CYCLOTRON-UNDULATOR COOLING OF A FREE-ELECTRON-LASER BEAM*

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Abstract

We propose methods of fast cooling of an electron beam, which are based on wiggling of particles in an undulator in the presence of an axial magnetic field. We use a strong dependence of the axial electron velocity on the oscillatory velocity, when the electron cyclotron frequency is close to the frequency of electron wiggling in the undulator field. The abnormal character of this dependence (when the oscillatory velocity increases with the increase of the input axial velocity) can be a basis of various methods for fast cooling of moderately-relativistic (several MeV) electron beams. Such cooling may open a way for creating a compact X-ray free-electron laser based on the stimulated scattering of a powerful laser pulse on a moderately-relativistic (several MeV) electron beam.

INTRODUCTION

Fast development of the technique of photo-cathode electron photoinjectors has resulted in creation of compact and accessible sources of moderately-relativistic (several MeV) dense (~1 nC in a ps pulse) bunches [1-3]. Methods for decrease of the energy spread (cooling) are actual from the point of view of various applications of such beams, including free-electron lasers (FELs). However, cooling methods are developed now basically for electron beams of significantly higher energies [4,5]. As for a moderately-relativistic high-dense short e-bunches, the strong Coulomb interaction of the particles results in a requirement for a short (~ 1 m and even less) length of a cooling system. In this situation, the cooling system should possess resonant properties, namely, a strong dependence of parameters of the particles inside the cooling system on their input energies.

We propose to provide cooling by the use of electron wiggling in a circular polarized "cooling" undulator in the presence of an axial magnetostatic field zB_0 (Fig. 1). If the bounce-frequency of electron oscillations in the undulator, $\Omega_u = V_{||} h_u$ is comparable with the electron cyclotron frequency, $\Omega_c = eB_0 / \gamma mc$ (here $V_{||}$ is the electron axial velocity, h_u is the undulator wavenumber, and γ is the relativistic electron mass factor). In this situation, the velocity of undulator oscillations V_u depends strongly on the initial axial velocity.

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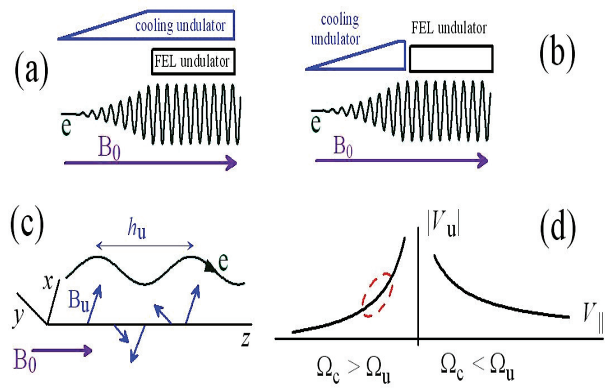

Figure 1: (a) and (b): Schematics of non-radiative cyclotron-undulator cooling systems with the operating FEL undulator placed inside and outside of the cooling system. (c): Electron motion in the circular polarized undulator with the uniform axial field. (d): Characteristic dependence of the undulator velocity on the axial electron velocity (the optimal range is shown schematically).

NON-RADIATIVE "AXIAL" COOLING

Non-radiative "axial" cooling is based on the fact that the axial velocity spread is the only factor important for the FEL operation. This spread can be decreased due to its "transformation" into the spread in the velocity of electron rotation in the cooling system. Electrons move along axial magnetic field and enter the cooling undulator with the adiabatically growing field in the input section, where each electron gets its own rotatory velocity (Fig. 1a). If at the input of the system every particle possesses only the axial velocity $V_0 = \overline{V} + \delta V$, then the axial velocity in the regular region of the undulator is determined by the energy conservation law: $V_{||}^2 \approx V_0^2 - V_u^2$. Thus,

$$V_{||}^2 \approx \overline{V}^2 + 2\overline{V}\delta V - V_u^2(\overline{V}) - \alpha\delta V \ , \quad \alpha = \partial V_u^2 / \partial V_{||} .$$

If $\alpha = 2\overline{V}$, then the spread in $V_{||}$ disappears. This condition is independent of the initial spread, δV. Evidently, we should use the range of parameters, where $\partial |V_u| / \partial V_{||} > 0$ (Fig. 1d), so that the initial axial velocity excess, δV, is compensated by the greater rotatory velocity, V_u.

If such a cooling system is used in a FEL, then the operating FEL undulator designed to produce optical radiation can be placed inside the regular section of the cooling undulator (Fig. 1a). Another way is to "switch off" the field of the cooling "undulator" sharply (Fig. 1

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b). Then, forced undulator oscillations of the particles are just transformed into free cyclotron oscillations possessing the same rotatory velocities, $V_\perp = V_u$ (and, therefore, the same axial velocities).

In the case of the adiabatically smooth entrance of electrons into the cooling undulator, the normalized velocity of the forced oscillations of the particles, $\beta_u = V_u / c$, is determined as follows:

$$\beta_u = K / \gamma\Delta . \quad (1)$$

Here, $K = eB_u / h_u mc^2$ is the undulator factor (the normalized transverse electron momentum, $K = \gamma V_u / c$ in the case, when the axial magnetic field is absent), and $\Delta = 1 - \Omega_c / \Omega_u$ is the mismatch between the electron cyclotron frequency and the bounce-frequency of electron oscillations in the undulator.

In the simplest situations, when electrons enter into an ideal cooling undulator along ideally rectilinear trajectories (so that they have no input transverse velocity), then the condition of axial velocity cooling has the following form [6]:

$$K^2 \approx -\Delta^3 . \quad (2)$$

In order to describe this spread, it is convenient to introduce the axial gamma-factor, $\gamma_{||} = 1/\sqrt{1-\beta_{||}^2}$, and use the spread $D(\gamma_{||}) \approx \gamma^2 D(\beta_{||})$, where D denotes the relative dispersion. The uncompensated axial spread is determined by three factors. First, there is the initial spread in transverse velocity, $0 < \beta_{\perp 0} < \overline{\beta}_{\perp 0}$. This leads to the following uncompensated axial spread:

$$D_\perp(\gamma_{||}) \approx \gamma_0^2 \beta_u \overline{\beta}_{\perp 0} / 2 . \quad (3)$$

Second, there is the spread in the transverse electron position, $0 \le r \le r_e$. As the undulator field is not uniform, $B_u(r) \approx 1 + (h_u r / 2)^2$, this spread induces the corresponding spread in the undulator velocity $0 \le \delta\beta_u \le 2\beta_u (h_u r / 2)^2$. This leads to estimation similar to the previous one:

$$D_r(\gamma_{||}) \approx \gamma_0^2 \beta_u^2 (r_e / \lambda_u)^2 / 2 . \quad (4)$$

The third source of the uncompensated spread is the non-ideal transformation of the axial spread [6]:

$$D_{||}(\gamma_{||}) \approx \left[D_0(\gamma_{||})\right]^2 (1 + \Delta^{-1}) . \quad (5)$$

Let us notice that an increase in the undulator parameter leads to the reduction in the Doppler frequency up-conversion factor $\sim \gamma_{||}^2$ due to the increase in the transverse electron velocity. However, according to Eqs. (1) and (2), β_u depends on K weakly in the optimal cooling regime, $\beta_u \approx K^{1/3} / \gamma$. At the same time, the undulator factor is related by Eq. (2) to the mismatch between the cyclotron undulator frequencies, $|\Delta| \approx K^{2/3}$. Evidently, K should be great enough to avoid the close-to-resonance situation, when it is difficult to provide the adiabatically smooth entrance into the undulator [7,8]. In addition, in the case of $|\Delta| << 1$ the system is very critical to the initial spread [see Eq. (5)].

Let us consider a 5 MeV electron bunch with the parameters typical for modern photo-injectors: energy spread $D_0(\gamma) \approx D_0(\gamma_{||}) \sim 1\%$, normalized emittance $\varepsilon \approx \pi$ mm mrad, and the bunch radius $r_e \sim 1$ mm. In the case of a cooling undulator with $\lambda_u = 5$ cm and $K = 0.2$, the condition (2) leads to $\Delta \approx 0.3$ and $\beta_u \approx 0.06$. In this case, the estimations (3)-(5) result in the similar uncompensated spreads in axial velocity $D_\perp(\gamma_{||}) \approx 3\times10^{-4}$, $D_r(\gamma_{||}) \approx 1\times10^{-4}$, $D_{||}(\gamma_{||}) \approx 4\times10^{-4}$. According to simulations of motion of a 5 MeV electron bunch in a cooling undulator with the period $\lambda_u = 5$ cm and with the adiabatically tapered entrance, the undulator length should amount tens cm. In this case, the non-relativistic cyclotron wavelength $\lambda_c = 2\pi c / (\gamma\Omega_c) \approx \lambda_u / \gamma = 5$ mm corresponds to the axial magnetic field $B_0 \approx 2$ T.

CYCLOTRON RADIATION COOLING

A disadvantage of the non-radiative "axial" cooling is that the operating undulator of the FEL should be placed inside the cooling system. Moreover, the uncompensated axial velocity spread is limited by the transverse velocity spread. Since the cooling system does not remove the transverse spread, it is impossible to improve axial cooling by the use of the additional second stage of the cooling system. An alternative method of cooling, namely, cyclotron radiation cooling, might be more attractive. In this case, the cooling system consists of two sections (Fig. 2), namely, the undulator section with the adiabatically tapered entrance, and the cyclotron radiation section (the region of the uniform magnetic field). The non-adiabatic exit of electrons from the undulator section is accompanied with transformation of the forced

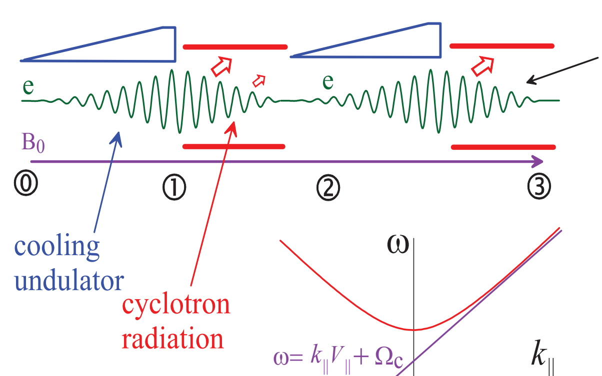

Figure 2: Schematic of the two-stage cooling system with cyclotron radiation sections, and electron-wave dispersion characteristics in the radiation sections.

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oscillations into the free cyclotron oscillations with the same oscillatory velocity. Then, in the radiation section, electrons lose their transverse momentums due to the cyclotron radiation. Optimally, at the output of the cooling system, electrons possess only the axial velocity.

Let us consider radiation of a particle, which performs free cyclotron rotations in the radiation section. We suppose that this section represents a waveguide, and the cyclotron resonance condition, $\omega = \Omega_c + k_{||}V_{||}$, is fulfilled for the lowest transverse mode in the so-called grazing regime, when the wave group velocity, $V_{gr} = c^2 k_{||} / \omega$, coincides approximately with the axial electron velocity (Fig. 2). The elementary act of radiation of this wave is emission of a photon with the energy $\hbar\omega$ and the axial momentum $\hbar k_{||}$. Thus, a radiation loss in the electron energy is related to the change in axial momentum as follows [9]: $dp_{||} / d\gamma = \beta_{gr}$, where $\beta_{gr} = V_{gr} / c$. Therefore, the cyclotron radiation does not perturb the axial velocity, $d\beta_{||} / d\gamma = \beta_{gr} - \beta_{||} = 0$. Thus, the radiation cooling operates similar to the non-radiative axial cooling, namely, the spread in $\beta_{||}$ is minimized due to its transformation into the spread in velocity of electron rotation in the undulator section, β_u. Then, in the radiation section, the rotatory velocity disappears.

As the cyclotron radiation does not perturb the axial electron velocity, the condition of cooling (2) and estimations (3)-(5) stay true for this scheme. Thus, from the viewpoint of the uncompensated spread, the single-stage radiation scheme has no advantages as compared to the non-radiative scheme. However, the important advantage of the radiation scheme is that the rotatory velocity is absent at the output. Therefore, a further decrease in the spread can be provided by organizing the downstream second cooling section (Fig. 2). Let us consider a situation, when at the entrance of the first cooling section the electron bunch possesses the axial spread $D_0(\gamma_{||})$=1% and a relatively large transverse velocity spread $\bar{\beta}_{\perp 0} = 5\times 10^{-4}$ (points 0 in Fig. 2). According to estimations (3)-(5) and simulations, in this case at the output of the first section (point 1) the axial spread is $D(\gamma_{||}) \sim 10^{-3}$. Therefore, at the input of the second section (points 2), the electrons have the axial spread $D_0(\gamma_{||})$=0.1% and a small transverse velocity spread. According to simulations, when the electrons have passed the second section (point 3 in Fig. 2), the uncompensated spread in axial velocity can be decreased down to the level of $D(\gamma_{||}) \sim 10^{-5}$.

The length of the radiation section can be estimated on the basis of the theory of the cyclotron autoresonance maser [9]. If the radiated electromagnetic pulse and the electron bunch propagate together, then one can use the amplifier equations, when the wave amplitude grows with the bunch coordinate. If the phase velocity of the wave is close to the speed of light, $\beta_{ph} = 1/\beta_{gr} \to 1$, then the phase of electrons with respect to the synchronous wave, $\theta = \omega t - k_{||}z - \int \Omega_c dt - \theta_{c0}$, varies slowly due to the autoresonance effect, so that $\theta(z) \approx \theta_0$. At the input of the radiation section, initial phases of different electrons are determined by their initial cyclotron phases and time of the entrance into the section. $\theta_0 = \omega t_0 - \theta_{c0}$. Cyclotron oscillations have sin-phase character [10], as they have arisen due to the transformation of the forced undulator oscillations. Thus, the phase size of the electron bunch, $\delta\theta_0 = 2\pi l_e / \lambda$, is less than 2π, if the length of the electron bunch, l_e, is smaller than the wavelength of the synchronous wave. The latter can be estimated by formula $\lambda \approx \lambda_u / \gamma^2$ following from $\omega \approx \gamma^2\Omega_c \approx \gamma^2\Omega_u$. As an example, in the case of $\lambda_u = 5\text{cm}$ and $\gamma = 10$, the wavelength is $\lambda = 0.5\text{mm}$, whereas for a 1 ps electron bunch $l_e = 0.3\text{mm}$. In this case, cyclotron radiation has the spontaneous character, as the electron bunch has a ready-to-radiate size. Estimations predict that the radiation section can be as short as tens of cms.

UBITRON RADIATION COOLING

The abnormal dispersion of the undulator velocity of electrons, $\partial|V_u| / \partial V_{||} > 0$, can be used to provide also a cooling method based on the ubitron radiation of electrons inside the regular part of the undulator with guiding axial magnetic field (Fig. 3). In this situation, electrons with higher initial energies have bigger undulator velocities and, therefore, lose more energy due to the radiation.

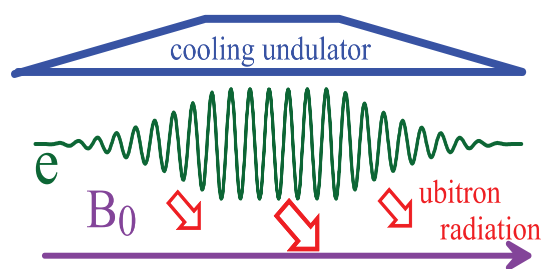

Figure 3: Ubitron radiation cooling system.

Let us assume, that at the input of the cooling undulator there is a spread in electron energy, so that the initial electrons mass-factors can be expressed as follows: $\gamma_0 = \bar{\gamma}_0 + \Delta\gamma_0$, where $\bar{\gamma}_0$ describes he averaged value of the initial electron energy and $\Delta\gamma_0$ describes the spread. Evidently, at the output of the cooling system, the electron energies become lower due to the ubitron radiation inside the undulator:

$$\gamma = \bar{\gamma}_0 + \Delta\gamma_0 - \Delta\gamma_{rad}. \qquad (6)$$

Here $\Delta\gamma_{rad}$ is the radiation loss. Similar to the cyclotron radiation cooling considered above, we assume

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that the radiated wave propagates together with the bunch ($V_{gr} = V_{||}$), and the bunch length is shorter that the wavelength of the radiated wave ($l_e < \lambda$). In this situation, the amplitude of the co-propagated wave increases in the process of the radiation (similar to the SASE FEL regime). The energy radiated during the trip of the e-bunch through the cooling undulator is proportional to the square of the undulator velocity: $\Delta\gamma_{rad} = \alpha V_u^2$, where $\alpha \propto IL^2$ is a coefficient proportional to the electron current and to the square of the undulator length. Since the undulator velocity depends of the initial electron energy, $\gamma_0 = \bar{\gamma}_0 + \Delta\gamma_0$, Eq. (6) can be represented as follows:

$$\gamma = \bar{\gamma}_0 + \Delta\gamma_0 - \alpha V_u^2(\bar{\gamma}_0) - \Delta\gamma_0 \times \alpha \left.\frac{\partial V_u^2}{\partial \gamma_0}\right|_{\gamma_0 = \bar{\gamma}_0}.$$

Thus, in the case of the abnormal dispersion of the undulator velocity, $\partial|V_u|/\partial\gamma_0 > 0$, the ubitron radiation leads to the decrease of the energy spread. Let us notice, that the condition of the cooling, $\alpha \times \partial V_u^2/\partial\gamma_0 = 1$, is independent on the value of the initial spread, $\Delta\gamma_0$. According to estimations, in the case of a 5 MeV e-bunch with radius $r_e \sim 1\,\mathrm{mm}$, the optimal cooling condition (2) is true for the undulator length amounting tens cm.

RF UNDULATOR COOLING

Finally, we discuss the possibilities to use cyclotron-undulator cooling schemes for electron bunches with higher energies. Since in the case of a magnetostatic cooling undulator the non-relativistic cyclotron wavelength is estimated as $\lambda_c \sim \lambda_u/\gamma$, an increase in the electron gamma-factor results in an increase of the axial magnetic field required. However, for high-energy electrons, instead of the magnetostatic cooling undulator, one can use an rf undulator (a powerful rf pulse), which co-propagates together with the bunch (Fig. 4). In this case, the condition of closeness of the cyclotron and undulator frequencies,

$$\Omega_c \sim \omega_u - k_{||u}V_{||} \approx \omega_u(1 - \beta_{gr,u}\beta_e),$$

leads to the following estimation: $\lambda_c \sim \lambda_u/\gamma(1 - \beta_{gr,u}\beta_e)$. If the group velocity of the undulator wave is close to the speed of light, $\beta_{gr,u} \to 1$, the optimal conditions for the cooling can be provided at a moderate magnetic field.

Let us consider axial cooling of electrons with $\gamma = 100$ in a rf pulse with $\lambda_u = 3\,\mathrm{cm}$ (Fig. 4). A super-radiant GW-power-level Cherenkov backward-wave oscillator [11] can be used as a source of such pulse. If this pulse is formed by the $TE_{1,1}$ transverse mode of a waveguide with the radius $R \approx \lambda_u$, then $\beta_{gr} \approx 0.95$, and the non-relativistic cyclotron wavelength $\lambda_c \approx \lambda_u/5 = 0.6$ cm corresponds to the axial magnetic field $B_0 \approx 1.8$ T. In this case, the rf pulse power $P_u = 0.5$ GW corresponds to the undulator parameter $K = 0.25$, whereas the rf pulse duration of 0.5 ns corresponds to the cooling system length of about 3 m only.

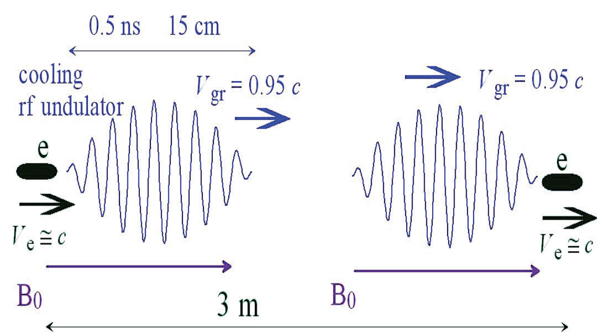

Figure 4: Schematic of the cooling system based on a short powerful rf pulse co-propagating with the e-bunch.

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