TRANSITION RADIATION OF AN ELECTRON BUNCH AND IMPRINT OF LORENTZ-COVARIANCE AND TEMPORAL-CAUSALITY

Gian Luca Orlandi*,
Paul Scherrer Institut, 5232 Villigen PSI, Switzerland

Abstract

The study of Transition Radiation (TR) of a bunch of N electrons offers a precious insight into the role that Lorentz-covariance and temporal-causality play in an electromagnetic radiative mechanism of a relativistic beam. The contributions of the N single electrons to the radiation field are indeed characterized by emission phases from the metallic surface which are in a causality relation with the temporal sequence of the N particle collisions onto the radiating screen. The Lorentz-covariance characterizing the virtual quanta field of the relativistic charge is also expected to imprint the radiation field and the related energy spectrum. The main aspects of a Lorentz-covariance and temporal-causality consistent formulation of the TR energy spectrum of an electron bunch will be described.

INTRODUCTION

The electromagnetic field of a relativistic charge in a rectilinear and uniform motion can induce the boundary interface between two media with different dielectric properties to emit radiation, the so called Transition Radiation (TR) [1, 2]. Forward and backward radiation is emitted with an angular distribution scaling down with the Lorentz factor γ of the relativistic charge ($\gamma = E/mc^2$) and a frequency bandwidth determined by the plasma frequency and the finite dimension of the radiating surface. TR finds various application in beam diagnostics of a particle accelerator: to monitor the beam transverse profile by imaging the visible light spot (OTR, Optical Transition Radiation) emitted by the charged beam while crossing a metallic foil [3] or to measure the length of the charged beam via spectroscopic analysis of the coherent enhancement of the radiation intensity [4]. For observation condition of the radiation in the visible or in the far infrared, the interface vacuum-metal practically behaves like an ideal conductor. The modelling of the radiator surface as an ideal conductor allows a better comprehension of the dynamics of the charged distributions involved in TR emission and of the attribute as "polarization radiation" that is sometimes given to TR. An ideal conductor surface can be indeed schematized as a double layer of charge. The collision of a bunch of N relativistic electrons with the metallic surface can be modelled as the interaction of an incident relativistic charge with a double layer of charge (a beam collision at a normal angle of incidence onto the radiator surface is supposed in the present paper). At the same time, the radiation emission can be interpreted as the result of the dipolar oscillation of the double layer of charge which is induced by the relativistic electron bunch, see Fig. 1. The observation of a backward emitted component in the TR can be indeed explained as the result of a dipolar oscillation of a double layer of charge. The propagation of the radiation field from the metallic surface can be interpreted in terms of the Huygens-Fresnel principle and formally expressed in terms of the Helmholtz-Kirchhoff integral theorem. Under far-field approximation, the Helmholtz-Kirchhoff integral theorem allows to express the single harmonic component of the radiation field as the Fourier transform - calculated with respect to the coordinates of the radiator surface - of the transverse component of the electric field of the relativistic charge, the so called *virtual quanta* field. The density-like Lorentz-covariance characterizing the electric field of the incident electron bunch is expected to characterize both the TR field and the corresponding energy spectrum. Signature of the Lorentz-covariance is the dependence of the radiation energy spectrum on the Lorentz-invariant distribution of the transverse coordinates of the N electrons. Besides Lorentz-covariance, the TR field and the related energy spectrum of the N electron bunch are also expected to bear the imprint of the temporal-causality constraint: the emission phases of the N single electron amplitudes composing the radiation field must be causality related to the temporal sequence of the N electron collision onto the metallic screen, which only depends on the distribution of the longitudinal coordinates of the N electrons. Lorentz-covariance and temporal-causality are physical constraints an electromagnetic radiative mechanism by a relativistic charged beam must fulfill. Both these two constraints are expected to imprint the TR energy spectrum of a bunch of N electrons. The failure in implementing the one of the two physical constraints in the formula of the TR energy spectrum of a bunch of N electrons implies necessarily also the defect of the other and vice-versa. In the present paper, a Lorentz-covariance and temporal-causality consistent formulation of the TR energy spectrum of a relativistic bunch of N electrons is presented and the agreement of the presented model with well-known results of the TR theory of a single electron is demonstrated [5–7]. The breaking point between a Lorentz-covariance and temporal-causality consistent or inconsistent theoretical model of TR of a bunch of N electrons is discussed.

TR ENERGY SPECTRUM

Causality and Covariance

The case of a bunch of N relativistic electrons at normal incidence onto a metallic screen is considered in the present paper. The N electrons are supposed to move with a common rectilinear and uniform velocity $\vec{w} = (0, 0, w)$ along the z-axis of the laboratory reference frame. The radiator

* gianluca.orlandi@psi.ch

ISBN 978-3-95450-133-5

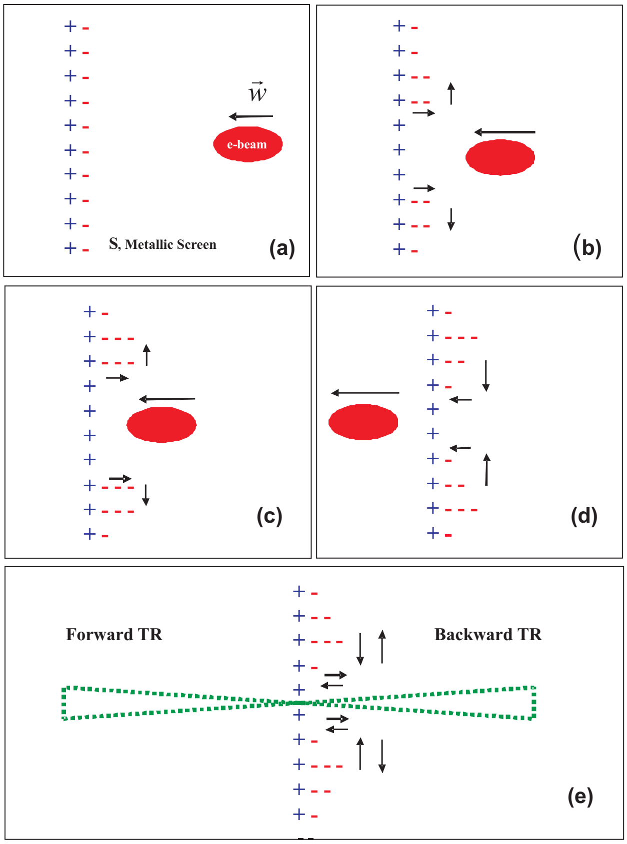

Figure 1: In the picture sequence $[(a) \rightarrow (e)]$, a simplified representation of the TR emission is sketched. The incident relativistic charge induces a dipolar oscillation on the double layer of charge, i.e., the metallic surface. With the charge approaching the metallic foil, the conduction electrons, initially at rest on the metallic surface, undergo a tangential displacement due to the electric component of the Lorentz force (the transverse component of the electric field is γ times stronger than the longitudinal one). Subsequent to the initial transverse motion, the conduction electrons are also displaced along the longitudinal direction because of the magnetic component of the Lorentz force. As a result of the dipolar oscillation, the double layer of charge generates the TR emission. Due to the simplicity of this pictorial representation, charge oscillation and radiation emission are artificially distinct in two different phases. In reality, such two phases are intrinsically and temporally indistinguishable.

surface is supposed to behave as an ideal conductor in the radiation frequency range of interest. The Fourier transform of the electric field travelling with the electron bunch reads

$$\vec{E}(\vec{k},\omega) = -i(8\pi^2 e)\frac{[\vec{k}-(\omega\vec{w}/c^2)]}{[k^2-(\omega/c)^2]} \times \times\left(\sum_{j=1}^{N} e^{-i\vec{k}\cdot\vec{r}_{0j}}\right)\delta(\omega-\vec{w}\cdot\vec{k}), \quad (1)$$

where $[\vec{\rho}_{0j} = (x_{0j}, y_{0j}), z_{0j}]$ $(j = 1,..,N)$ are the spatial coordinate vectors of the N electrons at the reference time $t = 0$. The (x, y) components of the Eq.(1) are normally referred as *virtual quanta* field. The harmonic component of the radiation field of the N electrons can be obtained by applying the Helmholtz-Kirchhoff integral theorem [1] to the electric field of the incident charge on the boundary metallic surface. Under far-field approximation, this reads

$$E^{tr}_{x,y}(\vec{\kappa},\omega) = \frac{k}{2\pi D}\int_S d\vec{\rho}\, E^{vq}_{x,y}(\vec{\rho},\omega)e^{-i\vec{\kappa}\cdot\vec{\rho}}, \quad (2)$$

where $\vec{\rho} = (x, y)$ are the spatial coordinates of the radiator surface S having an arbitrary size and shape, $k = \omega/c = 2\pi/\lambda$ is the wave number, $\vec{\kappa} = (k_x, k_y) = k\sin\theta(\cos\phi, \sin\phi)$ is the transverse component of the wave-vector and D is the distance from the radiator surface to the observation point. From Eqs.(1,2), the harmonic component of the TR field of the N electrons reads [5–7]

$$E^{tr}_{x,y}(\vec{\kappa},\omega) = \sum_{j=1}^{N} H_{x,y}(\vec{\kappa},\omega,\vec{\rho}_{0j})\, e^{-i(\omega/w)z_{0j}} \quad (3)$$

where, with $\alpha = \frac{\omega}{w\gamma}$,

$$H_{x,y}(\vec{\kappa},\omega,\vec{\rho}_{0j}) = \frac{iek}{2\pi^2 Dw} \times \times \int_S d\vec{\rho}\int d\vec{\tau}\frac{\tau_{x,y}\, e^{-i\vec{\tau}\cdot\vec{\rho}_{0j}}}{\tau^2+\alpha^2}e^{i(\vec{\tau}-\vec{\kappa})\cdot\vec{\rho}}. \quad (4)$$

TR energy spectrum can be finally obtained

$$\frac{d^2I}{d\Omega d\omega} = \frac{cD^2}{4\pi^2}\sum_{\mu=x,y}\left(\sum_{j=1}^{N}\left|H_{\mu,j}\right|^2 + \sum_{j,l(j\neq l)=1}^{N} e^{-i(\omega/w)(z_{0j}-z_{0l})}H_{\mu,j}H^{*}_{\mu,l}\right). \quad (5)$$

where $H_{\mu,j} = H_{x,y}(\vec{\kappa},\omega,\vec{\rho}_{0j})$ $(\mu = x, y)$, see Eq.(4). The evidence of the temporal-causality constraint in the expression of the TR field stays in the emission phases from the radiator of the N single electron radiation field amplitudes $H_{\mu,j}$ which are in a causality relation with the temporal sequence of the N electron collisions onto the metallic screen, only dependent on the distribution of the N electron longitudinal coordinates z_{0j} $(j = 1,..,N)$, see Eqs.(3,4). Temporal-causality consistent is the formula of the TR energy spectrum - see Eqs.(5) - where the reciprocal interference of N single electron radiation field amplitudes only depends on the reciprocal emission delays $(z_{0j} - z_{0l})$, $j, l = 1,..,N$. The covariant dependence of the TR energy spectrum - see Eqs.(3,4,5) - on the distribution of the N electron transverse coordinates (x_{0j}, y_{0j}) $(j = 1,..,N)$ appears evident after performing the integral calculus in the Eq.(4) [5]. In fact, for a round radiator surface S with a radius R larger than the beam transverse size $(R \gg \rho_{0j} = \sqrt{x^2_{0j}+y^2_{0j}})$, the TR

ISBN 978-3-95450-133-5

field - Eqs.(3,4) - reads

$$E^{tr}_{x,y}(\vec{\kappa},\omega) = \sum_{j=1}^{N} \frac{2iek}{Dw}\frac{\kappa}{\kappa^2+\alpha^2} e^{-i[(\omega/w)z_{0j}+\vec{\kappa}\cdot\vec{\rho}_{0j}]} \times \times \begin{pmatrix} \cos\phi \\ \sin\phi \end{pmatrix} \Big[\rho_{0j}\Phi(\kappa,\alpha,\rho_{0j}) - (R+\rho_{0j})\Phi(\kappa,\alpha,R+\rho_{0j})\Big], \quad (6)$$

where, with (J,K) Bessel function of 1*st* and 2*nd* kind,

$$\Phi(\kappa,\alpha,\rho_{0j}) = \alpha J_0(\kappa\rho_{0j})K_1(\alpha\rho_{0j}) + \frac{\alpha^2}{\kappa}J_1(\kappa\rho_{0j})K_0(\alpha\rho_{0j}). \quad (7)$$

From Eqs.(5,6,7), the formula of the TR energy spectrum of a N electron bunch at normal incidence onto a circular radiator with a finite radius R ($0 \le R < \infty$) follows

$$\frac{d^2I}{d\Omega d\omega} = \frac{d^2I_e}{d\Omega d\omega}\Big(\sum_{j=1}^{N}|A_j|^2 + \sum_{j,l(j\neq l)=1}^{N} A_j A_l^* e^{-i[(\omega/w)(z_{0j}-z_{0l})+\vec{\kappa}\cdot(\vec{\rho}_{0j}-\vec{\rho}_{0l})]}\Big) \quad (8)$$

where

$$\frac{d^2I_e}{d\Omega d\omega} = \frac{(e\beta)^2}{\pi^2 c}\frac{sin^2\theta}{(1-\beta^2 cos^2\theta)^2} \quad (9)$$

is the well-known Frank-Ginzburg formula and

$$A_j = \rho_{0j}\Phi(\kappa,\alpha,\rho_{0j}) - (R+\rho_{0j})\Phi(\kappa,\alpha,R+\rho_{0j}). \quad (10)$$

In Eqs.(8,9,10), the N electron transverse coordinates (x_{0j}, y_{0j}) ($j = 1,..,N$), on the one hand, contribute to determine the well-known three-dimensional form factor, on the other hand, leave a covariant mark on both the temporal incoherent and coherent components of the TR energy spectrum. The case of the TR emission from an infinite metallic surface ($S = \infty$) can be obtained from the results above under the limit $R \to \infty$. Under the limit $R \to \infty$, Eq.(10) - see also Eq.(12) - reads

$$A_j \to \rho_{0j}\Phi(\kappa,\alpha,\rho_{0j}). \quad (11)$$

Finally, with reference to Eqs.(8,9,11), the formula of the TR energy spectrum of N electrons hitting an infinite radiator ($S = \infty$) can be obtained. It is worth to stress that, in order to obtain the results of TR from an infinite radiator ($S = \infty$) - Eqs.(8,9,11) - the following formal procedure is applied: first, in the implicit expression of the TR field - see Eq.(3,4) - the integral calculus with respect to the radiator surface S is performed for a round screen with a finite radius R; finally, the limit $R \to \infty$ is applied to the resultant expression of the TR field. In the following subsection, the consequences of applying directly the limit $S \to \infty$ to the implicit expression of the radiation field - Eq.(3,4) - will be described. Numerical simulations of the angular distribution of the TR intensity - refer to the temporal incoherent part of the TR energy spectrum, see Eqs.(8,9,11) - are shown in Fig. 2 for a given value of the beam transverse size and different values of the observed wavelength and beam energy. A beam transverse size dependent diffractive cut-off clearly affects both the angular and spectral distributions of the TR intensity [5–7], see Fig. 2. Some relevant results already well-known in literature can be derived from the results reported above, see Eqs.(8,9,10). In the case of a single electron travelling along the z-axis, it can be indeed demonstrated [5] that under the limit $R \to \infty$ and $\rho_{01} \to 0$

$$\begin{cases} (R+\rho_{0j})\Phi(\kappa,\alpha,R+\rho_{0j}) \to 0 \\ \rho_{0j}\Phi(\kappa,\alpha,\rho_{0j}) \to 1 \end{cases} \quad (12)$$

the TR field - Eq.(6) - tends to

$$E^{tr,e}_{x,y}(\vec{\kappa},\omega) = \frac{2iek}{Dw}\frac{\kappa}{\kappa^2+\alpha^2}\begin{pmatrix} \cos\phi \\ \sin\phi \end{pmatrix} \quad (13)$$

from which the Frank-Ginzburg formula follows, see Eq.(9). Furthermore, in the case of a single electron with $\rho_{01} \to 0$ and a radiator with a finite radius R the well-known result of the TR field of a single electron hitting a round radiator can be obtained from Eq.(6,7,12)

$$E^{tr}_{x,y}(\vec{\kappa},\omega) = \frac{2iek}{Dw}\frac{\kappa}{\kappa^2+\alpha^2}\begin{pmatrix} \cos\phi \\ \sin\phi \end{pmatrix} \times \times\Big[1 - \alpha R J_0(\kappa R)K_1(\alpha R) - \frac{\alpha^2 R}{\kappa}J_1(\kappa R)K_0(\alpha R)\Big]. \quad (14)$$

See Eq.(14) and compare it with Eqs.(8,9) in [8].

Causality and Covariance Defect

In previous section, the formula of the TR energy of an N electron beam normally hitting an infinite metallic surface ($S = \infty$) - see Eqs.(8,9,11) - was derived according to the following procedure: first, the integral calculus in Eq.(3,4) with respect to a finite screen size ($S < \infty$) is performed; finally, the limit $S \to \infty$ is applied to the so obtained result. If this procedure is inverted, i.e., if the limit $S \to \infty$ is directly applied to the implicit expression of the TR field - see Eq.(3,4) - before the integral calculus with respect the radiator surface S is performed, then the TR field reads [7]

$$E^{tr}_{x,y}(\vec{\kappa},\omega) = \sum_{j=1}^{N} E^{tr,e}_{x,y}(\vec{\kappa},\omega)\, e^{-i[(\omega/w)z_{0j}+\vec{\kappa}\cdot\vec{\rho}_{0j}]}, \quad (15)$$

and the related formula of the TR energy spectrum reads:

$$\frac{d^2I}{d\Omega d\omega} = \frac{d^2I_e}{d\Omega d\omega}\Big(N + \sum_{j,l(j\neq l)=1}^{N} e^{-i[(\omega/w)(z_{0j}-z_{0l})+\vec{\kappa}\cdot(\vec{\rho}_{0j}-\vec{\rho}_{0l})]}\Big) \quad (16)$$

where $E^{tr,e}_{x,y}(\vec{\kappa},\omega)$ is given in Eq.(13). Looking at Eqs.(15,16), it appears evident the covariant role of the

ISBN 978-3-95450-133-5

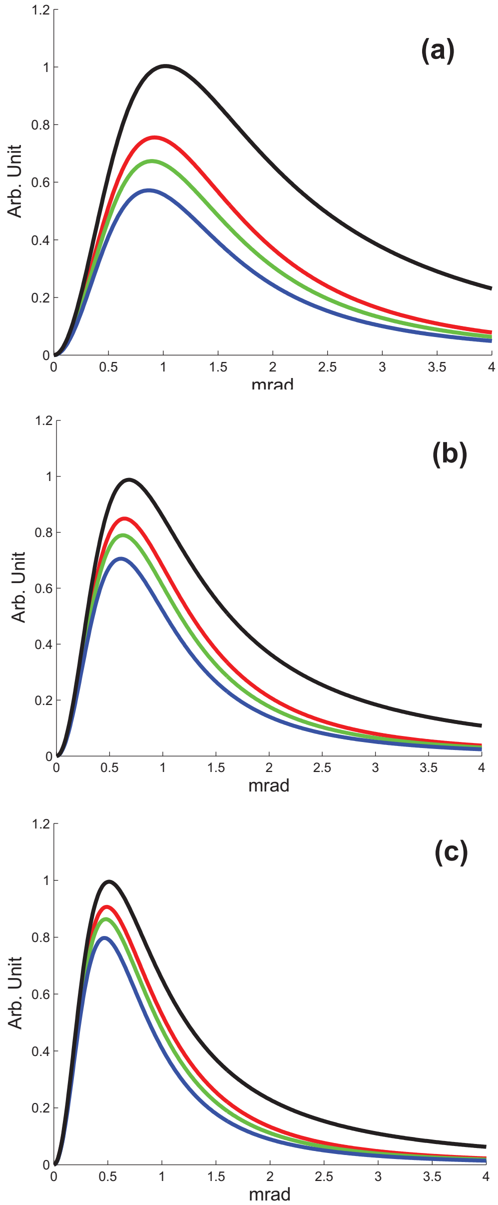

Figure 2: TR angular distribution for different beam energy: (a) 500, (b) 750 and (c) 1000 MeV; wavelength $\lambda = 680\ nm$ (Red curve), $\lambda = 530\ nm$ (Green curve), $\lambda = 400\ nm$ (Blue curve); Gaussian bunch of $N = 10^5$ electrons with $\sigma = 50\ \mu m$. Blue, Red and Green curves from Eqs.(8,9,11), (Black curve) from Eq.(9).

transverse coordinates of the N electrons is completely missed. In fact, in both the N single electron radiation field amplitudes and the temporal incoherent part of the TR energy spectrum any dependence on the Lorentz-invariant distribution of the N electron transverse coordinates (x_{0j}, y_{0j}) $(j = 1, .., N)$ disappeared. Moreover, looking at the formula of the TR field - Eq.(15) - it turns out that the causality role played by the N electron longitudinal coordinates z_{0j} $(j = 1, .., N)$ in determining the emission phases of the N single electron radiation amplitudes is completely mixed up (indistinguishable) with the role of the N electron transverse coordinates (x_{0j}, y_{0j}) $(j = 1, .., N)$. The transverse coordinates of the N electrons indeed do not determine the emission phases but only contribute to determine the observation phases as a function of the N electron distances from the z-azis of the reference frame. But, according to the formulation given in Eq.(15), the transverse coordinates of the N electrons seem to play the same causal role played by the N electron longitudinal coordinates.

CONCLUSIONS

Transition radiation (TR) emission of a bunch of N electrons is ruled by Lorentz-covariance and temporal-causality as other electromagnetic radiative mechanism by relativistic charged beams. In case of normal incidence, the Lorentz-covariance consistency for a model of TR means that the Lorentz-invariant dependence on the transverse coordinates of the N electrons is transmitted from the electric field of the relativistic bunch to the TR energy spectrum. At the same time, the clear and strict dependence of the emission phases of the N single electron radiation field amplitudes from the radiator surface on the distribution of the N electron longitudinal coordinates - i.e., on the temporal sequence of the N electron collision onto the metallic screen - is the signature of the temporal-causality in a model of TR. The formal procedure how to implement in the radiation field expression the integral calculus with respect to the radiator surface represents the breaking-point between a Lorentz-covariance and temporal-causality consistent or inconsistent model of TR.

REFERENCES

[1] M.L. Ter-Mikaelian, *High-Energy Electromagnetic Processes in Condensed Media*, Wiley, New York (1972).

[2] V.L. Ginzburg, V.N. Tsytovich, *Transition Radiation and Transition Scattering*, Adam Hilger, Bristol (1990).

[3] L. Wartski, S. Roland, J. Lasalle, M. Bolore, G. Filippi, J. Appl. Phys. **46** (1975) 8.

[4] U. Happek, A.J. Sievers, E.B. Blum, Phys. Rev. Lett. **67**, 21 (1991) 2962-2965.

[5] G.L. Orlandi, Optics Communications 311 (2013) 180-185.

[6] G.L. Orlandi, Paul Scherrer Institut Preprint PSI-PR-12-03, arXiv:1207.0644 [physics.acc-ph].

[7] G.L. Orlandi, Paul Scherrer Institut Preprint PSI-PR-13-10, arXiv:1306.0768 [physics.acc-ph].

[8] S. Casalbuoni, B. Schmidt,P. Schmüser, V. Arsov, S. Wesch, Phys. Rev. ST-AB **12**, 030705 (2009).

ISBN 978-3-95450-133-5