EVOLUTION OF A WARM BUNCHED BEAM IN A FREE DRIFT REGION

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Abstract

The state of the art of FELs development at present is "Table-Top X Ray Free Electron Lasers". Many such schemes involves a pre-bunched electron beam [1]. In this paper we will analyze the evolution and "survivability" of bunching introduced into the beam in the free drift region prior to the wiggler [2-6]. We examined analytically the first order degradation in beam bunching due to space charge effect. It will be shown that there is a limited interaction region, characterized by an exponential decay of the bunching factor, having a length inversely proportional to the square of the electron beam normalized temperature, followed by a stable bunch region. We will present examples of the effect for several schemes of X Ray and Tera Hertz FELs considered or being constructed presently.

INTRODUCTION

First, we present a solution for the evolution of a cold bunched continuous electron beam in a free drift region, based on a one-dimensional first order Vlasov equations including space charge effects [3-5]. Based on the first order cold beam solution, we expand the analysis for the evolution of a warm bunched electron beam in a free drift region, by assuming normal distributions for both transversal and longitudinal components of the momentum, independently [2,6]. Analytical solution is achieved by using a second order two-dimensional Taylor expansion of the exponent argument in the previously derived cold beam solution.

EVOLUTION OF A COLD BUNCHED ELECTRON BEAM IN A FREE DRIFT REGION

The analysis for cold electron beam is based on relativistic Vlasov equation for plasma:

$$\frac{\partial f}{\partial t} + \vec{v}\cdot\nabla f + \frac{d\vec{p}}{dt}\cdot\nabla_p f = 0 \quad (1)$$

where $f(\vec{r},\vec{p},t)$ is the distribution function of the plasma, $\vec{p}$ and $\vec{v}$ are the momentum and velocity vectors, respectively.

The time derivative of the momentum can be replaced by Lorentz force in the absence of an external magnetic field:

$$\frac{d\vec{p}}{dt} = -e\vec{E} \quad (2)$$

where e is the absolute value of the electron charge, and $\vec{E}$ is the electric field vector. In the current model there is no external electric field and no radiation field (since there is no external acceleration/deceleration, this is a good assumption). Therefore only the self-induced Coulomb field is considered (space charge). Thus the electric field can be derived out of the electric scalar potential:

$$\vec{E} = -\nabla\psi \quad (3)$$

where ψ is the electric scalar potential.

Substituting (2) and (3) into (1) results in:

$$\frac{\partial f}{\partial t} + \vec{v}\cdot\nabla f + e\nabla\psi\cdot\nabla_p f = 0 \quad (4)$$

where

$$\nabla_p \triangleq \hat{x}\frac{\partial}{\partial p_x} + \hat{y}\frac{\partial}{\partial p_y} + \hat{z}\frac{\partial}{\partial p_z} \quad (5)$$

In this formulation we restrict ourselves to a single (longitudinal) dimension. Hence, equation (4) becomes:

$$\frac{\partial f}{\partial t} + v_z\frac{\partial f}{\partial z} + e\frac{\partial\psi}{\partial z}\frac{\partial f}{\partial p_z} = 0 \quad (6)$$

p_z and v_z are the longitudinal components of the momentum and velocity, respectively.

Space charge effects are derived from the Poisson equation:

$$\nabla^2\psi = -\frac{\rho}{\varepsilon_0} \quad (7)$$

ρ is the charge density (per unit volume), and ε_0 is the permittivity of free space.

The charge density can be integrated out of the distribution function:

$$\rho(z,t) = -e\int_{-\infty}^{\infty} f(p_z,z,t)\,dp_z \quad (8)$$

Substituting (8) into (7) results in:

$$\frac{\partial^2\psi}{\partial z^2} = \frac{e}{\varepsilon_0}\int_{-\infty}^{\infty} f(p_z,z,t)\,dp_z \quad (9)$$

The charge density of the electron beam is modulated at the origin ($z = 0$), hence the distribution function at the origin can be expressed as:

$$f(z=0) = n_0(1+\alpha e^{j\omega t})\delta(p_z-\bar{p}_z) \quad (10)$$

where n_0 is the electrons density per unit volume, ∝ is the modulation factor, ω is the angular frequency of the modulation, δ is the Dirac delta function, and $\bar{p}_z$ is the average longitudinal momentum; since the electron beam is cold, the spread in longitudinal momentum is described by a Dirac delta function.

Using perturbation theory, we assume that both the distribution function and the electrical scalar potential can be expressed as an infinite series of terms, each proportional to a higher power of the modulation factor, ∝:

$$f = f_0 + \propto f_1 + \propto^2 f_2 + \cdots, \qquad \psi = \propto\psi_1 + \propto^2\psi_2 + \cdots \quad (11)$$

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$\boldsymbol{\psi_0 = 0}$, since in a free drift region there is no external electrical scalar potential applied to the electron beam.

In this section we are interested only in terms that are proportional to the modulation factor itself, i.e. linear to ∝.

Assuming that the perturbation is propagating along the electron beam in the positive longitudinal direction with an angular frequency equals to that of the modulation, **ω**, and a propagation coefficient **k**, then the form of the first order terms in (11) can be expressed as:

$$f_1 = f_{1\omega} e^{j(\omega t - kz)} \quad , \quad \psi_1 = \psi_{1\omega} e^{j(\omega t - kz)} \qquad (12)$$

where $f_{1\omega}$ and $\psi_{1\omega}$ are the frequency dependent amplitudes of the first order distribution function and the electrical scalar potential, respectively.

Substituting (12) in (6) and neglecting any terms that are not first order in ∝ results in:

$$f_{1\omega} = \frac{k \cdot e \cdot n_0}{\omega - v_z \cdot k} \psi_{1\omega} \frac{\partial}{\partial p_z} [\delta(p_z - \bar{p}_z)] \qquad (13)$$

Substituting (12), (13) in (9) and neglecting non-linear ∝-dependent terms results:

$$1 + \frac{e^2 \cdot n_0}{k \cdot \varepsilon_0} \int_{-\infty}^{\infty} \frac{1}{\omega - v_z \cdot k} \cdot \frac{\partial}{\partial p_z} [\delta(p_z - \bar{p}_z)] dp_z = 0 \quad (14)$$

In order to proceed from (14) we calculate the derivatives of the following expressions:

$$v_z = \frac{p_z}{m \cdot \gamma} \Longrightarrow \frac{dv_z}{dp_z} = \frac{1}{m \cdot \gamma^3} \quad ,$$

$$\gamma = \left(1 + \frac{{p_z}^2}{m^2 c^2}\right)^{-\frac{1}{2}} \Longrightarrow \frac{d\gamma}{dp_z} = \frac{p_z}{m^2 c^2 \gamma} \qquad (15)$$

The integral in (14) is carried by parts and the result is:

$$\int_{-\infty}^{\infty} \frac{1}{\omega - v_z \cdot k} \cdot \frac{\partial}{\partial p_z} [\delta(p_z - \bar{p}_z)] dp_z =$$

$$= -\frac{k}{m \tilde{\gamma}^3} \cdot \frac{1}{(\omega - \tilde{v}_z \cdot k)^2} \qquad (16)$$

where $\tilde{\gamma} \triangleq \gamma(p_z = \bar{p}_z)$ and $\tilde{v}_z \triangleq v_z(p_z = \bar{p}_z)$.

Substitute (16) in (14) leads to the dispersion relation:

$$k = \frac{\omega \pm \tilde{\omega}_p}{\tilde{v}_z} \qquad (17)$$

where $\omega_p \triangleq \left(\frac{e^2 n_0}{m \varepsilon_0 \gamma^3}\right)^{\frac{1}{2}}$ is the plasma frequency and $\tilde{\omega}_p \triangleq \omega_p(p_z = \bar{p}_z)$.

Therefore the first order distribution function for the cold electron beam is:

$$f = n_0 \left\{1 + \frac{\alpha}{2}\left[e^{j\left(\omega t - \frac{\omega + \omega_p}{v_z} z\right)} + e^{j\left(\omega t - \frac{\omega - \omega_p}{v_z} z\right)}\right]\right\} \cdot$$

$$\cdot \delta(p_z - \bar{p}_z) \qquad (18)$$

The physical interpretation of (18) is that the modulation in the charge density of the electron beam propagates in two waves, having phase velocities higher and lower than the longitudinal velocity of the electron beam, with no degradation in amplitude along the longitudinal direction. Superimposed these two waves results in a standing wave, having a wave number of $\frac{\omega_p}{v_z}$ and acting as an "envelope" (amplitude modulation) of a propagating wave (carrier), having an angular frequency ω, and a propagation coefficient $\frac{\omega}{v_z}$:

$$\left[e^{j\left(\omega t - \frac{\omega + \omega_p}{v_z} z\right)} + e^{j\left(\omega t - \frac{\omega - \omega_p}{v_z} z\right)}\right] \approx$$

$$\approx e^{j\left(\omega t - \frac{\omega}{v_z} z\right)} \cdot \cos\left(\frac{\omega_p}{v_z}\right) \qquad (19)$$

EVOLUTION OF A WARM PRE-BUNCHED BEAM

Calculation of Terms

Based on the first order cold beam solution for the distribution function, we expand the analysis for the evolution of a warm bunched electron beam in a free drift region, by replacing the one-dimensional Dirac delta function in (18) with normal distributions for both transversal and longitudinal components of the momentum, independently:

$$f_1 = \frac{1}{2} n_0 \cdot \left[e^{j\left(\omega t - \frac{\omega + \omega_p}{v_z} z\right)} + e^{j\left(\omega t - \frac{\omega - \omega_p}{v_z} z\right)}\right] \cdot$$

$$\cdot \left[\frac{1}{\sqrt{2\pi} \cdot \Delta p_\perp} e^{-\frac{(p_\perp)^2}{2 \cdot (\Delta p_\perp)^2}}\right] \cdot \left[\frac{1}{\sqrt{2\pi} \cdot \Delta p_z} e^{-\frac{(p_z - \bar{p}_z)^2}{2 \cdot (\Delta p_z)^2}}\right] \qquad (20)$$

where $p_\perp$ is the transversal component of the momentum, $\Delta p_\perp$ and Δp_z are the standard deviations of the transversal and longitudinal normal distributions, respectively; the average transversal momentum component is zero: $\bar{p}_\perp = 0$.

In expression (20) only terms that are proportional to the modulation factor itself, i.e. linear to ∝, are considered.

The influence on the charge density along the electron beam, caused by the perturbation, is given by the two-dimensional integration over the transversal and longitudinal components of the momentum:

$$\rho_1 = -e \int_{-\infty}^{\infty} dp_\perp \int_{-\infty}^{\infty} f_1 dp_z \qquad (21)$$

ρ_1 is the first order approximation of the charge density: $\rho = \rho_0 + \propto \rho_1 + \propto^2 \rho_2 + \cdots$

In order perform the integration in (21), we use a second order two dimensional Taylor expansion for the core of the cold beam solution, centered at the average values of the transversal and longitudinal components of the momentum; we define:

$$g^{\pm}(p_\perp, p_z) \triangleq \frac{\omega \pm \omega_p}{v_z} \qquad (22)$$

The second order approximation of the function $g^{\pm}$ is given by:

$$g^{\pm}(p_\perp, p_z) \approx g^{\pm}(0, \bar{p}_z) +$$

$$+ \left[\frac{dg^{\pm}}{dp_\perp}\Big|_{(0, \bar{p}_z)}\right] \cdot p_\perp + \left[\frac{dg^{\pm}}{dp_z}\Big|_{(0, \bar{p}_z)}\right] \cdot (p_z - \bar{p}_z) +$$

$$+ \frac{1}{2}\left[\frac{d^2 g^{\pm}}{d{p_\perp}^2}\Big|_{(0, \bar{p}_z)}\right] \cdot {p_\perp}^2 +$$

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$$+\left[\frac{d^2g^\pm}{dp_\perp dp_z}\Big|_{(0,\bar{p}_z)}\right]\cdot p_\perp\cdot(p_z-\bar{p}_z)+$$

$$+\frac{1}{2}\left[\frac{d^2g^\pm}{dp_z{}^2}\Big|_{(0,\bar{p}_z)}\right]\cdot(p_z-\bar{p}_z)^2 \qquad (23)$$

Basic relations:

$$v_z=\frac{p_z}{m\cdot\gamma} \quad ; \quad \gamma=\left(1+\frac{p_\perp{}^2+p_z{}^2}{m^2c^2}\right)^{\frac{1}{2}} \quad ;$$

$$\omega_p=\left(\frac{e^2n_0}{m\varepsilon_0\gamma\gamma_z{}^2}\right)^{\frac{1}{2}} \qquad (24)$$

$$\gamma=(1-\beta^2)^{-\frac{1}{2}} \; ; \quad \gamma_z=(1-\beta_z{}^2)^{-\frac{1}{2}} \; ;$$

$$\gamma_z=\gamma\cdot\left(1+\frac{p_\perp{}^2}{m^2c^2}\right)^{-\frac{1}{2}} \qquad (25)$$

First order derivatives:

$$\frac{d\gamma}{dp_\perp}=\frac{p_\perp}{m^2c^2\gamma} \quad ; \quad \frac{d\gamma}{dp_z}=\frac{p_z}{m^2c^2\gamma} \quad ;$$

$$\frac{d\gamma_z}{dp_\perp}=-\frac{p_\perp p_z{}^2\gamma_z{}^3}{m^4c^4\gamma^4} \quad ; \quad \frac{d\gamma_z}{dp_z}=\frac{p_z\,\gamma_z}{m^2c^2\gamma^2} \qquad (26)$$

$$\frac{dv_z}{dp_\perp}=-\frac{p_\perp p_z}{m^2c^2\gamma^3} \quad ; \quad \frac{dv_z}{dp_z}=\frac{1}{m\,\gamma\gamma_z{}^2} \quad ;$$

$$\frac{d\omega_p}{dp_\perp}=\frac{\omega_p p_\perp}{m^2c^2\gamma^2}\left(\gamma_z{}^2-\frac{3}{2}\right) \; ; \quad \frac{d\omega_p}{dp_z}=-\frac{3}{2}\frac{\omega_p p_z}{m^2c^2\gamma^2} \qquad (27)$$

$$\frac{dg^\pm}{dp_\perp}=\frac{p_\perp}{m\,c^2\gamma p_z}\left[\omega\pm\omega_p\left(\gamma_z{}^2-\frac{1}{2}\right)\right] \quad ;$$

$$\frac{dg^\pm}{dp_z}=-(\pm)\frac{3}{2}\frac{\omega_p}{m\,c^2\gamma}-\frac{m\gamma}{p_z{}^2\gamma_z{}^2}(\omega\pm\omega_p) \qquad (28)$$

Second order derivatives:

$$\frac{d^2g^\pm}{dp_\perp{}^2}=\frac{1}{m\,c^2p_z}\cdot\frac{p_\perp{}^2}{m^2c^2\gamma^2}\cdot\left[\pm\omega_p\left(\frac{5}{4}-\gamma_z{}^2-\gamma_z{}^4\right)-\omega\right]+$$

$$+\frac{1}{m\,c^2p_z}\cdot\left[\omega\pm\omega_p\left(\gamma_z{}^2-\frac{1}{2}\right)\right] \qquad (29)$$

$$\frac{d^2g^\pm}{dp_\perp dp_z}=\frac{p_\perp}{m^3c^4\gamma^3}\left[\pm\omega_p\left(\frac{7}{4}-\frac{3}{2}\gamma_z{}^2\right)-2\omega\right]$$

$$-\frac{p_\perp}{m\,c^2\gamma\gamma_z{}^2p_z{}^2}\left[\omega\pm\omega_p\left(\gamma_z{}^2-\frac{1}{2}\right)\right] \qquad (30)$$

$$\frac{d^2g^\pm}{dp_z{}^2}=\pm\frac{15}{4}\frac{\omega_p p_z}{m^3c^4\gamma^3}+2\frac{m\gamma}{p_z{}^3\gamma_z{}^2}(\omega\pm\omega_p)+$$

$$+\frac{\omega\pm\frac{5}{2}\omega_p}{m\,c^2\gamma\gamma_z{}^2p_z} \qquad (31)$$

Substituting the average values of the transversal and longitudinal components of the momentum in expressions (24)-(31) leads to the following definitions of constants:

$$\tilde{\gamma}\triangleq\gamma(p_\perp=0,p_z=\bar{p}_z)=\left(1+\frac{\bar{p}_z{}^2}{m^2c^2}\right)^{\frac{1}{2}} \quad ;$$

$$\tilde{\gamma}_z\triangleq\gamma_z(p_\perp=0,p_z=\bar{p}_z)=\tilde{\gamma} \qquad (32)$$

$$\tilde{\omega}_p\triangleq\omega_p(p_\perp=0,p_z=\bar{p}_z)=\left(\frac{e^2n_0}{m\varepsilon_0\tilde{\gamma}^3}\right)^{\frac{1}{2}} \quad ;$$

$$\tilde{v}_z\triangleq v_z(p_\perp=0,p_z=\bar{p}_z)=\frac{\bar{p}_z}{m\cdot\tilde{\gamma}} \qquad (33)$$

$$A^\pm\triangleq g^\pm(0,\bar{p}_z)=\frac{\omega\pm\tilde{\omega}_p}{\tilde{v}_z} \qquad (34)$$

$$\left[\frac{dg^\pm}{dp_\perp}\Big|_{(0,\bar{p}_z)}\right]=0 \quad ; \quad B^\pm\triangleq\left[\frac{dg^\pm}{dp_z}\Big|_{(0,\bar{p}_z)}\right]=$$

$$=-(\pm)\frac{3}{2}\frac{\tilde{\omega}_p}{m\,c^2\tilde{\gamma}}-\frac{m}{\bar{p}_z{}^2\tilde{\gamma}}(\omega\pm\tilde{\omega}_p) \qquad (35)$$

$$C_1{}^\pm\triangleq\left[\frac{d^2g^\pm}{dp_\perp{}^2}\Big|_{(0,\bar{p}_z)}\right]=\frac{1}{m\,c^2\bar{p}_z}\left[\omega\pm\tilde{\omega}_p\left(\tilde{\gamma}^2-\frac{1}{2}\right)\right] \; ;$$

$$\left[\frac{d^2g^\pm}{dp_\perp dp_z}\Big|_{(0,\bar{p}_z)}\right]=0 \qquad (36)$$

$$C_2{}^\pm\triangleq\left[\frac{d^2g^\pm}{dp_z{}^2}\Big|_{(0,\bar{p}_z)}\right]=$$

$$=\pm\frac{15}{4}\frac{\tilde{\omega}_p\bar{p}_z}{m^3c^4\tilde{\gamma}^3}+2\frac{m}{\bar{p}_z{}^3\tilde{\gamma}}(\omega\pm\tilde{\omega}_p)+\frac{\omega\pm\frac{5}{2}\tilde{\omega}_p}{m\,c^2\tilde{\gamma}^3\bar{p}_z} \qquad (37)$$

Using the above definitions of the constants, the second order approximation of $g^\pm$ as given in (23) becomes:

$$g^\pm(p_\perp,p_z)\approx A^\pm+B^\pm\cdot(p_z-\bar{p}_z)+$$

$$+\frac{1}{2}\left[C_1{}^\pm\cdot p_\perp{}^2+C_2{}^\pm\cdot(p_z-\bar{p}_z)^2\right] \qquad (38)$$

With the definition:

$$h(p_\perp,p_z,z)\triangleq\frac{p_\perp{}^2}{2\cdot\Delta p_\perp{}^2}+\frac{(p_z-\bar{p}_z)^2}{2\cdot\Delta p_z{}^2}+$$

$$+jz\cdot g^\pm(p_\perp,p_z) \qquad (39)$$

The integration in (21) can be written as:

$$\rho_1{}^\pm=-\frac{en_0}{4\pi\cdot\Delta p_\perp\cdot\Delta p_z}e^{j\omega t}\cdot$$

$$\cdot\int_{-\infty}^{\infty}dp_\perp\int_{-\infty}^{\infty}e^{-h(p_\perp,p_z,z)}dp_z \qquad (40)$$

Substituting (38) in (39) results in:

$$h(p_\perp,p_z,z)\approx\frac{p_\perp{}^2}{2\cdot\Delta p_\perp{}^2}+\frac{(p_z-\bar{p}_z)^2}{2\cdot\Delta p_z{}^2}+$$

$$+jzA^\pm+jzB^\pm\cdot(p_z-\bar{p}_z)+$$

$$+\frac{1}{2}\left[jzC_1{}^\pm\cdot p_\perp{}^2+jzC_2{}^\pm\cdot(p_z-\bar{p}_z)^2\right] \qquad (41)$$

We examine the coefficients of the terms $p_\perp{}^2$, $p_z{}^2$ and p_z; we suggest the following definitions:

$$\sigma_\perp(z)\triangleq\frac{\Delta p_\perp}{\sqrt{1+jzC_1{}^\pm\cdot\Delta p_\perp{}^2}} \quad ;$$

$$\sigma_z(z)\triangleq\frac{\Delta p_z}{\sqrt{1+jzC_2{}^\pm\cdot\Delta p_z{}^2}} \quad ;$$

$$\bar{p}(z)\triangleq\bar{p}_z-jzB^\pm\cdot[\sigma_z(z)]^2 \qquad (42)$$

Substitute (42) in (41) results in:

$$h(p_\perp,p_z,z)\approx\frac{p_\perp{}^2}{2\cdot[\sigma_\perp(z)]^2}+\frac{[p_z-\bar{p}(z)]^2}{2\cdot[\sigma_z(z)]^2}+$$

$$+jzA^\pm+\frac{1}{2}z^2(B^\pm)^2[\sigma_z(z)]^2 \qquad (43)$$

The first order charge density can now be calculated in (40) by using the approximation for $h(p_\perp,p_z,z)$ in (43):

$$\rho_1{}^\pm=-\frac{en_0\cdot\sigma_\perp(z)\cdot\sigma_z(z)}{2\cdot\Delta p_\perp\cdot\Delta p_z}e^{j\omega t}e^{-\left\{jzA^\pm+\frac{1}{2}z^2(B^\pm)^2[\sigma_z(z)]^2\right\}}=$$

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$$=-\frac{en_0}{\sqrt{\left(1+jzC_1^{\pm}\cdot\Delta p_\perp^2\right)\left(1+jzC_2^{\pm}\cdot\Delta p_z^2\right)}}\cdot\frac{1}{2}\cdot e^{j\left(\omega t-\frac{\omega\pm\tilde{\omega}_p}{\tilde{v}_z}z\right)}$$

$$\cdot e^{-\frac{\left(B^{\pm}\right)^2\Delta p_z^2}{2}\cdot\frac{z^2}{1+jzC_2^{\pm}\cdot\Delta p_z^2}} \quad (44)$$

Physical Interpretation and Significance for Low Energy X-Ray FEL

The factor $\frac{1}{2}e^{j\left(\omega t-\frac{\omega\pm\tilde{\omega}_p}{\tilde{v}_z}z\right)}$ is proportional to the cold beam solution.

In order to evaluate the degradation in amplitude of the first order charge density along the longitudinal direction, we separate the other exponential expression in (44) into real and imaginary parts:

$$e^{-\frac{\left(B^{\pm}\right)^2\Delta p_z^2}{2}\cdot\frac{z^2}{1+jzC_2^{\pm}\cdot\Delta p_z^2}}=$$

$$=e^{j\frac{z^3\cdot C_2^{\pm}\cdot\left(B^{\pm}\right)^2\cdot\Delta p_z^4}{2\left[1+z^2\cdot\left(C_2^{\pm}\right)^2\cdot\Delta p_z^4\right]}}\cdot e^{-\frac{\left(B^{\pm}\right)^2\cdot\Delta p_z^2}{2}\cdot\frac{z^2}{1+z^2\cdot\left(C_2^{\pm}\right)^2\cdot\Delta p_z^4}} \quad (45)$$

The physical interpretation of the result in (45) is that there is a limited length region, at the vicinity of the modulation starting point, characterized by an exponential decay in the charge density modulation, described by the expression $e^{-\frac{\left(B^{\pm}\right)^2\cdot\Delta p_z^2}{2}z^2}$,with typical length of $\Delta z\approx\frac{1}{|C_2^{\pm}|\cdot\Delta p_z^2}$, followed by a stable modulation region.

If the frequency of the modulation divided by the factor $\tilde{\gamma}^2$ is significantly higher than the plasma frequency, i.e. $\frac{\omega}{\tilde{\gamma}^2}\gg\tilde{\omega}_p$, the terms $(B^{\pm})^2$ and $C_2^{\pm}$ are simplified:

$$(B^{\pm})^2\approx\frac{\omega^2}{m^2c^4\tilde{\gamma}^6} \quad ; \quad C^{\pm}\approx\frac{3\omega}{m^2c^3\tilde{\gamma}^4} \quad (46)$$

Note that $\frac{\omega}{c}=\frac{2\pi}{\lambda}$ and for $\tilde{\gamma}>10$ the average longitudinal momentum is approximately: $\bar{p}_z=m\tilde{\gamma}\tilde{v}_z\approx m\tilde{\gamma}c$; it is convenient to define the normalized temperature of the electron beam:

$$T_n\triangleq\frac{\Delta p_z}{\bar{p}_z} \quad (47)$$

The expression describing the exponential decay in (45) becomes:

$$exp\left[-\frac{\left(B^{\pm}\right)^2\cdot\Delta p_z^2}{2}\cdot\frac{z^2}{1+z^2\cdot\left(C_2^{\pm}\right)^2\cdot\Delta p_z^4}\right]\approx$$

$$\approx exp\left[-\frac{2\pi^2}{\tilde{\gamma}^4}\cdot T_n^2\cdot\frac{z^2}{\lambda^2}\cdot\frac{1}{1+\frac{36\pi^2}{\tilde{\gamma}^4}\cdot T_n^4\cdot\frac{z^2}{\lambda^2}}\right] \quad (48)$$

Therefore, the typical length of region characterized by the exponential decay, normalized with respect to the wavelength, is inversely proportional to the square of the normalized temperature:

$$\frac{\Delta z}{\lambda}\approx\frac{\tilde{\gamma}^2}{6\pi}\cdot\frac{1}{T_n^2} \quad (49)$$

The attenuation in the amplitude of the first order charge density in the stable region, with respect to the initial modulation factor is:

$$\left|\frac{\rho_1^{\pm}(z\gg\Delta z)}{\rho_1^{\pm}(z=0)}\right|\approx e^{-\frac{1}{18\cdot T_n^2}} \quad (50)$$

Typical normalized length for the attenuation in the amplitude of the first order charge density (the point where the attenuation decreases down to about $\frac{1}{e}$ of its initial value) is no less than:

$$\left|\frac{\rho_1^{\pm}(z=\Delta z)}{\rho_1^{\pm}(z=0)}\right|\approx\frac{1}{e}\ \Rightarrow\ \frac{\Delta z}{\lambda}>\frac{\tilde{\gamma}^2}{\sqrt{2}\cdot\pi}\cdot\frac{1}{T_n} \quad (51)$$

The actual normalized length in (51) may increase for high values of the normalized temperature, due to the term in the denominator of (48), which becomes: $1+18T_n^2$ at that point.

The table below depicts the estimate typical normalized lengths of the attenuation in the amplitude of the first order charge density, as calculated in (51). This was calculated for two different operating ranges, (Tera Hertz and X Ray) in various cases:

λ	$\tilde{\gamma}$	T_n	Δz
100μm	10	0.1	22.5mm
10nm	80	0.001	14.4mm
10nm	400	0.01	36mm

CONCLUSIONS

For a cold electron beam in a free drift region, a first order perturbation in charge density was calculated. The result is shown to be a propagation wave in the longitudinal direction at the excitation frequency. The phase velocity of the density wave equals to that of the beam velocity plus an amplitude modulation caused by a standing wave acting as a slow-varying "envelope". The standing wave has a wave number equals to the plasma frequency divided by the beam velocity, but with no attenuation in charge density modulation along the longitudinal direction.

Introducing two dimensional velocity spreads causes the first order charge density modulation of a warm electron beam in a free drift region to decrease rapidly (exponential decay) in the longitudinal direction. The decrease has a typical normalized length proportional to $\tilde{\gamma}^2$, and inversely proportional to the normalized temperature.

In the Tera Hertz range, a typical normalized length (the effective length without losing the bunching) of the order of tens of millimeters can be achieved with $\tilde{\gamma}\approx 10$ and a normalized temperature of 0.1; the same order of magnitude of typical normalized length in the X Ray range can be achieved with $\tilde{\gamma}\approx 80$ and a normalized temperature of 0.001, or $\tilde{\gamma}\approx 400$ and a normalized temperature of 0.01.

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