SUPPRESSION OF THE CSR-INDUCED EMITTANCE GROWTH IN ACRHOMATS USING TWO-DIMENSIONAL POINT-KICK ANALYSIS*

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Abstract

Coherent synchrotron radiation (CSR) effect causes transverse emittance dilution in high-brightness light sources and linear colliders. Suppression of the emittance growth induced by CSR is essential and critical to preserve the beam quality and to help improve the machine performance. To evaluate the CSR effect analytically, we propose a novel method, named "two-dimensional point-kick analysis". In this method, the CSR-induced emittance growth in an n-dipole achromat can be evaluated with the analysis of only the motion of particle in (x, x') two-dimensional plane with n-point kicks, which can be, to a large extent, counted separately. To demonstrate the effectiveness of this method, the CSR effect in a two-diople achromat and a symmetric TBA is studied, and generic conditions of suppressing the CSR-induced emittance growth, which are independent of concrete element parameters and are robust against the variation of initial beam distribution, are found. These conditions are verified with the ELEGANT simulations and can be rather easily applied to real machines.

INTRODUCTION

Electron beams with low transverse normalized emittance (at the μm.rad or sub-μm.rad scale), short bunch length (at the sub-picosecond scale), and high peak current (up to thousands of Amperes) are generated or expected in high-brightness light sources and linear colliders. In these machines as beams pass through bending magnets, the emission of the coherent synchrotron radiation (CSR) leads to beam quality degradation, by inducing increased beam energy spread and causing transverse emittance dilution. Suppressing this effect is necessary and important to preserve the expected machine performance that is evaluated without considering the CSR effect. This has stimulated extensive analytical, numerical, and experimental studies [1-20] on the CSR effect in the past few decades. One important topic among these studies is to suppress the CSR-induced emittance growth. It has been shown that the CSR effect can be suppressed through optical balance method [4, 19], CSR-kick matching [12, 13], shielding [17], and pulse shaping [18].

In this paper we present a novel method of analysing the net CSR kick after passage through an achromatic cell, named "two-dimensional (2D) point-kick analysis" [20]. With this method, the CSR-induced emittance growth in a n-dipole achromat can be evaluated with the analysis of only the motion of particle in (x, x') 2D plane with n-point kicks, which can be, to a large extent, counted separately. In addition, the beam line between adjacent dipoles is treated as a whole and is formulated with a 2-by-2 transfer matrix. As a result, general CSR-cancellation (in linear regime) conditions can be obtained with this method. In the following, we will introduce the CSR point-kick model, and then use the 2D point-kick analysis to study the CSR effect in a two-dipole achromat and a symmetric TBA, respectively.

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CSR 2D POINT-KICK MODEL

It has been shown that for an electron bunch of Gaussian temporal distribution, the rms energy spread caused by CSR is [12, 13]

$$\Delta E_{rms} = 0.2459 \frac{eQ\mu_0 c_0^2 L_b}{4\pi\sigma_z^{4/3}\rho^{2/3}}, \tag{1}$$

where Q is the bunch charge, μ_0 is the permeability of vacuum, c_0 is the speed of light, L_b is the particle bending path in a dipole, σ_z is the rms bunch length, and ρ is the bending radius. Note that ΔE_{rms} is proportional to both L_b and $\rho^{-2/3}$, or namely, $\Delta E_{rms} \propto \rho^{1/3}\theta$, with θ being the bending angle. Therefore the CSR effect can be linearized by assuming $\delta(csr) = k\rho^{1/3}\theta$, where $\delta(csr)$ is the CSR-induced particle energy deviation, k depends only on the bunch charge Q and the bunch length σ_z, and is in unit of $m^{-1/3}$. It reveals [20] through ELEGANT simulations that this relation applies well to the cases with θ ranging from 1 to 12 degrees and ρ ranging from 1 to 150 m. With the so-called R-matrix method [12], the coordinate deviations of a particle relative to the ideal path after passage through a sector bending magnet can be evaluated,

$$\Delta X = \begin{pmatrix} D \\ D' \end{pmatrix} \delta_i + \begin{pmatrix} \zeta \\ \zeta' \end{pmatrix} k, \tag{2}$$

where $D = \rho(1\text{-}\cos\theta)$ and $D' = \sin\theta$ are the momentum dispersions, and $\zeta = \rho^{4/3}(\theta - \sin\theta)$ and $\zeta' = \rho^{1/3}(1-\cos\theta)$ are the "CSR-dispersions".

Through theoretical derivations, we find that the CSR effect in a dipole can be simplified as a point-kick. This kick occurs at the center of the dipole, and is in the form of [20]

$$X_k = \begin{pmatrix} \rho^{4/3}k[\theta\cos(\theta/2) - 2\sin(\theta/2)] \\ \sin(\theta/2)(2\delta + \rho^{1/3}\theta k) \end{pmatrix}. \tag{3}$$

After each kick, the particle coordinates increase by X_k, and in addition, the particle energy deviation increases by $kL_B/\rho^{2/3}$ (or $k\rho^{1/3}\theta$).

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Next we will show that with this point-kick model, the analysis of the CSR-induced emittance growth can be greatly simplified.

TWO-DIPOLE ACRHOMAT

For a two-dipole achromat, as illustrated in Fig. 1, one needs only to consider two CSR kicks at the dipole centers. To avoid dependency of the analysis on concrete optics design, the beam line between two dipoles (actually the beam line between the centers of the two dipoles) is treated as a whole and is formulated in a general form

$$M_{c2c} = \begin{pmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{pmatrix}. \tag{4}$$

For simplicity, it is assumed that the particle starts from the entrance of the achromat with initial energy deviation of δ_0 and with initial coordinates of $x_0 = x'_0 = 0$. The coordinates remain zero until the particle experiences the CSR kick at the center of the first dipole.

Right after the first kick,

$$\begin{aligned} X_{1+} &= X_{1-} + X_{k,1} \\ &= \begin{pmatrix} 0 \\ 2S_1 \end{pmatrix} \delta_0 + \begin{pmatrix} \rho_1 (C_1\theta_1 - 2S_1) \\ S_1\theta_1 \end{pmatrix} \rho_1^{1/3} k, \\ \delta_{1+} &= \delta_0 + k\rho_1^{1/3}\theta_1, \end{aligned} \tag{5}$$

where $S_1 = \sin(\theta_1/2)$ and $C_1 = \cos(\theta_1/2)$.

Similarly one can obtain the particle coordinates and energy deviation right after the second kick,

$$\begin{aligned} X_{2+} &= M_{c2c} X_{1+} + X_{k,2} = M_{c2c} X_{k,1} + X_{k,2}, \\ \delta_{2+} &= \delta_0 + k\rho_1^{1/3}\theta_1 + k\rho_2^{1/3}\theta_2, \end{aligned} \tag{6}$$

where X_{2+} is just the orbit deviation of particle relative to the ideal path, i.e., $\Delta X = X_{2+}$. It contains two terms, $\Delta X(\delta_0)$ and $\Delta X(k)$, which are omitted here due to lengthy expressions. The achromatic condition can be derived by solving $\Delta X(\delta_0) = 0$, and then the CSR-cancellation conditions are obtained by solving $\Delta X(k) = 0$.

The achromatic condition is

$$M_{c2c} = \begin{pmatrix} -S_1/S_2 & 0 \\ m_{21} & -S_2/S_1 \end{pmatrix}, \tag{7}$$

where $S_2 = \sin(\theta_2/2)$, $C_2 = \cos(\theta_2/2)$.

And the CSR-cancellation conditions are in the form

$$\begin{aligned} L_1\theta_1^2 &\cong L_2\theta_2^2, \\ m_{21} &\cong \frac{12}{L_1}\frac{S_2}{S_1}. \end{aligned} \tag{8}$$

In an actual circumstance, due to various reasons (e.g., random errors, other optics constraints), $M_{c2c}(2, 1)$ may be close to, rather than at the exact optimal value. Therefore it is useful to investigate the scaling of the emittance growth when the first condition in Eq. (8) is fulfilled while the second is not satisfied. It is in the form of

$$\begin{aligned} &\Delta\varepsilon_n |_{r=r^*} \approx 2\gamma\beta k_{rms}^2 S_1^2 \theta_1^2 \rho_1^{2/3} \beta_1 \cdot \\ &[1 - M_{c2c}(2,1) / M_{c2c}^*(2,1)]^2. \end{aligned} \tag{9}$$

where $r \equiv \rho_2/\rho_1$, γ is the Lorentz factor, β is the particle velocity relative to the speed of light, β_1 is the horizontal beta function at the center of the first dipole, and an asterisked quantity means the quantity leads to a zero emittance growth in linear regime.

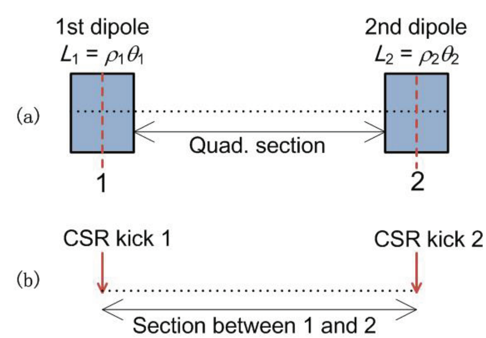

Figure 1: Schematic layout of a two-dipole achromat (a) and physical model for the 2D point-kick analysis of the CSR effect (b). The points 1 and 2 indicate the centers of the first and the second dipole, respectively.

To verify the found conditions, a two-dipole achromat with θ_1 of 6 degrees, ρ_1 of 8 m and θ_2 of 4 degrees, is considered. From Eq. (8), the optimal parameters resulting in zero emittance growth in linear regime can be determined, such as $\rho_2^* \cong 27$ m, and $M^*_{c2c}(2, 1) \cong 30/\pi$. Such an achromat is designed with four families of quadrupoles located between the dipoles, whose optics can be varied flexibly. The emittance growth in presence of the CSR wake is simulated with the ELEGANT program, where an electron bunch with typical parameters of initial normalized emittance of 2 μm.rad, mean energy of 1 GeV, energy spread of 0.05%, bunch charge of 500 pC, and bunch length of 30 μm is tracked.

The variation of the growth in normalized emittance, $\Delta\varepsilon_n$, with $M_{c2c}(2, 1)$ (while fixing $r = r^*$) and r [while fixing $M_{c2c}(2, 1) = M^*_{c2c}(2, 1)$] is investigated. The results are presented in Fig. 2. It shows that the found conditions result in a minimum $\Delta\varepsilon_n$, and are quite robust against the variation of the initial Courant-Snyder (C-S) parameters (or namely, the initial beam distribution in phase space). In addition, the variation of $\Delta\varepsilon_n$ with M_{c2c} (2, 1) agrees pretty well with the analytical prediction from Eq. (9) with k_{rms} of 0.0012 $m^{-1/3}$ (dashed lines in Fig. 2), which indicates the validity of the 2D point-kick analysis.

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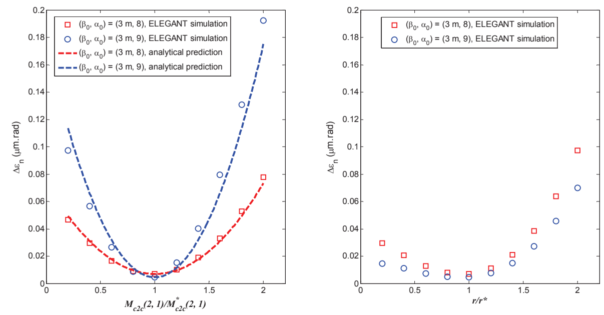

Figure 2: Variation of the emittance growth $\Delta\varepsilon_n$ due to CSR with r and M_{c2c} (2, 1) for the cases with initial C-S parameters (β_0, α_0) of (3 m, 8) and (3 m, 9) in a two-dipole achromat, obtained by ELEGANT simulations. The dashed lines are the analytical prediction from Eq. (9) and with a shift of the minimum $\Delta\varepsilon_n$.

Note that only the linear effect of the CSR wake in the dipole is considered in the 2D point-kick analysis. It is necessary to investigate the effects of the nonlinear components of the CSR wake in a dipole (denoted by *n .c. CSR*), the transient CSR at the edges of the dipole (denoted by *tr. CSR*), and the CSR wake in drift spaces following dipoles (denoted by *d.s. CSR*). Simulation results are summarized in Table 1. It shows that these effects are very weak relative to the linear effect of the CSR wake in dipoles, and rarely affect the performance of the proposed cancellation conditions.

Table 1: Emittance Growth in Presence of the Effects of *n.c. CSR*, *tr. CSR*, and *d.s. CSR* at the Optimal Condition

CSR effects	$\Delta\varepsilon_n/\varepsilon_{n0}$
n.c. CSR	1.5×10^{-3}
n.c. CSR + *tr.* CSR	2.8×10^{-3}
n.c. CSR + *tr.* CSR + *d.s.* CSR	5.95×10^{-3}

TBA WITH SYMMETRIC LAYOUT

The schematic layout of a TBA and the corresponding physical model of the CSR effect in a TBA are presented in Fig. 3. To control the number of the variables and to obtain explicit formulation of the net CSR kick, a TBA with symmetric layout (where all the dipoles have the same bending radii ρ, while the first and the third bending angles are θ_1 and the second bending angle is θ_2) is considered.

For such a TBA, the transfer matrix of the section between point 1 and 2 can be given by

$$M_{12} = \begin{pmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{pmatrix}, \tag{10}$$

and the transfer matrix of the section between point 2 and 3 is in the form [21]

$$M_{23} = \begin{pmatrix} m_{22} & m_{12} \\ m_{21} & m_{11} \end{pmatrix}. \tag{11}$$

With similar procedure to that in the above section, the particle coordinates deviation after passage the TBA can be evaluated by

$$\Delta X = M_{23}(M_{12}X_{k,1} + X_{k,2}) + X_{k,3}, \tag{12}$$

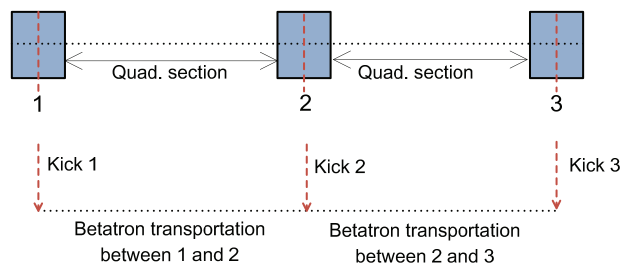

Figure 3: Schematic layout of a symmetric TBA and the corresponding physical model of the CSR effect in a TBA with three point-kicks.

Thorough straightforward derivations, the achromatic and the CSR-cancellation condition require the transfer matrix M_{12} in the form of

$$M_{12} = \begin{pmatrix} -\dfrac{q_2\rho + 2m_{12}(\theta_1+\theta_2)S_1}{2q_1\rho} & m_{12} \\ \dfrac{1}{m_{12}}\left(\dfrac{q_2S_2}{4q_1S_1} + \dfrac{m_{12}(\theta_1+\theta_2)S_2}{2q_1\rho} - 1\right) & -\dfrac{S_2}{2S_1} \end{pmatrix}, \tag{13}$$

where $q_1 = 2S_1 - C_1\theta_1$ and $q_2 = 2S_2 - C_2\theta_2$.

To verify the found conditions, we consider a TBA consisting of three identical dipoles with the bending radii of 7 m and bending angles of 3 degrees. The dependency of $\Delta\varepsilon_n$ on m_{11} is investigated by fixing $m_{12} = -0.261$ and -1.058, respectively. The results are shown in Fig. 4. It shows that $\Delta\varepsilon_n$ reaches minimum as m_{11} is on or close to the optimal value, which agrees reasonably well with the analytical prediction.

Furthermore, in the case with $\theta_2 = 0$, the TBA reduces to a DBA, and the transfer matrix between the first and the third dipole centers turns out to be

$$\begin{aligned} M_{13}\,|_{\theta_2=0} &= M_{23}M_{12}\,|_{\theta_2=0} \\ &= \begin{pmatrix} 0 & m_{12} \\ -1/m_{12} & -m_{12}\theta_1S_1/q_1\rho \end{pmatrix}\begin{pmatrix} -m_{12}\theta_1S_1/q_1\rho & m_{12} \\ -1/m_{12} & 0 \end{pmatrix} \\ &= \begin{pmatrix} -1 & 0 \\ 2\theta_1S_1/q_1\rho & -1 \end{pmatrix} \cong \begin{pmatrix} -1 & 0 \\ 12/L_1 & -1 \end{pmatrix}, \end{aligned} \tag{14}$$

which is the same as the CSR-cancellation conditions for a DBA [see Eqs. (7) and (8) with $\theta_1 = \theta_2$].

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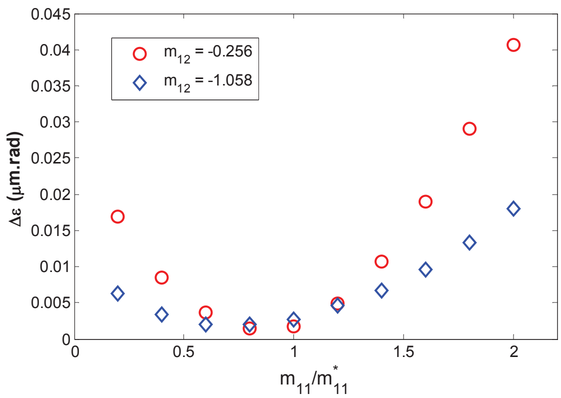

Figure 4: Variation of the emittance growth $\Delta\varepsilon_n$ through TBA with respect to $m_{11}/\ m^*_{11}$, for the cases with m_{12} fixed as -0.261 (red points) and -1.058 (blue points).

CONCLUSION

In this paper we introduce the 2D point-kick method, and with this method we obtain conditions to cure the linear effect of the CSR wake in a two-dipole achromat and a symmetric TBA. The found conditions impose general rather than specific constrains on the optics of the beam line between adjacent dipoles and can be easily achieved by tuning the strengths (and the position if necessary) of the quadupoles. In addition, the found conditions are quite robust against the variation of the initial beam distribution. The presented results are useful and easily followed in the optical design of the future FEL/ERL light sources and linear colliders.

At last it is worth mentioning that it assumes constant bunch charge and bunch length in the analysis, the results presented in this paper are more appropriate to the transport system with a small momentum compaction (R_{56}) than to that with a large R_{56}, e.g., a specified functional bunch compressor. Further study is being carried out to cover the cases where the bunch length has a large variation.

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