MaRIE

(Matter-Radiation Interactions in Extremes)

Controlling the Emittance Partitioning of High-Brightness Electron Beams

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Overview

- LANL motivation for an X-ray FEL and electron beam requirements – hard X-ray FELs require tiny emittances
- Using eigen-emittances to increase beam quality
- Flat-beam transforms (FBTs) and other eigen-emittance applications
- Four eigen-emittance schemes with potential to achieve very low emittances

MaRIE

(**Ma**tter-**R**adiation **I**nteractions in **E**xtremes)

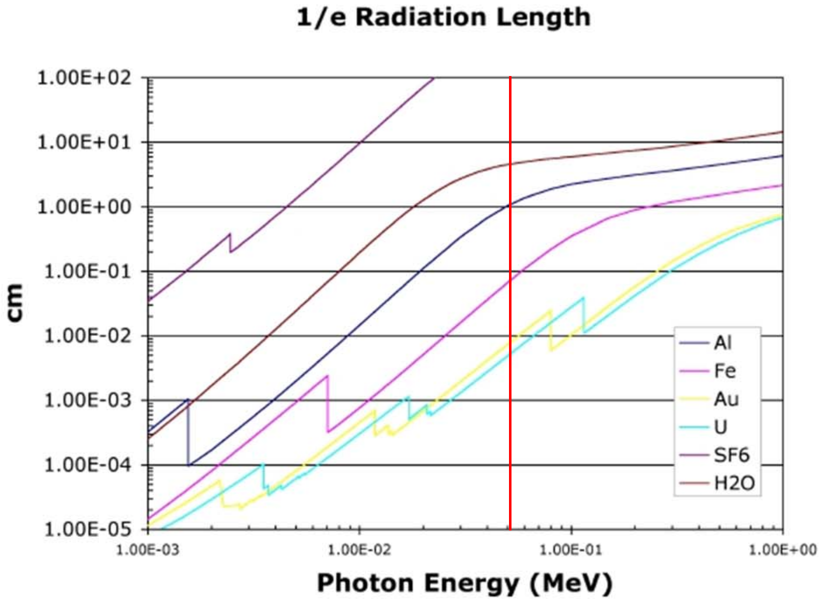
- The Multi-probe Diagnostic Hall will *provide unprecedented probes of matter.*
  - X-ray scattering capability at high energy and high repetition frequency with simultaneous proton imaging.
- The Fission and Fusion Materials Facility will *create extreme radiation fluxes.*
  - Unique in-situ diagnostics and irradiation environments comparable to best planned facilities.
- The M4 Facility dedicated to making, measuring, and modeling materials will *translate discovery to solution.*
  - Comprehensive, integrated resource for controlling matter, with national security science infrastructure.

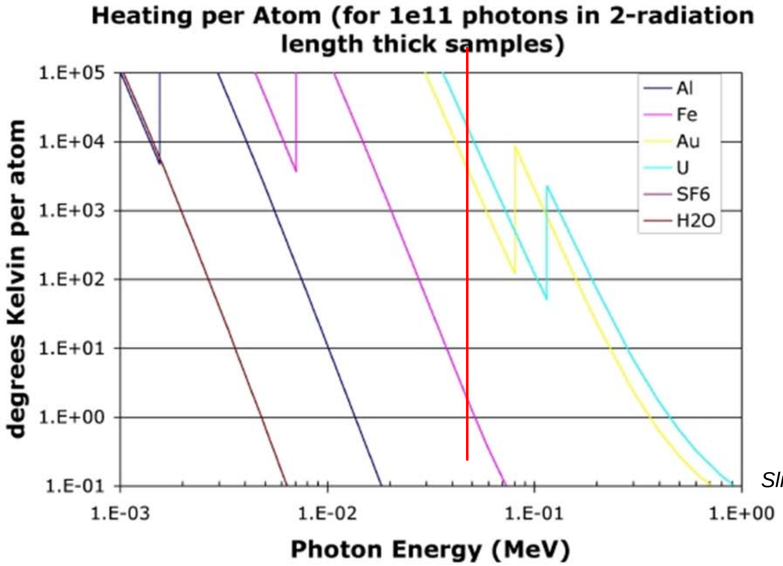
LANSCE Accelerator

Why 50 keV XFEL?

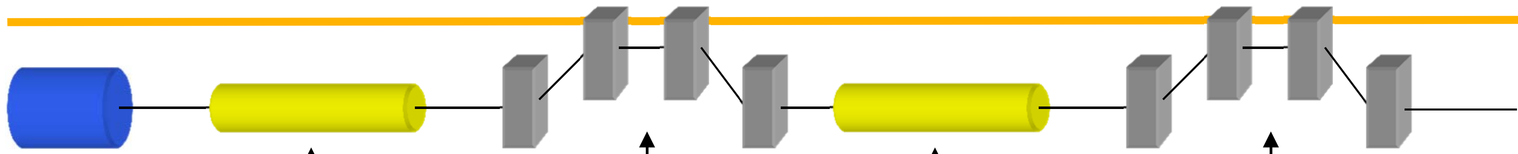
MaRIE seeks to probe *inside* multigranular samples of condensed matter that represent bulk performance properties with sub-granular resolution. With grain sizes of tens of microns, "multigranular" means 10 or more grains, and hence samples of few hundred microns to a millimeter in thickness. For medium-Z elements, this requires photon energy of 50 keV or above.

This high energy also serves to reduce the absorbed energy per atom per photon in the probing, and allows multiple measurements on the same sample. Interest in studying transient phenomena implies very bright sources, such as an XFEL.

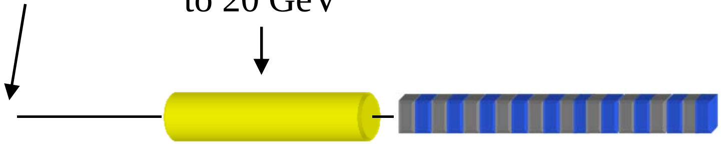


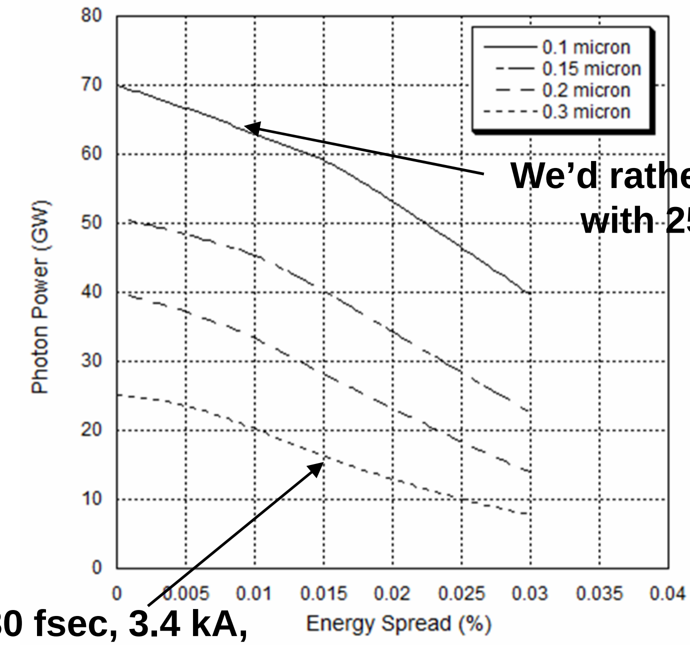


Baseline Design is at 100 pC Because of Brightness Limitations Using Current Technology



$$\varepsilon_n \sim 1\ \mu m (q/\mathrm{nC})^{1/2}$$



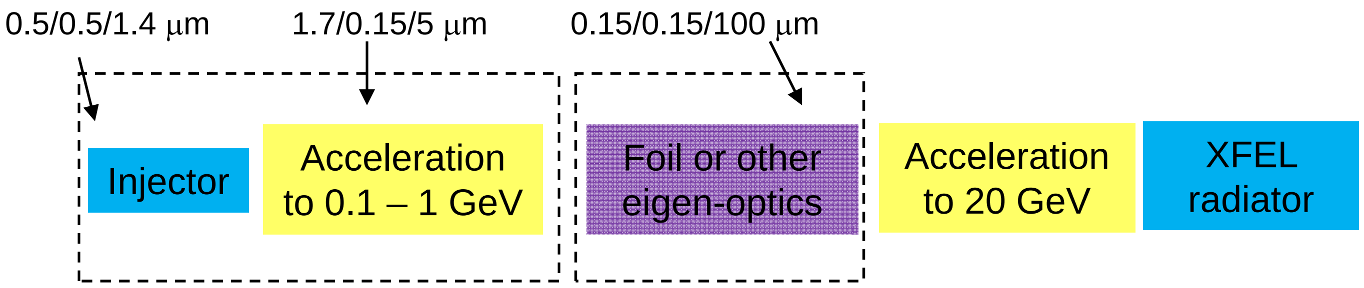


We'd rather be here with 250 pC

100 pC, 30 fsec, 3.4 kA, 0.015% energy spread 0.30 μm emittance

Duffy, TUPA28

Concept of Emittance Partitioning to Increase Charge and Decrease Emittance (and one possible implementation)



There is enough "spare" area in the longitudinal phase space to move excess area from the transverse phase spaces (250 pC numbers above)

The key controlling feature is how small the longitudinal energy spread can be kept; there is likely some significant overall increase in total volume

Two-stage approach (shown) might work, has significant advantages for maintaining brightness in photoinjector

Eigen-Emittance Concept Can Be Used To Control Phase Space Partitioning

- Let σ denote the beam second moment matrix

$$J = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & -1 & 0 \end{pmatrix}$$

- The eigenvalues of $J\sigma$ are called eigen-emittances
- Eigen-emittances are invariant under all *linear symplectic* transformations, which include all ensemble electron beam evolution in an accelerator

  however, the eigen-emittances can be *exchanged* among the x-p_x, y-p_y, z-p_z phase planes

- We can control the formation of the eigen-emittances by controlling correlations when the beam is generated (demonstrated in Flat-Beam Transforms (FBTs))
- *We recover the eigen-emittances as the beam rms emittances when all correlations are removed*

We Use Consistent Units to Describe Eigen-Emittances

$$\varsigma_{can}^{T} = (x, p_x, y, p_y, t, p_t)$$

$$\vec{p} = \vec{p}_{mech} + q\vec{A} \quad p_t = -\gamma mc^2$$

Canonical variables with arbitrary normalization

$$\varsigma_{can}^{T} = (x, (\gamma\beta_x/\gamma_0\beta_0), y, (\gamma\beta_y/\gamma_0\beta_0), c\Delta t, (\Delta\gamma/\gamma_0\beta_0))$$

or: $$\varsigma^{T} = (x, x', y, y', c\Delta t, \Delta(\gamma\beta)/\gamma_0)$$

Canonical variables with the "proper" (traditional) normalization

We use symplectic transformations along beamline:

$$\sigma_2 = R\sigma_1 R^T$$

$$\vec{\varsigma}_2 = R\vec{\varsigma}_1$$

$$J_6 = R^T J_6 R$$

$$J_6 = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & -1 & 0 \end{pmatrix}$$

What does Symplectic Mean in an RMS or Linear Sense?

- Lorentz force law follows from a Hamiltonian:

$$H = c\sqrt{\left(\vec{p} - q\vec{A}(\vec{r},t)\right)^2 + m^2c^2} + q\phi(\vec{r},t)$$

- All electrodynamic motion satisfies Liouville's theorem

- If the Hamiltonian is quadratic in beam coordinates (transformation is lienar), then

$$J_6 = R^T J_6 R$$

- If the Hamiltonian is higher order in beam coordinates, the *rms* symplectic condition no longer follows:

$$J_6 \neq R^T J_6 R$$

We Use a Similar Formalism to Define Correlations

We define a correlation matrix *C*:

$$\sigma_{corr} = (I+C)\sigma_0(I+C)^T$$

$$\det(I+C) = 1$$

$$C_1 = \begin{pmatrix} 0 & 0 & 0 & c_{14} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad \sigma_{C1} = \begin{pmatrix} \sigma_x^2 + c_{14}^2\sigma_{y'}^2 & 0 & 0 & c_{14}\sigma_{y'}^2 \\ 0 & \sigma_{x'}^2 & 0 & 0 \\ 0 & 0 & \sigma_y^2 & 0 \\ c_{14}\sigma_{y'}^2 & 0 & 0 & \sigma_{y'}^2 \end{pmatrix}$$

$$C_2 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ c_{14} & 0 & 0 & 0 \end{pmatrix} \quad \sigma_{C2} = \begin{pmatrix} \sigma_x^2 & 0 & 0 & c_{41}\sigma_x^2 \\ 0 & \sigma_{x'}^2 & 0 & 0 \\ 0 & 0 & \sigma_y^2 & 0 \\ c_{41}\sigma_x^2 & 0 & 0 & \sigma_{y'}^2 + c_{41}^2\sigma_x^2 \end{pmatrix}$$

We can stack correlations multiplicatively. The order doesn't necessarily commute.

$$I + C_{total} = \prod_{i=1}^{n}(I + C_i)$$

PRSTAB 14, 050706, 2011, upcoming *NIMA* article

Examples of Symplectic and Non-Symplectic Correlations

Axial field on the cathode (magnetized photoinjector) is an example of a nonsymplectic correlation (once the beam leaves the field region)

$$I + C_{total} = \left(I + \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -a & 0 & 0 & 0 \end{pmatrix} \right) \left(I + \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & a & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \right) = \left(I + \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & a & 0 \\ 0 & 0 & 0 & 0 \\ -a & 0 & 0 & 0 \end{pmatrix} \right)$$

$$\mathbf{L} = \left| \langle xy' - yx' \rangle \right| / 2 = |a| \left(\sigma_x^2 + \sigma_y^2 \right) / 2$$

$$a = \frac{e}{2\gamma\beta mc} B_{cath} \left(\frac{R_{cath}}{R_{beam}} \right)^2$$

$$\sigma_{axial\ \ field} = \begin{pmatrix} \sigma_x^2 & 0 & 0 & -a\sigma_x^2 \\ 0 & \sigma_{x'}^2 + a^2\sigma_y^2 & a\sigma_y^2 & 0 \\ 0 & a\sigma_y^2 & \sigma_y^2 & 0 \\ -a\sigma_x^2 & 0 & 0 & \sigma_{y'}^2 + a^2\sigma_x^2 \end{pmatrix}$$

A skew-quad is an example of a symplectic transformation:

$$R_{skew} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & a & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ a & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\sigma_2 = R_{skew} \sigma_0 R_{skew}^T = \begin{pmatrix} \sigma_x^2 & 0 & 0 & a\sigma_x^2 & 0 & 0 \\ 0 & \sigma_{x'}^2 + a^2\sigma_y^2 & a\sigma_y^2 & 0 & 0 & 0 \\ 0 & a\sigma_y^2 & \sigma_y^2 & 0 & 0 & 0 \\ a\sigma_x^2 & 0 & 0 & \sigma_{y'}^2 + a^2\sigma_x^2 & 0 & 0 \\ 0 & 0 & 0 & 0 & \sigma_z^2 & 0 \\ 0 & 0 & 0 & 0 & 0 & \sigma_{z'}^2 \end{pmatrix}$$

Easy to Write 4-D Eigen-Emittance Solution

Can find the eigen-emittances using the conservation of the 4-D determinant and of the "Raj" trace (KJK PRSTAB 6, 104002, 2003)

$$-\frac{1}{2}Tr(J\sigma J\sigma)$$

We can always make beam waists, eigen-emittances are then:

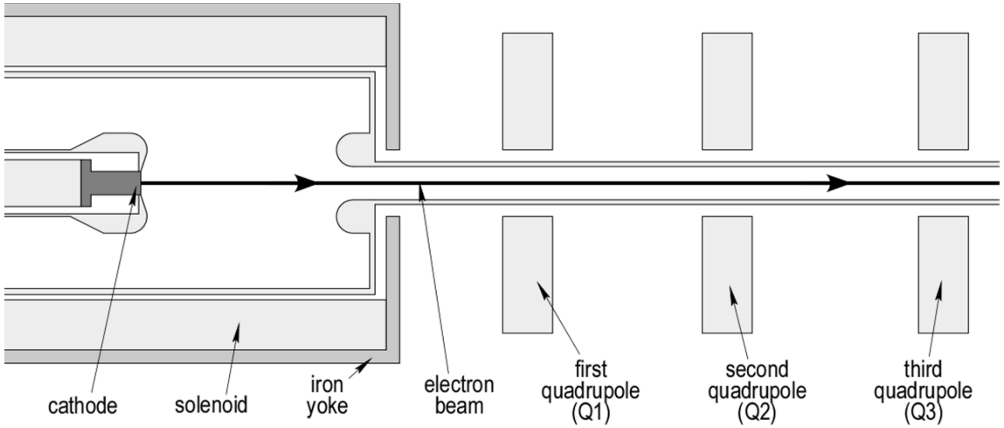
$$\sigma_{beam} = \begin{pmatrix} \bar{\sigma}_1^2 & 0 & D & B \\ 0 & \bar{\sigma}_2^2 & E & F \\ D & E & \bar{\sigma}_3^2 & 0 \\ B & F & 0 & \bar{\sigma}_4^2 \end{pmatrix}$$

$$\boxed{\varepsilon^2_{eig,\pm} = U \pm V}$$

where:

$$U = \frac{1}{2}\left(\bar{\sigma}_1^2\bar{\sigma}_2^2 + \bar{\sigma}_3^2\bar{\sigma}_4^2 - 2BE + 2FD\right) \quad V^2 = \frac{1}{4}\left(\bar{\sigma}_1^2\bar{\sigma}_2^2 + \bar{\sigma}_3^2\bar{\sigma}_4^2 - 2BE + 2FD\right)^2 - \left(\bar{\sigma}_1^2\bar{\sigma}_2^2\bar{\sigma}_3^2\bar{\sigma}_4^2 - F^2\bar{\sigma}_1^2\bar{\sigma}_3^2 - E^2\bar{\sigma}_1^2\bar{\sigma}_4^2 - D^2\bar{\sigma}_2^2\bar{\sigma}_4^2 - B^2\bar{\sigma}_2^2\bar{\sigma}_3^2 + D^2F^2 + E^2B^2 - 2EBDF\right).$$

The Flat Beam Transform (FBT) is a 2-D Example



$$\sigma_{axial\ field} = \begin{pmatrix} \sigma_x^2 & 0 & 0 & -a\sigma_x^2 \\ 0 & \sigma_{x'}^2 + a^2\sigma_x^2 & a\sigma_x^2 & 0 \\ 0 & a\sigma_x^2 & \sigma_x^2 & 0 \\ -a\sigma_x^2 & 0 & 0 & \sigma_{x'}^2 + a^2\sigma_x^2 \end{pmatrix}$$

Observed emittances: $\varepsilon_{beam} = \sqrt{\varepsilon_0^2 + \mathbf{L}^2}$

$$\mathbf{L} = \frac{e|B_{cath}|R_{cath}^2}{8\gamma\beta cm} = |a|\sigma_x^2 = \frac{1}{2}\left|\langle xy' - yx' \rangle\right|$$

Eigen-emittances: $\varepsilon_{eig,-} = \frac{\varepsilon_0^2}{2\mathbf{L}}$

$\mathbf{L}^2 >> \varepsilon_0^2$

$\varepsilon_{eig,+} = 2\mathbf{L}$

FBT is protected from nonlinearities by symmetry and conservation of canonical angular momentum

These are always zero

$$\sigma_{XY} = \begin{pmatrix} \langle xy \rangle & \langle xy' \rangle \\ \langle x'y \rangle & \langle x'y' \rangle \end{pmatrix}$$

So you can always define intrinisic emittances so the FBT equations hold

$$\varepsilon_0 = \sqrt{\varepsilon_{beam}^2 - \mathbf{L}^2}$$

We Have Thought About Four Ways to Get Low Emittances

1. Thin pancake with axial field (KJK, AAC08)

2. Asymmetric beam with laser tilt

3. Magnetized photoinjector and nonsymplectic foil/undulator (using ISR or Bremstrahlung)

4. General three-dimensional couplings

We are currently evaluating these options

We typically consider an "ideal" photoinjector with nominal emittances (x,y,z) of 0.5/0.5/1.4 μm, with target eigen-emittances of 0.15/0.15/90 μm (250 pC), but 4:1 ratio in final transverse emittances almost as good

The problem comes down to how low the energy spread (and longitudinal emittance) can be maintained

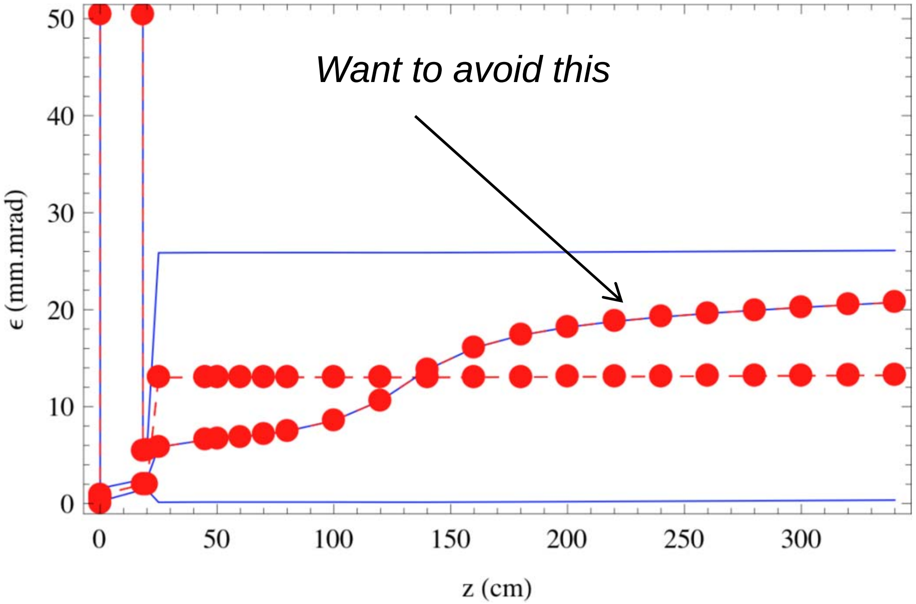
Super-Thin Pancake Off Photocathode

1. Start with a super-short pancake of charge, emittances of 1.5/1.5/0.15 μm, all in a magnetized photoinjector

2. Use a FBT to adjust these numbers to 0.15/15/0.15 μm

3. Use an EEX to swap y and z and end up with 0.15/0.15/15 μm

Problem with this approach is that the phase space volume is not conserved in conventional photoinjectors (initial product increases with pancake shape)

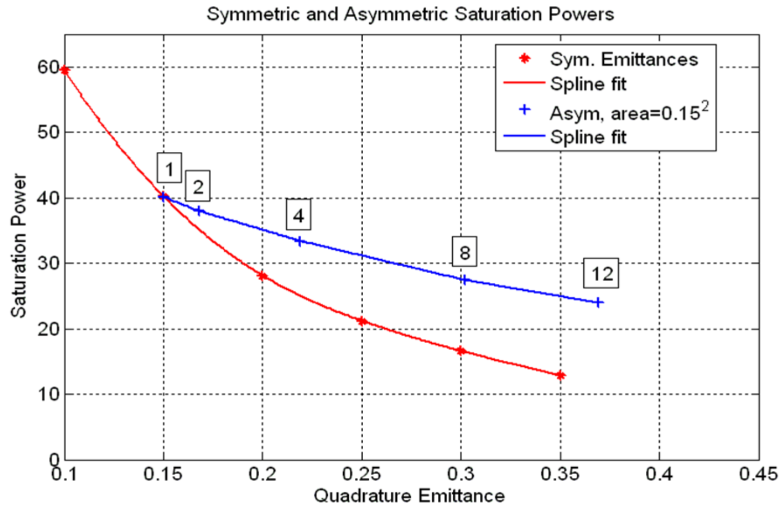


Asymmetric Beam with Titled Drive Laser

1. Start with 5.3:1 ellipticity at cathode (1.61/0.3/1.4 μm) (500 pC)

2. Use a 83° laser tilt (2.3/0.43-mm radius cathode, 3.3-psec long pulse)

3. Eigen-emittances are: 0.075/0.3/30 μm, about 15% decrease in x-ray flux:



Problem with this approach is that there is no conservation property that helps us and space charge nonlinearities may be an issue, we're studying this. Initial simulations (IMPACT-T) show the product of the transverse emittances is mostly conserved but complicated.

Magnetized Photoinjector and Nonsymplectic Element

1. Start with round beam at cathode (0.5/0.5/1.4 μm)

2.FBT in the usual way gives 1.7/0.15/1.4 μm (or 1.0/0.25/1.4 μm)

3.Can use ISR from an undulator or wedge-shaped foil to generate correlation between *x* and energy (transverse beam size ~ cm, undulator length ~ m)

ISR: $\Delta E[\mathrm{MeV}] = 6.31\,10^{-4}\ E^2[\mathrm{GeV}]\ B^2[\mathrm{T}]\ L[\mathrm{m}]$ leads to too long undulators if under a few GeV, wedge-shaped foil may work at lower electron beam energies

4. Use a wedge-shaped foil at 1 GeV to provide roughly 100 keV more attenuation at one horizontal end of the beam than the other, final eigen-emittances might be 0.25/0.25/90 μm (there is an emittance hit)

Problem with this approach is that there is both a transverse emittance growth and an energy spread (both from scattering), but it looks promising

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REDUCTION OF BEAM EMITTANCE BY A TAPERED-FOIL TECHNIQUE*

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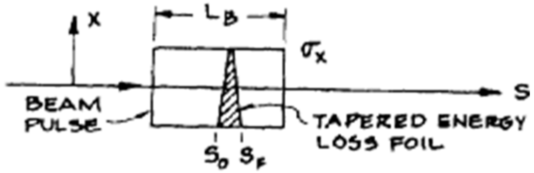


Figure 1. Geometry of the Tapered Energy-Loss Foil

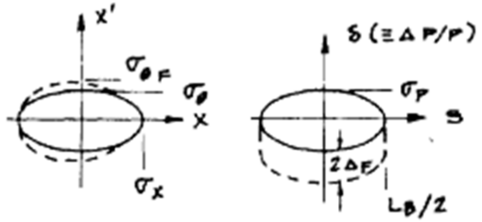


Figure 2. Transverse and Longitudinal Phase Spa Occupied by Beam Before (solid) and After (dashed) the Tapered Energy-Loss Foil.

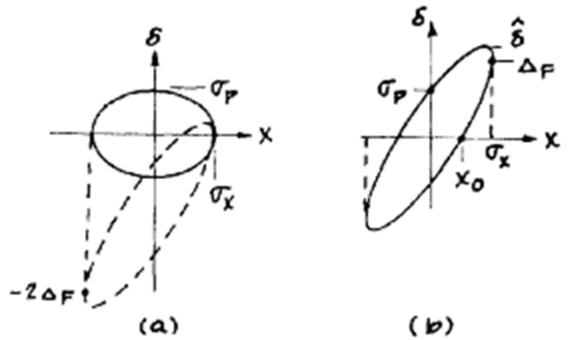


Figure 3. (a) x,δ Phase Space Occupied by Beam Before (solid) and After (Dashed) the Tapered Foil (b) After the Foil with Momentum Renormalized to the Center Momentum.

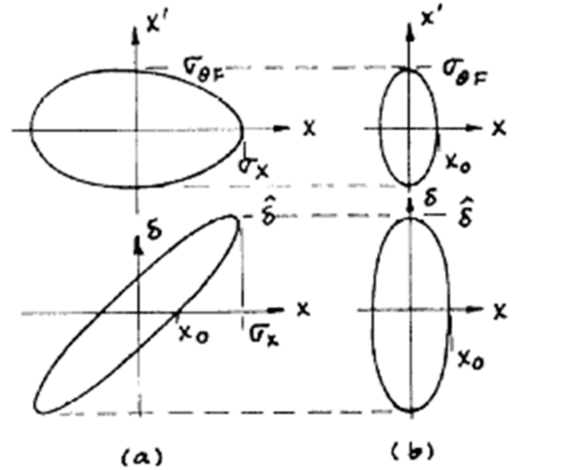


Figure 5. x,x' and x,δ Phase Space Occupied (a) At S_F, the Exit Side of the Tapered Foil and (b) At S1, the Image Point with Zero Dispersion.

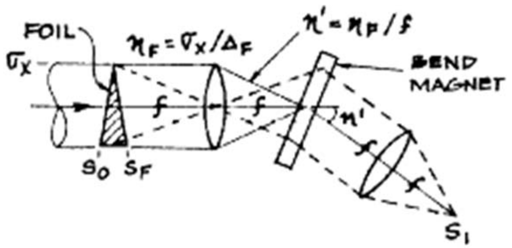


Figure 4. One-Dimensional Beam Transport System that Removes Dispersion and Has One-to-One Imaging of S_F at S_1. Focal length of each lens is f.

Peterson's optics has the same horizontal emittance reduction but does not recover the third eigen-emittance (we probably need both). Peterson's optics also point out the value of an alternative x'-z' transform.

First reported by Claud Bovet LBL-ERAN89, June 1970.

Foil Idea May Work, Stimulating Other Concepts

We nominally start with a magnetized photoinjector to get $\varepsilon_{x,n} / \varepsilon_{y,n} / \varepsilon_{z,n}$ = 4.0/0.25/1.4 μm at 1 nC

Non-symplectic element separates issues and simplifies design.

Induced angular scattering and increased energy spread limit effectiveness, still might get factors of ten improvement

Intrinsic energy spread and emittance

$$\varepsilon_{x,\,final} = \frac{\left(\left(\frac{\Delta\gamma}{\gamma}\right)_{ind}^{2} + \left(\frac{\Delta\gamma}{\gamma}\right)_{int}^{2}\right)^{1/2}}{\left(\frac{\Delta\gamma}{\gamma}\right)_{slew}} \left(\varepsilon_{ind}^{2} + \varepsilon_{x,int}^{2}\right)^{1/2} \qquad \varepsilon_{z,\,final} = \gamma\left(\frac{\Delta\gamma}{\gamma}\right)_{slew} \sigma_z$$

You can do an exact eigen-emittance recovery, if you wish, but it's hard, prone to second-order effects, and you don't need to – simple asymmetric chicane works fine

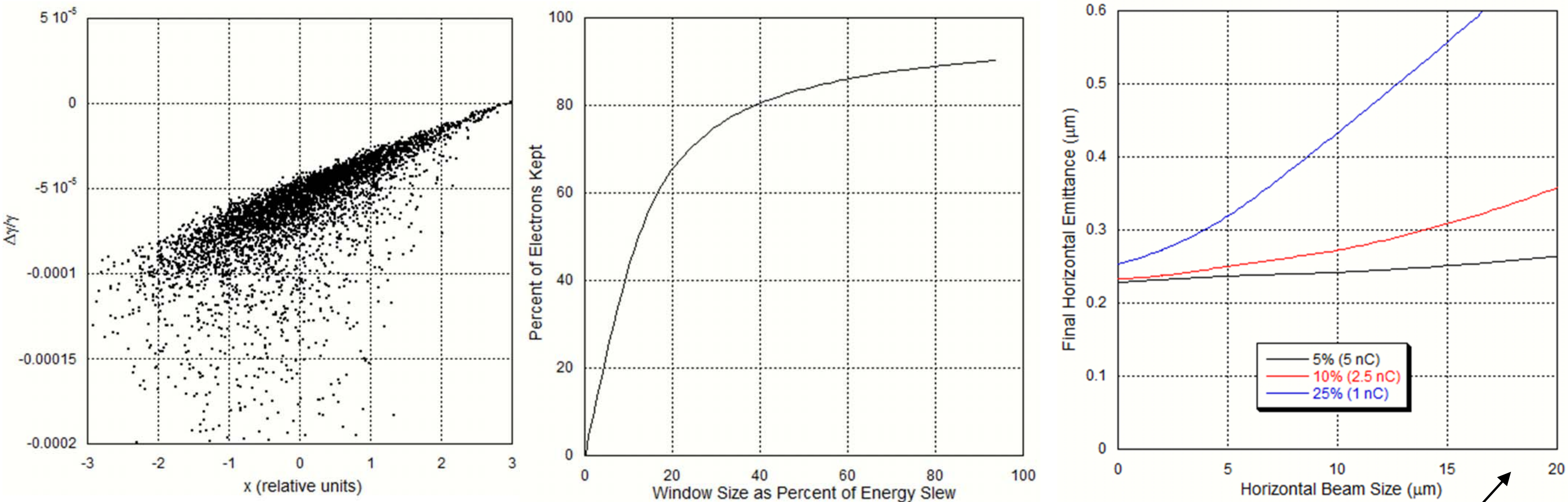
$$M_{\text{s-chicane}} = \begin{pmatrix} 1 & L_1 & 0 & \eta_1 \\ 0 & 1 & 0 & 0 \\ 0 & \eta_1 & 1 & \varepsilon_1 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & L_2 & 0 & -\eta_2 \\ 0 & 1 & 0 & 0 \\ 0 & -\eta_2 & 1 & \varepsilon_2 \\ 0 & 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & L_t & 0 & \Delta\eta \\ 0 & 1 & 0 & 0 \\ 0 & \Delta\eta & 1 & \varepsilon_t \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$L_t = L_1 + L_2$$
$$\Delta\eta = \eta_1 - \eta_2$$
$$\varepsilon_t = \varepsilon_1 + \varepsilon_2$$
$$\Delta\eta = 0.049 \text{ m}$$

$$\varepsilon_x^2 \varepsilon_z^2 = \varepsilon_{x0}^2 \varepsilon_{z0}^2 + \eta^2 \left(\varepsilon_{x0}^2 + \varepsilon_{z0}^2\right)\left\langle x_0'^2 \right\rangle\left\langle z_0'^2 \right\rangle + \eta^4 \left\langle x_0'^2 \right\rangle^2 \left\langle z_0'^2 \right\rangle^2$$

The growth in the product of the emittances of only about 1%.

Wedge Foil Results with 250 pC of Final Charge



Requires some amount of scraping. Fairly insensitive to fraction kept (20% or less), energy (100 MeV to 1 GeV), and factors of a few for magnitude of energy slew (emittance target is 0.25/0.25/90 μm).

1-GeV case, wedge is 80 μm by ~400 μm

Dominated by beam's intrinsic slice energy spread.

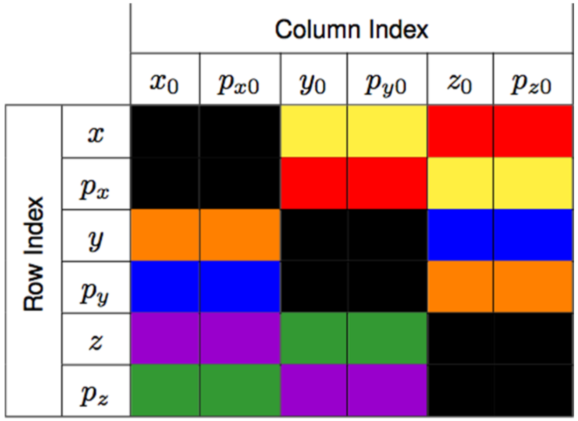
(More details at Bishofberger, THPB18)

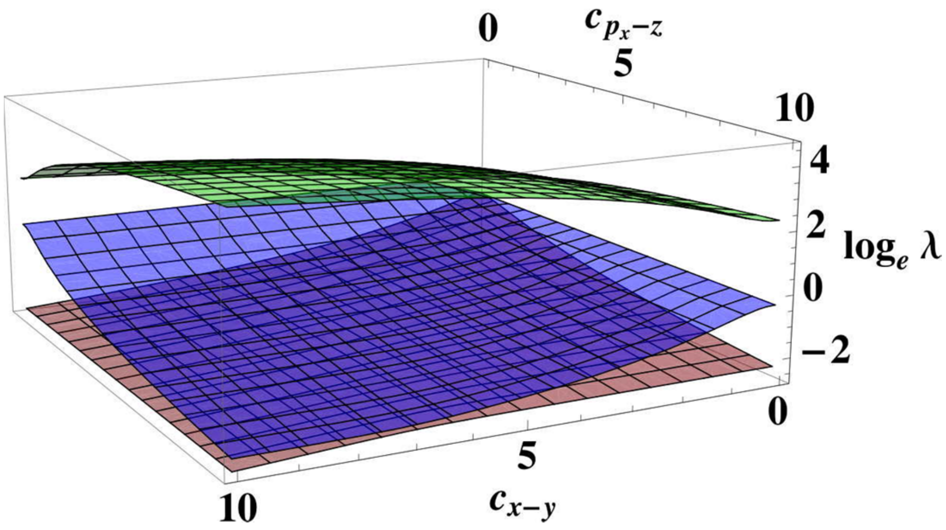
You Can Also Consider General 6-D Couplings

1. Start with round beam at cathode (0.5/0.5/1.4 μm)

2. Pick combination where row index is function of column index; issue here is to identify some combinations that are least sensitive to photoinjector nonlinearities, ongoing research

3. We have developed an algorithm to determine what combination of 3 and more correlations lead to 2 small eigen-emittances (Duffy et al, NIMA in press)





Summary

- Future XFEL designs will require higher brightness electron beams
- Exploiting eigen-emittances may lead to a new way of achieving very low transverse emittances by moving excess transverse phase space into the longitudinal dimension
- Two-stage generation of beam correlations (using a non-symplectic beamline element) may be a practical application of eigen-emittances
- Asymmetric beams/multiple initial correlations may also lead to practical applications
- Eigen-emittance recovery optics don’t have to exactly diagonalize beam matrix