SPECTRAL LIMITS AND FREQUENCY SUM-RULE OF CURRENT AND RADIATION NOISE MEASUREMENT*

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Abstract

The current noise spectrum of an electron beam is generally considered white and expressed by the shot-noise formula (eI_0). It is possible to control the spectral energy of a random electron beam current by longitudinal space charge micro-dynamics and dispersive transport. Both noise suppression (relative to eI_0) and noise enhancement have been demonstrated, exhibiting sub/super-Poissonian particle distribution statistics, respectively. We present a general theory for the current noise of an e-beam and its radiation emission in the entire spectrum. The measurable current noise spectrum is not white. It is cut-off at high frequencies, limited by the measurement length and the beam axial momentum spread (fundamentally limited by quantum uncertainty). We show that under certain conditions the current noise spectrum satisfies a frequency sum-rule: exhibiting noise enhancement in one part of the spectrum when suppressed at another part and vice versa. The spontaneous emission (radiation noise) into a single radiation mode or single direction in any scheme (OTR, Undulator etc.) is sub-radiant when the beam current is sub-Poissonian and vice versa, but the sum-rule does not apply.

INTRODUCTION

Electron beam current-noise is an inherent property of any particulate current resulting from the microscopic discontinuity of charge flow in a charged particles beam. The conventional assumption regarding the current noise in an accelerated electron beam is that it is limited by the Shot-Noise formula:

$$S_{I_{\text{shot}}} = eI_0 \qquad (-\infty < f < \infty), \tag{1}$$

where I_0 is the average current of a continuous coasting electron beam and $S_I(f)$ is the power spectral density (PSD) of the current. This expression is a direct consequence of the assumption that the beam particles positions are random uncorrelated uniform variables, so that the number of particles in each interval satisfies the Poisson statistics.

It has been known [1] that as an electron beam propagates, the Coulomb interaction between the particles results in correlation between the particle positions and therefore, the particles statistics, as well as their current PSD may deviate from the shot-noise formula (1). In particular it has been shown that Eq. (1), is not even a lower limit for random electron beam noise, and taking advantage of the Coulomb interaction effect, or the longitudinal space charge (LSC) interaction of random bunching, it is possible to suppress the current noise below the shot-noise level (current noise suppression) at least in part of the spectrum.

Recently it has been shown theoretically [2–8] and experimentally [9, 10] that shot-noise suppression is possible in high quality e-beams at optical frequencies. This noise suppression process can be achieved by transport of the beam along a drift section of quarter plasma oscillation length [9] or by transport through a drift section and a subsequent dispersive section [10]. In the later case, if the dispersion effect (presented by the parameter R_{56}) is large, the opposite effect - noise gain - is achieved, at least in part of the spectrum (micro-bunching instability [11]).

Current noise is the source of incoherent spontaneous emission of radiation (radiation noise) in all electron-beam radiation sources, and in particular Undulator radiation and SASE (Self Amplified Spontaneous Emission) in FEL [8]. For this reason controlling radiation noise is one of the reasons of interest in controlling electron-beam current noise at short wavelengths. In innovative temporally coherent seed-injected FELs [12] incoherent SASE radiation limits the coherence of the FEL output and imposes stringent demands on the power level of the seed radiation source [8]. Hence current noise suppression of the e-beam before injection into the wiggler is desirable. On the other hand, control over the e-beam current shot-noise can also be useful for the opposite purpose: enhancing SASE radiation [13]. This process has been recently demonstrated by Marinelli et al [14] and may possibly be used to produce high power radiation in SASE FELs with shorter wigglers.

MEASUREMENTS OF CURRENT-NOISE SPECTRUM

Determining the spectral limits of noise-control, and particularly noise-suppression, is a task of prime interest in connection to electron-beam transport in applications of electron beams for emission of coherent radiation (FEL). Of primary importance is the short wavelength limit, as there is significant interest in developing coherent (low noise) X-UV FELs.

The short wavelength limit of current noise suppression by drift over a quarter plasma wavelength is the Debye condition: $\lambda > \lambda_D$, where the Debye wavelength λ_D is determined by the axial velocity spread of the beam due to finite emittance or energy spread [15]. A similar condition applies also to the drift/dispersion noise suppression scheme where the dispersive section enhances the optical phase-spread at short wavelength [15]. The LSC noise-suppression effect is also limited at low frequencies (though this limit is of less interest). The low frequency limitation of LSC interaction

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is due to the 3-D fringing effect in a finite cross-section beam [3].

While the shot-noise formula (1) predicts a "white noise" spectrum for a continuous uncorrelated electron beam as a direct result from the assumption of Poisson statistics, the noise-suppressed (or noise-enhanced) electron beam is governed by more complex statistics, and its spectrum is certainly not "white". To understand the nature of this spectrum we take a close look into the derivation of the current noise formulae, and in particular pay attention to physical limitations associated with the measurement of the spectrum.

Since optical frequency current cannot be measured directly, the only way to measure the current optical frequency noise spectrum is by measuring the radiation that the e-beam current emits in any kind of radiation scheme, in which the optical radiation spectrum is proportional to the current noise spectrum. Most measurements of this kind were done using Optical Transition Radiation (OTR) [16]. Another obvious scheme is Undulator radiation, which is the spontaneous emission radiation of an FEL. In any case it is important to point out that any physical measurement is done during a finite time, which in the present case is the transit time of the electron through the radiation device $T_{\mathrm{tr}} = L_{\mathrm{tr}}/(\beta c)$, where L_{tr} is the radiative emission length, namely the formation length $L_f = \beta\gamma\lambda$ in the case of OTR and the Undulator length L_u in the case of Undulator radiation.

The finite measurement time and the finite radiation emission length have implications on the applicability of the current noise spectral measurement of a correlated electron beam as well as an uncorrelated electron beam (shot-noise): the interaction length must be short enough in order to make sure that one measures spontaneous emission only and not SASE, and that the current noise stays constant along the transit length, and that transition of current noise to velocity noise though LSC micro-dynamic processes [8, 15] is negligible.

Under these limiting assumptions, a general formulation was presented in [17] for the calculation of the spectral radiation energy emission by a finite pulse of N_e electrons entering the radiative interaction region at ordered times (pre-bunched) or random times. The current of the electron pulse is represented by:

$$I(t) = -e\sum_{j=1}^{N_e}\delta(t - t_{0j}), \tag{2}$$

where t_{0j} is the time at which electron j is at $z = 0$. Its spectral components are

$$\breve{I}(f) = -e\sum_{j=1}^{N_e} e^{i2\pi f t_{0j}} \tag{3}$$

where we define the Fourier transform

$$\breve{I}(f) = \int_{-\infty}^{\infty} dt\, I(t)\exp(i2\pi f t), \tag{4}$$

The energy spectral density (ESD) of the finite pulse is

$$p_I(f) = \langle|\breve{I}(f)|^2\rangle = e^2\left\langle\left|\sum_{j=1}^{N_e} e^{i2\pi f t_{0j}}\right|^2\right\rangle, \tag{5}$$

here $\langle\rangle$ represents statistical average over the arrival times t_{0j}. For a continuous coasting beam one usually defines the power spectral density (PSD) by $S_I(f) = \lim_{T\to\infty}\frac{1}{T}\langle|\breve{I}_T(f)|^2$, where $\breve{I}_T(f)$ is the Fourier transform in the time window T, which reduces for the case of a rectangular finite pulse of time T_b to

$$S_I(f) = \frac{1}{T_b}\langle|\breve{I}(f)|^2\rangle = \frac{p_I(f)}{T_b}, \tag{6}$$

The spontaneous emission formulation [17] is based on a modal expansion of the radiation field in terms of any complete set of eigenmodes, where the set may be discrete or continuous modes of free space (plane-waves).

The derivation leads to a general proportionality relation of the ESD of the emitted radiation energy per mode (in units of Joule/Hz) and the ESD of the current. The proportionality factor can be calculated specifically for any free electron radiation scheme like Undulator radiation and OTR [9, 17] then be used for the calculation of the current ESD as long as one measures the radiation energy per mode, or in the case of free space plane waves - the far field radiation ESD per unit solid angle.

As shown in the next sections the detailed analysis of the current noise and the consequent radiation noise shows that the ESD of the current and the radiation deviate from the simple white shot-noise of formula (1) even when the e-beam is random and uncorrelated. At low frequencies (long wavelengths)

$$f < 1/T_b \tag{7}$$

(where T_b is the pulse duration) one expects enhancement of the current noise due to the coherent contribution (DC part) of the pulse shape function to the Fourier transform. This effect gives rise to supper-radiant emission at low frequencies [17].

Another effect colors the measurable spectral energy of the current noise, and cuts it off at high frequencies. This effect is associated with the inherent uncertainty in the particle position during the measurement of the current noise. Because noise measurement is never instantaneous and because there is always inevitable uncertainty in the position σ or crossing time $\sigma_t = \sigma/(\beta c)$, of the particles either because of initial axial velocity spread, or fundamentally, because of quantum (Heisenberg) momentum uncertainty, the position of each particle in the calculation of the current ESD (5) must be averaged over the probability function of the position uncertainty (or t_{0j}). This is done in the following sections, and also resolves the problem of a seemingly infinite noise energy when one integrates the ESD of point particles (5) over all frequencies.

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An interesting observation, that was noted by Alex Chao [18] for current-noise spectrum, suggests a "sum-rule theorem", namely that the integral of the noise spectrum stays constant even when the noise is suppressed or enhanced in some part of the spectrum. This theorem and its limitations are examined in the following sections, using our finite noise energy model. Simple examples for noise suppression or enhancement (sub-Poissonian or super-Poissonian distributions) are examined.

THE DERIVATION OF THE SHOT-NOISE FOR A CONTINUOUS COASTING BEAM OF POINT CHARGES

We consider a bunch of particles, each one of charge e, all moving with a uniform velocity $v = \beta c$ in the $\hat{z}$ direction. The bunch length is $L_b \rightarrow \infty$, the number of charges is $N_e \rightarrow \infty$, so that N_e/T_b is finite, resulting in the average DC current $I_0 = eN_e/T_b$. Each charge is uniformly distributed in the interval $[0, L_b]$, i.e. has equal probability to be located anywhere in the bunch. To each space coordinate we associate a time interval coordinate, so that $T_b \equiv L_b/v$ is the time interval of the entire bunch. Hence counting charges in a space interval Δz at a given time is equivalent to counting charges that pass through a given point z during a time interval $\Delta t = \Delta z/v$. The bunch length T_b is divided in sub-intervals Δt, so that the current measured in the interval Δt is $I = en/\Delta t$, where n is a random variable representing the number of charges in the given interval Δt.

The probability to find a charge in an interval $\delta t \ll \Delta t$ is $p = N_e \delta t/T_b$. Because the probability p is extremely small, one may neglect the possibility of having 2 or more charges in the interval δt, considering the occurrence of the event of having a charge in δt as $N = \Delta t/\delta t$ Bernoulli trials, so that n is binomial distributed with N trials and probability p, with average $m_n = Np$ and variance $\sigma_n^2 = Np(1-p)$. Because N is very large and p is very small, n may be approximated by Poisson distribution with parameter $\lambda = Np = N_e \Delta t/T_b$, so that its average and variance are $m_n = \sigma_n^2 = \lambda$.

It follows that when the *location* is distributed *uniformly*, the number of charges in *any* interval Δt is *Poisson* distributed, i.e. having the mean equal to the variance.

So if one measures the current I in a time interval Δt, one obtains $I = en/\Delta t$, so that the current is the random variable n multiplied by $e/\Delta t$. Therefore the average current is $m_I = em_n/\Delta t = e\lambda/\Delta t \equiv I_0$ and its variance is $\sigma_I^2 = (e/\Delta t)^2 \sigma_n^2 = (e/\Delta t)^2 \lambda = eI_0/\Delta t$. Considering $\Delta t = 1/(2B)$ to be the Nyquist interval, where $2B$ is the two-sided bandwidth, we obtain $\sigma_I^2 = 2eI_0 B$. For point charges, the current at time t is independent from the current at time $t + \tau$, so the auto-covariance matrix is diagonal $C_I(\tau) \equiv C_{I(t)I(t+\tau)} = eI_0\delta(\tau)$ and its Fourier transform is eI_0 having the area of σ_I^2 in the bandwidth range $-B < f < B$. Therefore the auto-covariance function is

$$C_I(\tau) = eI_0\delta(\tau), \tag{8}$$

and from random processes theory, the auto-correlation function is the DC square plus the auto-covariance function:

$$R_I(\tau) = I_0^2 + eI_0\delta(\tau), \tag{9}$$

and the PSD is the Fourier transform of the above

$$S_I(f) = I_0^2\delta(f) + eI_0 \equiv I_0^2\delta(f) + S_{I_{\text{shot}}}, \tag{10}$$

where the flat spectral density is $S_{I_{\text{shot}}}$ in formula (1).

We saw in this section that if the charges are *uniformly* distributed over the interval T_b, the *number* of charges is *Poisson* distributed in *each* sub-interval Δt. The opposite is not necessarily true: if the number of charges is Poisson distributed in a sub-interval Δt we only know that it is also Poisson distributed in any sub-interval *bigger* than Δt (i.e. for frequencies smaller than $1/(2\Delta t)$), but about smaller intervals (higher frequencies) we don't know anything, and this depends on how the charges are distributed inside Δt. It is to be mentioned that the beam drift dynamics establishes the distribution of the charges locations and this *location distribution* establishes the statistics of the current, i.e. the statistics of the random variable n. In this work we do not use beam dynamics, but we would examine how changes in the statistics of n influence the spectrum.

THE FINITE PULSE ESD AND THE SUM RULE

Expression (10) describes an ideal Shot noise PSD, for a coasting beam. In this section we analyze the energy spectral density (ESD) for a finite beam of duration T_b and the connection to the coasting beam PSD.

Expanding Eq. (5), we get

$$p_I(f) = e^2 \left\langle \sum_{j=1}^{N_e} \sum_{k=1}^{N_e} e^{i2\pi f(t_{0j}-t_{0k})} \right\rangle = e^2 N_e + e^2 \left\langle \sum_{j=1}^{N_e} \sum_{\substack{k=1 \\ k\neq j}}^{N_e} e^{i2\pi f(t_{0j}-t_{0k})} \right\rangle \tag{11}$$

The first part of the result was obtained by summing over $j = k$ and represents the independent frequency ESD Shot noise term:

$$p_{I\,(\text{shot})} \equiv e^2 N_e = T_b e I_0, \tag{12}$$

which results in the PSD Shot noise term eI_0 in Eq. (10) if divided by T_b . The second part should average to 0 for $f \neq 0$, but it has a contribution for $f \simeq 0$. This contribution may be approximated for $N_e \gg 1$ by replacing the sum by an integral on dt and setting $dt \simeq T_b/N_e$, as follows

$$e^2 \left\langle \sum_{j=1}^{N_e} \sum_{\substack{k=1 \\ k\neq j}}^{N_e} e^{i2\pi f(t_{0j}-t_{0k})} \right\rangle \simeq e^2 \left| \frac{N_e}{T_b} \int_0^{T_b} e^{i2\pi f t} dt \right|^2 = I_0^2 T_b^2 \,\text{sinc}^2(fT_b). \tag{13}$$

This part, divided by T_b represents the $\delta(f)$ in Eq. (10), given that $\lim_{T_b\rightarrow\infty} T_b \,\text{sinc}^2(fT_b) = \delta(f)$.

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In addition, as mentioned before, there is always uncertainty in the particle axial momentum - either because of technical momentum spread of the electron beam, or fundamentally because of quantum limits (Heisenberg's uncertainty principle). Therefore we shall consider the size of the charge to be finite, so that $\delta(t-t_{0j})$ in Eq. (2) is replaced by $\frac{1}{\sqrt{2\pi}\sigma_t}\exp\left(-\frac{(t-t_{0j})^2}{2\sigma_t^2}\right)$, so that σ_t represents the uncertainty in the arrival time of a charge.

Repeating the Fourier transform on the current for charges of finite size, the $\exp(i2\pi f t_{0j})$ are replaced by $\exp(i2\pi f t_{0j})\exp\left[-\frac{1}{2}f^2(2\pi\sigma_t)^2\right]$, so that the Fourier transform of the current should be multiplied by the factor $\exp\left[-\frac{1}{2}f^2(2\pi\sigma_t)^2\right]$.

This implies that the ESD in Eq (5) should be multiplied by the square of the above term: $e^{-f^2(2\pi\sigma_t)^2}$, so that the ESD of the current is described by:

$$p_I(f) = e^2[N_e^2\,\mathrm{sinc}^2(fT_b) + N_e]e^{-f^2(2\pi\sigma_t)^2}. \tag{14}$$

This means that very close in time measurements of current are *dependent*. For this purpose let us consider the $T_b\,\mathrm{sinc}^2(fT_b) \simeq \delta(f)$ in Eq. (14), divide by T_b and inverse Fourier the non delta part. This gives a more accurate autocovariance function

$$C_{I(\text{finite size charges})}(\tau) = eI_0\frac{1}{2\sqrt{\pi}\sigma_t}\exp\left(\frac{-\tau^2}{4\sigma_t^2}\right), \tag{15}$$

which means that neighbor measurements of current done on time intervals of the order of the arrival uncertainty σ_t are dependent.

The sum rule

The total energy in the ESD may be analyzed using Parseval theorem. If $f(t)$ and $\breve{f}(f)$ are a Fourier transform pair

$$\int_{-\infty}^{\infty}|f(t)|^2dt = \int_{-\infty}^{\infty}|\breve{f}(f)|^2df \tag{16}$$

Averaging on both sides and using the ESD definition in Eq. (5), we obtain from Parseval theorem

$$\left\langle\int_0^{T_b} I^2(t)dt\right\rangle = \int_{-\infty}^{\infty}p_I(f)df = 2\int_0^{\infty}p_I(f)df \tag{17}$$

where the last equality uses the fact that the ESD is an even function. For point particles (canceling e^2 on both sides) it becomes:

$$\left\langle\int_0^{T_b}\left(\sum_{j=1}^{N_e}\delta(t-t_{0j})\right)^2dt\right\rangle = 2\int_0^{\infty}\left|\sum_{j=1}^{N_e}\exp(i2\pi f t_{0j})\right|^2df \tag{18}$$

This sum rule, was noted by Alex Chao [18]. It implies that the LHS is insensitive to particles positions (being δ^2 for point particles), hence the RHS must be constant, and therefore if there is noise suppression (homogenization) in one part of the spectrum, there must be noise enhancement in another part.

However, the problem for point particles is that the above constant is infinite, because the δ^2 function is ill defined. The solution: any physical measurement of current noise spectrum (OTR, Undulator radiation) takes place during a finite beam transit time, so that during the measurement there is inherent uncertainty in the particle location (σ) either because of momentum spread or fundamentally because of quantum (Heisenberg) principle. This consideration sets the constant to be finite, but sets a limit on the validity of the sum-rule (particles overlap).

Therefore the sum-rule is valid under the limitations of the overlap condition, and we may evaluate both sides of (17) for the case of finite size charges. The LHS of Eq. (17) results in

$$\left\langle\int_0^{T_b} I^2(t)dt\right\rangle = \int_0^{T_b}dt\left(\sum_{j=1}^{N_e}\frac{e^2}{\sqrt{2\pi}\sigma_t}\exp\left(-\frac{(t-t_{0j})^2}{2\sigma_t^2}\right)\right)^2 \simeq \frac{N_e e^2}{2\sqrt{\pi}\sigma_t}, \tag{19}$$

for any realization of $I(t)$, provided the gaussians do not overlap, i.e. if

$$\sigma_t \ll T_b/N_e. \tag{20}$$

or using the definition of I_0 the condition becomes

$$I_0 \ll e/\sigma_t, \tag{21}$$

hence is valid for low currents. The RHS of Eq. (17) results in

$$2\int_0^{\infty}p_I(f)df \simeq e^2N_e\left(\frac{1}{2\sqrt{\pi}\sigma_t}+\frac{N_e}{T_b}\right), \tag{22}$$

provided the $T_b\,\mathrm{sinc}^2(fT_b)$ in Eq. (14) is narrow enough to be approximated by $\delta(f)$ and hence not to be affected by the Gaussian decay $e^{-f^2(2\pi\sigma_t)^2}$. Indeed the results in Eqs (19) and (22) coincide if condition (20) is satisfied. It is easy to check that one full overlap of 2 charges increases the area by $\frac{1}{\sqrt{\pi}\sigma_t}$, so that partial overlaps increase by less than that.

SIMULATIONS OF THE SUM-RULE CONSERVATION

In Figure 1 we calculate the normalized ESD $p_I(f)/p_{I\,(\text{shot})}$ vs normalized frequency fT_b for a bunch of $N_e = 500$ charges, where the bunch duration is $T_b = 500$ time units and the arrival uncertainty is $\sigma_t = \frac{T_b/N_e}{80} = 0.0125$.

Next we examine sub and super Poissonian distributions. The interval T_b is divided into M sub-intervals, so given $N_e = 500$ the average number of charges in each sub-interval is N_e/M. Choosing for the number of charges in each sub-interval a Poisson random variable with average N_e/M, or a Gaussian random variable with average N_e/N and standard deviation $\sqrt{N_e/M}$, and spreading the charges by uniform distribution inside the sub-interval, gives a Poissonian realization. The PSD is calculated with Eq. 6, and the averaging is done over 200 realizations, resulting in Poissonian

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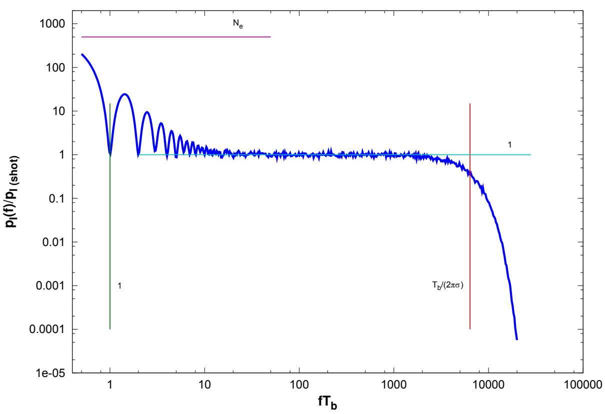

Figure 1: The normalized PSD of the current $p_I(f)/p_{I\text{ (shot)}}$. The location for each charge is randomly chosen by uniform distribution in the interval $[0, T_b]$, hence the number of charges in each sub-interval is Poisson distributed. The ESD is calculated with Eq. 5, and the averaging is done over 200 realizations. In the flat region we get 1 by definition. At $fT_b = 1$ we see the first 0 of the sinc^2 behavior for low frequencies, and for $fT_b \to 0$ (left to the plot) the normalized ESD tends to $I_0^2 T_b^2 / p_{I\text{ (shot)}} = N_e$. At $fT_b = T_b/(2\pi\sigma_t)$ the normalized ESD decays to e^{-1}. The total area under the curve is 11392, which compares well with $\frac{N_e e^2}{2\sqrt{\pi}\sigma_t} = 11284$.

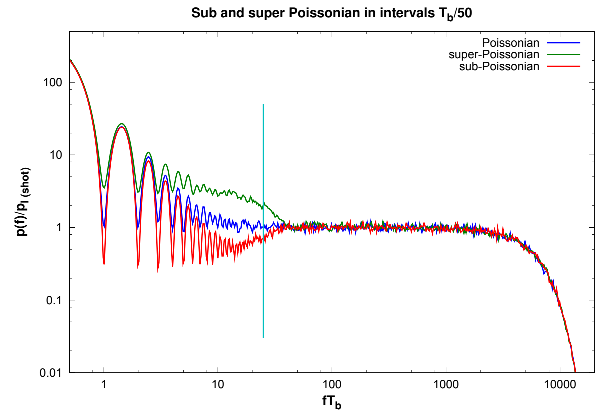

Figure 2: Super and sub-Poissonian statistics in sub-intervals $T_b/50$. The interval T_b is divided into 50 sub-intervals, so given $N_e = 500$ the average number of charges in each sub-interval is 10. The Poissonian, super-Poissonian and sub-Poissonian PSDs are shown by the blue, green ad red curves respectively. The standard deviation for the super-Poissonian case is $1.8\sqrt{10}$ instead of the chosen $2\sqrt{10}$, while the sub-Poissonian case has a standard deviation very close to $0.5\sqrt{10}$. The sub and super-Poissonian behavior are for frequencies $fT_b < 50/2 = 25$ (marked with vertical line), and the super and sub Poissonian PSD values are established by the relative variances, i.e. 1.8^2 and 0.5^2. The areas under the spectra are 11523, 11392, 11396 for the super-Poissonian, Poissonian, and sub-Poissonian, respectively.

PSD, which is identical to the PSD in Figure 1. Using the same procedure, with a Gaussian distribution of standard deviation $2\sqrt{N_e/M}$ or $0.5\sqrt{N_e/M}$, results in the super and sub-Poissonian PSDs. Because the number of charges in an interval cannot be negative, we cut the distribution below 0, and to keep the correct average we also cut the distribution over $2N_e/M$. Therefore the variances obtained are not exactly the ones chosen. The sub and super-Poissonian behaviors are in sub-intervals bigger or equal the basic sub-interval T_b/M (but not too big to compare with T_b !), i.e. for frequencies smaller than $M/(2T_b)$ but not 0.

Those sub and super-Poissonian PSDs are implemented in Figures 2 and 3 for $M = 50$ and 100 respectively.

In Figure 4 we check how the sum rule described above is satisfied. When creating sub or super-Poissonian distribution for frequencies below some threshold, to satisfy the sum rule, the distribution should be of the opposite type in the complementary range. Unfortunately, this is not easy to show, because for the super-Poissonian cases created here, the overlap between charges increases, hence increasing the total area under the PSD. In addition the frequency regions for the sub and super-Poissonian are very narrow (below normalized frequency $fT_b = 25$ or 50 for the cases described in Figures 2 and 3 respectively), relative to the whole normalized spectrum range of about $fT_b = 10000$, described in those cases. Hence in the complementary region one gets the opposite case, but very close to Poissonian. Also, the small fluctuations of the PSD make the comparison difficult. To show how the sum rule works we created a "very" sub Poissonian distribution in sub-intervals $T_b/50$, having a standard deviation close to 0 and compared it with the Poissonian distribution. The comparison required to smooth the lines, hence the crossing point is not accurate. We show the difference between the sub-Poissonian PSD and the Poissonian PSD around the crossing point. Left to the crossing point, the value is negative and right to the crossing point the value tends to be small positive.

CONCLUSION

We presented in this work a statistical and spectral analysis for the current of an electron beam. We showed that even for random and uncorrelated distribution of the electrons locations, the ESD is not exactly the shot-noise, but has supper-radiant properties at low frequencies, and is cut off at high frequencies due to the uncertainty in the arrival time of the electrons. The last conclusion solves the "infinite" energy dilemma of the shot-noise.

We showed that under some conditions, the ESD satisfies a sum-rule, so when noise is suppressed in some spectral region it is enhanced in another region. However the validity of the sum-rule is limited to low current beam electron only (21), such that the overlap of the uncertainties of the particles location during transit is smaller than the average spacing between the particles.

We calculated ESD for different cases of sub or super Poissonian, and checked the sum-rule, i.e. the (partial) conservation of the total energy.

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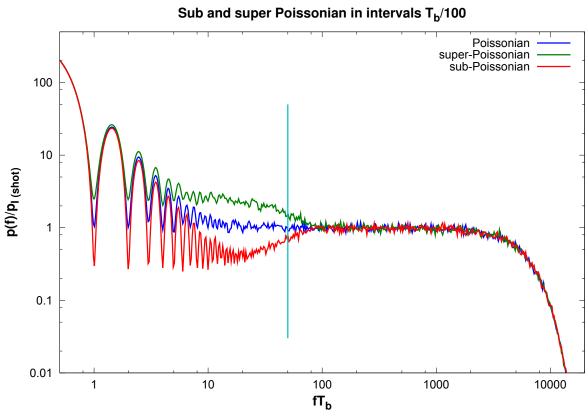

Figure 3: Super and sub-Poissonian statistics in sub-intervals $T_b/100$. The interval T_b is divided into 100 sub-intervals, so given $N_e = 500$ the average number of charges in each sub-interval is 5. The Poissonian, super-Poissonian and sub-Poissonian PSDs are shown by the blue, green ad red curves respectively. The standard deviation for the super-Poissonian case is $1.54\sqrt{10}$ instead of the chosen $2\sqrt{10}$, while the sub-Poissonian case has a standard deviation very close to $0.5\sqrt{10}$. The sub and super-Poissonian behavior are for frequencies $fT_b < 100/2 = 50$ (marked with vertical line) and the super and sub Poissonian PSD values are established by the relative variances, i.e. 1.54^2 and 0.5^2. The areas under the spectra are 11599, 11392, 11315 for the super-Poissonian, Poissonian, and sub-Poissonian, respectively.

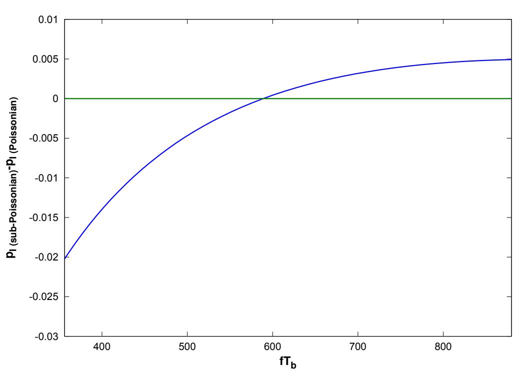

Figure 4: The difference between the PSD for a sub-Poissonian distribution (at low frequencies) and a Poissonian distribution. Because of the sum rule, sub-Poissonian spectrum becomes higher than the Poissonian spectrum at high frequencies.

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