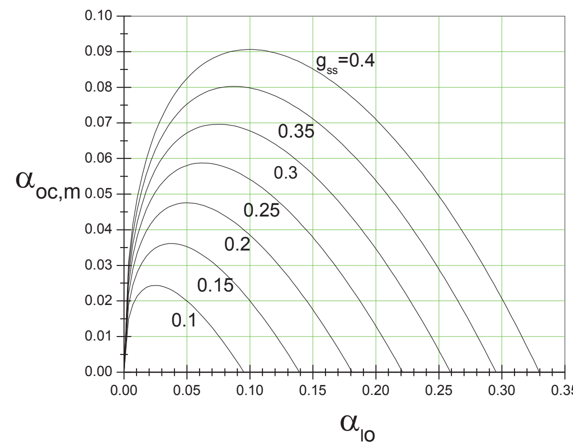

Figure 3: the variation of optimum out-coupling fraction with the cavity loss for different initial gain.

The corresponding maximum output power is:

$$P_{out,m} = (\alpha-\alpha_l)P_s = (\sqrt{1-e^{-g_{ss}}}-\sqrt{\alpha_{lo}})^2 P_c \, . \quad (11)$$

For comparison, an example of the output power with the out-coupling fraction is shown in Fig. 4, it is from ref. [2]. the optimum out-coupling fraction in the figure is 6%, while using Eq. 10 it is 5.8%. Comparing with the results in references [1,3], the results given by our formula (Eq. 10) are also in good agreement with them.

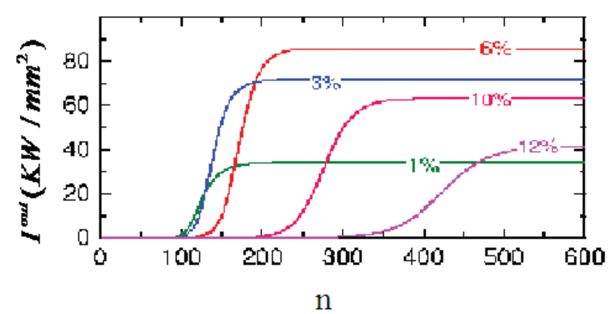

Figure 4: From ref. [2], the outcoupled intensity is shown as a function of the number of roundtrips calculated for different output couplers. The small-signal gain was assumed to be 25% and 5% losses were assumed.

The Eq.10 gives optimum out-coupling fraction 5.8%.

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SUMMARY

In summary a simple formula is given for the optimum out-coupling fraction and the corresponding maximum output power of an oscillator FEL, it will be helpful to design and optimization of an oscillator FEL. The obtained formula show that the optimum out-coupling fraction is related with initial gain and the passive loss of the light in the optical cavity.

It should be pointed out that when we get the formula, we assumed that the passive loss is not varied with the out-coupling fraction, which holds under certain conditions. For example, in the hole-coupling mode when the hole is small. But if the hole is too large, the optical field mode will be changed that will have effect on the passive loss. Another assumption is that the initial gain is the small signal gain, which is applicable to the case of the undulator length less than the three gain length, i.e. in the low gain regime.

The results from the formula agree well with the results in the references. The more detail comparison with numerical simulation will be carried out to check the extent of its validity.

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