

A NOVEL MODELING APPROACH FOR ELECTRON BEAMS IN SASE FELs

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Abstract

We have recently shown that the Wheeler-Feynman analysis of the interaction of a moving charge with distant absorbers provides a perfect match to the energy radiated by two coherently oscillating charged particles (a heretofore unsolved problem in classical electrodynamics). Here we explain the need to include the Wheeler-Feynman coherent radiation reaction force as an integral part of the solution to the boundary value problem for free electron lasers (FELs) that radiate into "free space". We will also discuss how the advanced field of the absorber can interact with the radiating particles at the time of emission. Finally we will introduce and explore the possibility of improving the temporal coherence of self amplified spontaneous emission (SASE) FELs as well as the possibility of optimizing the spectrum of their emitted radiation via altering the structure of their targets by using the Wheeler-Feynman coherent radiation reaction force in the analysis of FEL operation.

to the radiated energy in the case of two or more coherently oscillating charged particles [5].

These developments seem likely to clarify our understanding and contribute to the further advancement of FEL light sources, and possibly improve the temporal coherence of SASE FELs. With the rapid advancement and reliance upon these sources, this is a good time to consider the effect of the Wheeler-Feynman approach on current technology. We also hope that this approach will provide further insight into physics underlying the behavior of the of these powerful devices as well as additional means to control the spectral intensity and bandwidth in addition to the operating wavelength.

The description of advanced interactions as set forth in the Wheeler Feynman paper has led us to explore both the engineering implications for the design of SASE FEL systems and possible new approaches for the investigation of the distribution of distant matter in the universe as well [6].

INTRODUCTION

In the past few years the successful operation of x-ray FELs based on the SASE principle has made them a powerful new tool for addressing fundamental questions in biology, chemistry and nano-technology [1]. On the other hand the fundamental principle that the SASE FELs rely on, the theory of radiation in classical electrodynamics, has a few unresolved questions of its own. For instance, since classical field theory has, for about a century, failed to provide a non-diverging solution and origin for the radiation reaction field, our understanding of the process of coherent radiation in the classical limit with respect to conservation of energy is not complete. The radiation reaction field is the electric field responsible for energy conservation in the process of radiation.

The analysis provided by Wheeler and Feynman, in their 1945 paper "Interaction with the Absorber as the Mechanism of Radiation," for the first time yields an exact match between the radiated power of oscillating particles and the rate of change of the particle's kinetic energy [2]. The conceptual backbone of the Wheeler and Feynman model has been debated for many years and raises a number of questions about the nature of interactions of fields and particles. But it has been shown that the model is consistent with the quantum electrodynamics [3] and Dirac's theory of single particle radiation reaction force [4]. In their paper, Wheeler and Feynman did not consider the coherent radiation emitted by multiple accelerated charges. However, we have shown that the Wheeler-Feynman derivation for a moving charge interacting with the absorber also provides a perfect match

APPLYING WHEELER FEYNMAN ANALYSIS TO COLLECTION OF COHERENTLY OSCILLATING PARTICLES

Wheeler and Feynman were able to demonstrate that, when formulated in the language of covariant action-at-a-distance, the solution to the boundary value problem corresponding to an oscillating particle within a spherical absorbing shell of arbitrary density is dominated by the interference of the retarded and advanced forces originating in the accelerated and absorbing particles. This leads to a force on the accelerated particle exactly equal to that needed to match the power carried by radiation to the particles in the absorbing shell. In their 1949 paper, their quantitative findings also show that these interactions are only evident in the immediate vicinity of the radiating/absorbing charges, and converge to the conventional retarded electrodynamics at larger distances from the radiating charge [7].

Case of Two Coherently Oscillating Particles

Consider two coherently oscillating charged particles displaced by distance " r " in an arbitrary direction. If the displacement r has an angle α with respect to the direction of motion of the coherently oscillating charged particles, the integral of Poynting vector (the radiated power) is defined Equation 1. When the charges oscillate perpendicular to the direction of their separation vector, $\alpha = \pi/2$, the power radiated is given by Equation 2.

$$P_{\text{Radiated}}(\alpha) \propto 2 \int_0^{2\pi} \int_0^\pi \sin^3(\theta) \cos^2 \times \left(\frac{kr(\cos(\theta) \cos(\alpha) + \sin(\theta) \sin(\alpha) \cos(\phi))}{2} \right) d\theta d\phi \quad (1)$$

and $P_{\text{Radiated}}(\alpha = \pi/2) \propto$

$$2 \int_0^{2\pi} \int_0^\pi \sin^3(\theta) \cos^2 \left(\frac{kr(\sin(\theta) \cos(\phi))}{2} \right) d\theta d\phi \quad (2)$$

The work done on the radiating charges by the electric field they generate must equal the radiated power calculated by Poynting's vector in Equation 1 and Equation 2.

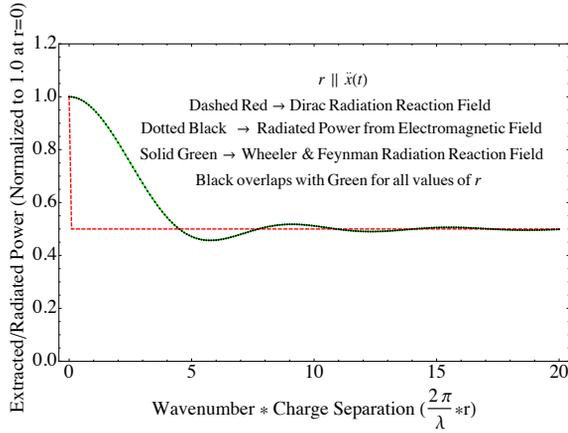


Figure 1: Comparison of the power extracted by the Dirac coherent radiation force and the amplitude of the Wheeler-Feynman coherent radiation force with the power radiated by the two oscillating charged particles' for displacements parallel to their vector accelerations. The non-local component of the Dirac coherent radiation reaction force falls to zero for any finite displacement of the two charges along this direction, leaving only each particle's single point radiation reaction force to oppose their oscillating velocities (adapted from Reference [5]).

As indicated in Figure 1 and 2, the Wheeler and Feynman model is particularly useful when dealing with coherently radiating particles in free-space. Figure 3 compares the radiated power and energy extracted by the Wheeler-Feynman coherent radiation reaction force for all angles of α .

Case of Coherent Beams in SASE FELs

The coherent radiation emitted by tightly bunched electron beams plays a critical role in the analysis and operation of free electron lasers. For conducting or reflecting (resonator mirror) FELs, a normal mode analysis of operation already includes the relevant boundary conditions. However, in order to arrive at a comprehensive first-principles field-based analysis of the intense of radiation emitted into free-space by devices that work based on the SASE principle, including all target interactions, we also need to include the effect of boundary condition at the target.

Kimel and Elias [8] were able to show it is possible to use the Lienard-Wiechert description of FEL interaction to

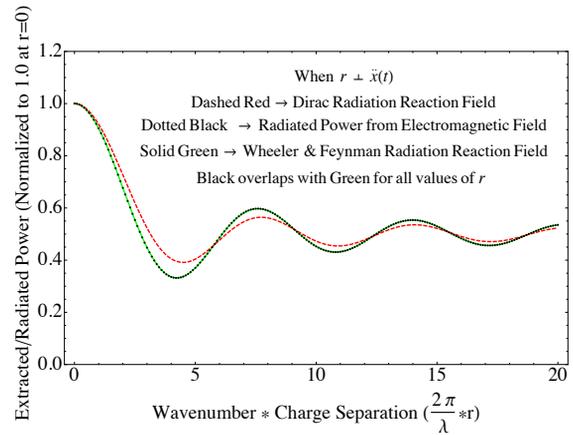


Figure 2: Comparison of the power extracted by the Dirac coherent radiation force and the amplitude of the Wheeler-Feynman coherent radiation force with the power radiated by the two oscillating charged particles' for displacements perpendicular to their vector accelerations (adapted from Reference [5]).

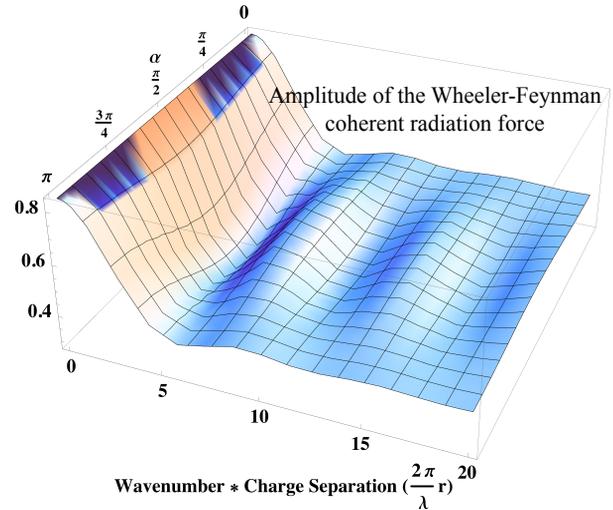


Figure 3: Amplitude of the Wheeler-Feynman coherent radiation reaction force for two oscillating charged particles' with displacement r and angle α (between the displacement and direction of oscillation).

include a viable model the coherent radiation reaction in covariant form valid for radiation into free-space.

In the Wheeler Feynman model, to calculate the effect of absorber in vicinity of an accelerating source charge, the retarded field of the source charge traveling outbound is first used to calculate the motions of absorber particles. Then the sum of the absorber's advanced fields near the source is calculated. Finally it is shown that the addition of this field to the half advanced plus half retarded field of the source gives the expected fully retarded field of the source, while also producing the correct radiation reaction force.

Applying the same principle to the beam traveling in a SASE FEL, we start by considering both the advanced and

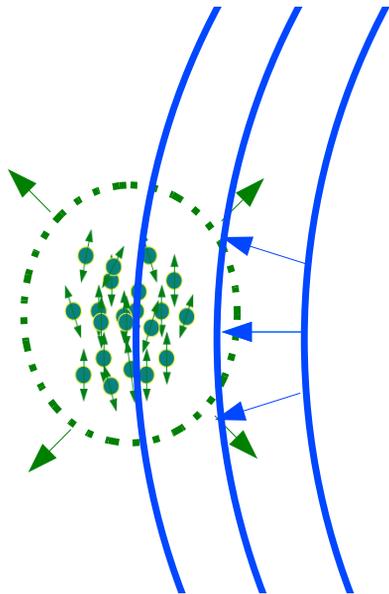


Figure 4: Diagram showing the advanced field (solid blue) of the absorber due to retarded field (dashed green) of emitting electrons acting on the oscillating electrons.

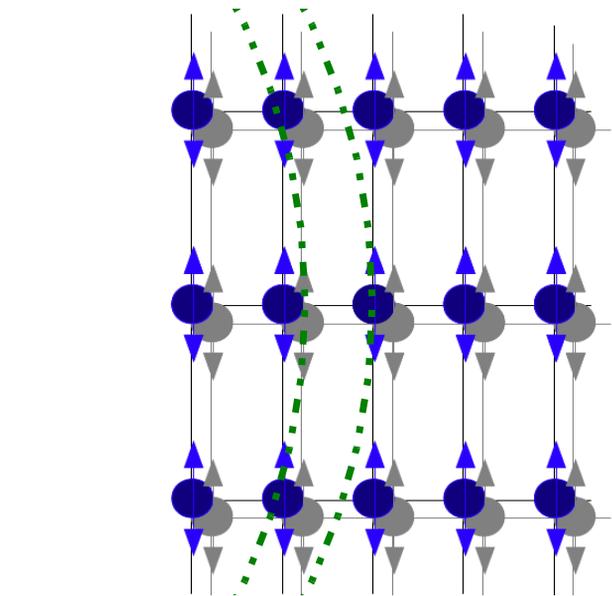
retarded field/potential of both the target (the absorber) and the electrons (the emitter). Now we must include half the retarded (outgoing) field of the emitters and the half the advanced (incoming) field of the target. The interaction of the advanced field of the target with the radiating electrons will insure energy conservation on the one hand, and on the other hand imposes the fields and forces initiated from the target on the source (Figure 4).

This feature allows for improving the spectral bandwidth and temporal coherence of the output of SASE FELs by manipulation of the target, as long as the target can resonate with the source and impose higher degrees of coherence.

MÖSSBAUER EFFECT

In order to measure and check the validity of this model it is essential to have a target that can verify the role of the absorber. In the past century, the Mössbauer effect has been used to investigate special electronic structures in matter, as well as biology and chemistry. In recent years, Mössbauer spectrometry has also been utilized for measurement of coherent scattering and diffraction of nuclear radiation [9]. Due to its nature, the Mössbauer absorber can be approximated stationary oscillating dipoles (Figure 4).

Since the energy distribution of the utilized radiation is extremely narrow, a Mössbauer absorber, for instance Fe^{57} above the Curie point [10], is an effective tool for high energy resonance spectroscopy. Including a Mössbauer absorber as part of the target of the SASE FEL will allow the advanced field of the target to have interactions with the emitting electrons and yield a narrow spectral bandwidth. Further analysis on use of Mössbauer solids as target for SASE FELs discusses the characteristic and advantages of concave target, one similar to the concave resonator mirrors



of a cavity FEL, for measurement of the coherence in these sources [11].

CONCLUSION

Here we discussed the results and implications of Wheeler-Feynman analysis of the interaction of a moving charge with distant absorbers. We showed that it provides a perfect match to the energy radiated by two coherently oscillating charged particles, and described the reasoning behind including the Wheeler-Feynman coherent radiation reaction force as an integral part of the solution of the boundary value problem of FELs that radiate into free-space. Finally we conclude that the Wheeler-Feynman model enables the development of a transparent first principles model of the instantaneous effects of boundary conditions on the radiation emitted by oscillating charges in systems subject to those boundary conditions enabling both the attainment of energy conservation and the modification of the relevant boundary conditions to optimize system performance.

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