TWO-COLOR FREE-ELECTRON LASER VIA TWO ORTHOGONAL UNDULATORS

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Abstract

An amplifier Free electron Laser (FEL) including two orthogonal polarized undulators with different periods and field intensities is able to emit two color radiations with different frequency and polarization while the total length of device does not change respect to usual single color FELs. The wavelengths of two different colors can be changed by choosing different periods, while variation in the magnetic strengths can be used to modify the gain lengths and saturation powers.

INTRODUCTION

Recently generation of free-electron laser radiation with two or more simultaneous colors opens new promising chapter in applications [1, 2] and in the study of the underlying physics. The packets contain two different spectral lines with adjustable time separation between them. Applications exist over a broad range of wavelengths involving pump-probe experiments, multiple wavelength anomalous scattering, or any process where there is a large change in cross section over a narrow wavelength range [3].

In order to produce this type of radiation several schemes have been proposed, and many promising theoretical proposals have been so far investigated. Some of the initially proposed designs were based on the use of staggered undulator magnets having different values of deflecting parameters to achieve lasing at two distinct wavelengths [4–7]. In this way, the length of the FEL undulator is essentially doubled and a complex scheme is required to reach saturation and power levels comparable with the single color configuration. A different technique involving the use of either a chirped or a two-color seed laser, is recently demonstrated at the FERMI soft X-ray FEL. It initiates the FEL instability at two different wavelengths within the modulator gain bandwidth [8, 9]. Another option is relying on injection of multi-energy electron beam in the FEL undulator [10] resonating at two different wavelength, allowing the control of frequency and time separation ranges of the FEL pulses, while maintaining similar saturated power levels and minimal undulator length [11, 12]. In this configuration, the SASE lasing occurs from separated and nearly independent electron distributions [13].

Recently a new proposal with a further different scheme has been presented in reference [14, 15]. In this case the FEL emission is obtained from two orthogonally polarized undulators with different polarized and field intensities. The two radiations have not only different frequencies, but also different polarizations, while the total length of the device does not change with respect to usual single color FELs. Producing two waves with orthogonal polarizations with comparable intensities is very important because it opens various possibilities to get insights into and to control the internal organization and orientation in space of molecules, taking advantage of the selective excitation of the molecular fluorescence by differently polarized beams.

This paper presents a brief overview over the main theory of the production and properties of two-color radiation generated by two orthogonal undulators.

MODEL EQUATION IN AN AVERAGED AND NON AVERAGED SVEA TREATMENT

The FEL undulator is assumed to be composed by two linear undulators orthogonally polarized with periods given respectively by λ_{01} and λ_{02}. and deflection parameters $K_{1,2} = \left|eB_{1,2}\lambda_{01,02}/mc^2\right|$. The undulator magnetic field, in the paraxial approximation, is described by the following expression

$$\mathbf{B}_w = -B_{w2}\sin(k_{02}z)\hat{e}_x + B_{w1}\sin(k_{01}z)\hat{e}_y, \qquad (1)$$

where $k_{01,02} = 2\pi/\lambda_{01,02}$.

Following the Colson's analysis [16], the zero order dimensionless velocity components can be written as

$$\beta_{x,y} = -\frac{K_{1,2}}{\gamma_0}cos(k_{01,02}z) \approx -\frac{K_{1,2}}{\gamma_0}cos(\omega_{01,02}t), \qquad (2)$$

$$\beta_z = -\frac{1}{4}\left[\left(\frac{K_1}{\gamma}\right)^2\cos(2k_{01}z) + \left(\frac{K_2}{\gamma}\right)^2\cos(2k_{02}z)\right] + \beta_0 \qquad (3)$$

with $\beta_{x(y),j} = v_{x(y),j}/c$ and $\beta_0 = 1/\sqrt{1-\gamma_0^2}$. From Eq (3) the following resonance conditions can be found.

$$\lambda_{1,2} = \frac{\lambda_{01,02}}{2\gamma_0^2}(1 + K_1^2/2 + K_2^2/2). \qquad (4)$$

The trajectories of the electrons inside the undulator takes the form:

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$$\mathbf{r} = \beta_0 ct\hat{e}_z - \frac{K_1}{k_{01}\gamma_0} sin(\omega_{01}t)\hat{e}_x - \frac{K_2}{k_{02}\gamma_0} sin(\omega_{02}t)\hat{e}_y \quad (5)$$
$$-\frac{\lambda_{01}}{16\pi}\frac{K_1^2}{\gamma_0^2} sin(2\omega_{01}t)\hat{e}_z - \frac{\lambda_{02}}{16\pi}\frac{K_2^2}{\gamma_0^2} sin(2\omega_{02}t)\hat{e}_z.$$

The longitudinal motion at zero order is described by:

$$z = \beta_0 ct - \frac{\xi_1}{k_1} sin(2\omega_{01}t) - \frac{\xi_2}{k_2} sin(2\omega_{02}t), \quad (6)$$

with $\xi_{1,2} = \frac{K_{1,2}^2}{4(1+K_1^2/2+K_2^2/2)}$.

The the proportional one-dimensional vector potential is assumed as

$$\mathbf{A} = -i\left[A_1 e^{i(k_1 z - \omega_1 t)}\hat{e}_x + A_2 e^{i(k_2 z - \omega_2 t)}\hat{e}_y\right], \quad (7)$$

$A_{1,2}$ are slow complex amplitudes, $k_{1,2} = 2\pi/\lambda_{1,2}$. In the following, we will assume $n\lambda_{01} = m\lambda_{02}$; in this way, we will treat both the case of an harmonic relation between λ_1 and λ_2, and the case where m/n is a generic rational number, describing all other situations.

In order to write the FEL equation in universal scaling notation [17], we define the normalized fields as $a_{1,2} = \frac{\omega_1}{\omega_p\sqrt{\rho_1\gamma_0}}\frac{eA_{1,2}}{mc^2}$ where

$$\rho_{1,2} = \frac{1}{\gamma_0}\left[\frac{\omega_p K_{1,2}\mathcal{F}_{1,2}}{8\omega_{01,02}}\right]^{2/3} \quad (8)$$

is the FEL parameter and ω_p is the plasma frequency. In terms of the scaled quantity $\Gamma_i = \frac{\gamma_i-\gamma_0}{\rho_1\gamma_0}$, and, the phases $\theta_{1,2} = \omega_{01,02}t + k_{1,2}\beta_z ct - \omega_{1,2}t$ the equations are therefore:

$$\frac{d\Gamma_i}{d\tau} = e^{i\theta_{1i}}a_1 + \frac{\mathcal{F}_2}{\mathcal{F}_1}\frac{K_2}{K_1}\frac{k_{w2}}{k_{w1}}e^{i\theta_{2i}}a_2 + cc. \quad (9)$$

$$\frac{\partial a_1}{\partial\zeta} + \frac{1}{c}\frac{\partial a_1}{\partial\tau} = \sum e^{-i\theta_{1i}}. \quad (10)$$

$$\frac{\partial a_2}{\partial\zeta} + \frac{1}{c}\frac{\partial a_2}{\partial\tau} = \frac{K_2 k_{w1}}{K_1 k_{w2}}\frac{\mathcal{F}_2}{\mathcal{F}_1}\sum e^{-i\theta_{2i}}, \quad (11)$$

where $\mathcal{F}_{1,2} = J_0(\frac{k_{1,2}\xi_{2,1}}{k_{2,1}})[J_0(\xi_{1,2}) - J_1(\xi_{1,2})]$ are the Bessel factors modified for the case of two undulators. The phases in field equations are

$$\frac{d\theta_{1i}}{d\tau} = \Gamma_i, \quad \frac{d\theta_{2i}}{d\tau} = \frac{k_{02}}{k_{01}}\Gamma_i. \quad (12)$$

Therefore the gain length of the two polarization, when $m \neq n$ are

$$L_{g1} = \frac{\lambda_{01}}{4\pi\sqrt{3}\rho_1} = (\frac{\lambda_{01}}{\lambda_{02}})^{1/3}(\frac{\mathcal{F}_2}{\mathcal{F}_1})^{2/3}L_{g2}. \quad (13)$$

In a non averaged orbit approximation, Lorentz force equations are employed directly. Then the electron momentum equations for j^{th} electron are

$$\frac{dp_{x,y}}{dt} = e\beta_z B_{1,2} sin k_{01,02}z - ek_{1,2}(1-\beta_z)[A_{1,2}e^{i\alpha_{1,2}} + cc]$$

$$\frac{dp_z}{dt} = -e\beta_y B_2 sin k_{02}z - ek_2\beta_y[A_2 e^{i\alpha_2} + cc] \quad (14)$$
$$-e\beta_x B_1 sin k_{01}z - ek_1\beta_x[A_1 e^{i\alpha_1} + cc]$$

where $\alpha_{1,2} = k_{1,2}z - \omega_{1,2}t$. By writing the transverse current in terms of the particle density $\bar{n}$ as $J_{x,y} = -\sum ec\beta_{x,y}\bar{n}\delta(z - z_j)$ in Maxwell's equation and by using the Slowly Varying Envelope Approximation (SVEA), we obtain the two following independent differential equations

$$\frac{\partial}{\partial z}A_{1,2} + \frac{1}{c}\frac{\partial}{\partial t}A_{1,2} = \frac{2\pi e\bar{n}}{k_{1,2}}\sum\beta_{x(y),j}\delta(z - z_j)e^{-i\alpha_{1,2}} \quad (15)$$

TWO PULSES SASE FEL EMISSION

In the approximation of the time independent scheme, the set of non averaged (14-15) and averaged (9-12) equations have been integrated numerically with independent codes. Both codes employ the forth-order Runge-kutta method to demonstrate the evaluations of FEL system. In non averaged code the Runge-Kutta step size must be small enough to demonstrate the particles motion in the wiggler. At the first step the particles are assumed to be unbunched and monoenergytic.

The parameters chosen for the simulations, similar to the SPARC's [18, 19], are: $\lambda_{01} = 2.8\,cm$, $K_1 = 2.1$, $\gamma_0 = 300$, and the electron current I has been fixed at 100 A, for a value of $\rho_1 = 5.47 \times 10^{-3}$. In simulation thermal and diffraction effect are ignored.

The comparison between the solutions of averaged (blue curves) and non averaged (red curves) equations is demonstrated in Fig. 1 for (a) $\lambda_{02} = 1.5\lambda_{01}$, (b) $\lambda_{02} = 2\lambda_{01}$ and (c) $\lambda_{01} = 10\lambda_{02}$, with the two magnetic strengths fixed at the same value $K_1 = K_2$.

The agreement is indeed significant along all the growth up to the onset of saturation. Discrepancies arise, instead, once that the saturation is reached, particularly when the two waves have similar intensity (case (a)), probably due to the differences in the sets of equations and to the different method of integration. Black curves, labeled with (T), indicate the logistic map proposed in [20], and given by

$$P_{1,2} = \frac{P_{01,2}A_{1,2}e^{(0.223t/Z_{1,2})}}{(1 + \frac{P_{01,2}}{P_{s1,2}}(A_{1,2} - 1))}. \quad (16)$$

$$A_{1,2} = (\frac{1}{3} + \frac{2}{9}\cosh(\frac{t}{L_{g1,2}}) + \frac{4}{9}\cos(\frac{\sqrt{3}t}{2L_{g1,2}})\cosh(\frac{t}{2L_{g1,2}}))$$

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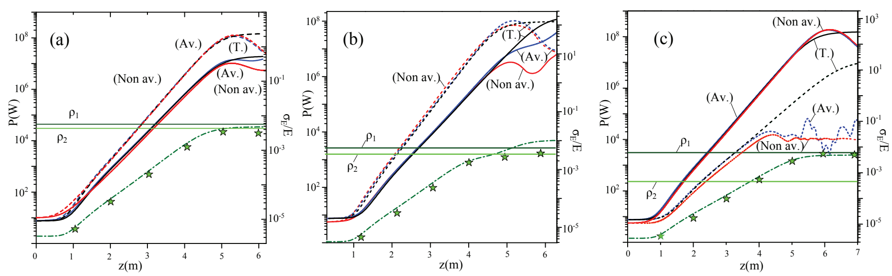

Figure 1: Power $P(W)$ in the x (solid curves) and y (dashed curves) polarizations vs $z(m)$. Comparison between non averaged (red curves) and averaged (blue) model for (a) $n/m = 1.5$ and (b) $n/m = 2$ and (c) $m/n = 10$. The black line is the logistic map, Eq (16). In green σ_E/E, as given by Eq (18). Green starts: energy spread computed by the phase spaces

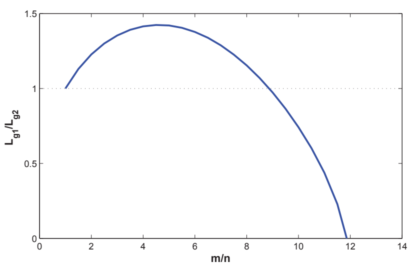

Figure 2: Ratio of gain length of both pulses vs m/n , $K_1 = K_2 = 2.1$

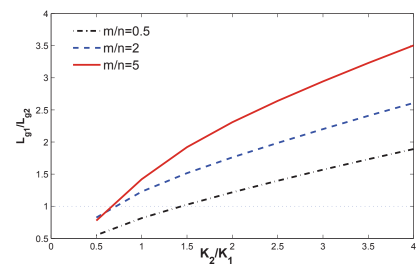

Figure 3: Ratio of gain length of both pulses vs K_2/K_1 , for different values of m/n.

with $Z_{1,2} = 1.066 L_g log(9\frac{P_{s1,2}}{P_{01,2}})$ and $P_{s1,2} = 1.42\rho_{1,2}P_b$, being the saturation power as a function of the beam power P_b. As can be seen, formula (16) fits very accurately lethargy, growth and gain lengths of both waves, while the saturation value is of the same order only for one of the two polarizations. This is due to the interaction between the two waves occurring when the power in the two polarizations is large enough, an effect which is not accounted in Eq (16). In fact, through electron interaction an induced increase in the energy spread σ_E occurs, as can be seen in Fig. 1, where the relative value σ_E/E , computed by the phase space, is presented together with the analytical formula [20]

$$\frac{\sigma_E}{E} = \sqrt{\left[\frac{\sigma_1}{E}\right]^2 + \left[\frac{\sigma_2}{E}\right]^2} \quad (17)$$

where:

$$\frac{\sigma_{1,2}}{E} = \frac{3}{2}\sqrt{\frac{\rho_{1,2}P_{01,2}}{P_b}}\sqrt{\frac{A}{1 + 1.24\frac{P_{01,2}}{P_{s1,2}}(A_{1,2} - 1)}}. \quad (18)$$

The growth of the energy spread continues up to the value of ρ_2, and then, saturates, producing also the anticipated saturation of the radiation with the longer gain length.

The graph of the coefficient L_{g1}/L_{g2} versus different value of the m/n while the two magnetic strengths are fixed at the same value $K_1 = K_2$ is shown in Fig. 2. It shows from $m = 1$ to about $m = 9$ the largest frequency wave has a shorter gain length, while for m larger than 9 the opposite occurs.

Figure 3 shows the ratio of gain length of both pulses versus $K_2/K_1$with fixed $m/n = 0.5, 2, 5$. The slop of the ratios increases as well as the value of the m/n increases.

As a result the wavelengths of two different colors can be changed by setting different periods, while variations in the magnetic strengths have the effect of modifying the gain lengths.

According to general FEL theory the saturation power and length are dependent on the FEL parameter $\rho_{1,2}$, however the numerical simulations show that the interaction between the two waves can change the level of the saturation power. When one wave reaches saturation, the electrons are strongly influenced by its electric field, so the growth of the other one is affected. In order to balance the level of the power in the two different polarization at the end of the undulator, the magnetic strength of one of the two waves can be varied. We fixed the vertical undulator properties ($K_1 = 2.1$ and $\lambda_{01} = 2.8$cm), while K_2 is varied for different n/m. The ratio of the power of the pulses at the first saturation point is reported vs K_2 in Fig. 4 for various values of n/m between 0.5 and 2. When $n/m = 2$, since $\rho_1 > \rho_2$, the first wave going in saturation was the x-polarization and the power amount of $P_{s1} = 168\,MW$ is reached in $z_s = 7.2\,m$. The power of the y-polarization in this same point can be varied by using different values of K_2. In the case $n/m = 1.5$, both x and y-polarizations saturate at $z_{1s} \approx 6\,m$, and the

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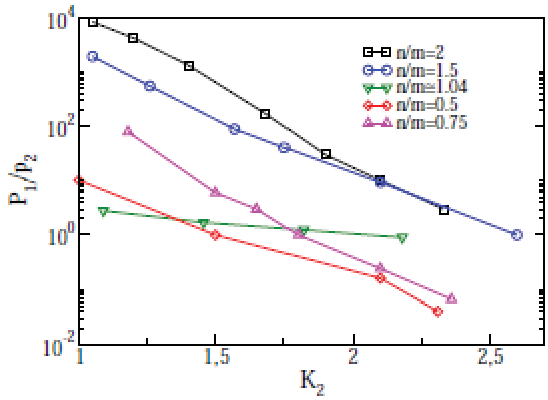

Figure 4: The power ratio of x-polarization to y-polarization for different value of n/m, while $K_1 = 2.1$ and $\lambda_{01} = 2.8$cm.

x-polarization reaches the value $P_{s1} = 168\,MW$. In the case n/m close but not equal to 1, the waves saturate in close positions, the saturation length does not depend strongly on K_2, while, instead, the power ratio depends on it. For $n/m = 1$, the gain length follows

$$L_{g1} = L_{g2} = \frac{\lambda_{01}}{4\pi\sqrt{3}(1 + (\frac{\mathcal{F}_2}{\mathcal{F}_1}\frac{K_2}{K_1})^2)^{1/3}} \qquad (19)$$

and the ratio between the powers has a different trend. For the cases $n/m < 1$, since the ratio between the FEL parameters is less than one ($\rho_1/\rho_2 < 1$), the first wave going to saturation is the y-polarization. If m/n is integer (as, for instance the case $n/m = 0.5$) the waves saturate in different points. In otherwise (like $n/m = 0.75$) both waves saturate in same position but in different power levels.

CONCLUSION

Emission of two pulses from two orthogonal undulators with different polarizations and periods have been discussed. Non averaged and averaged equations have been present. The agreement between these two models as regards lethargy, growth and gain length of the radiation, with discrepancies appear in saturation have been demonstrated. The advantage of this kind of device is production of two color radiation with an easy control of the frequencies and opposed polarizations, while the total length of the device does not change respect to usual single color FELs. The possibility of changing independently the strength of the two magnetic fields allows to control the final power and the saturation length.

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