SASE FEL PULSE DURATION ANALYSIS FROM SPECTRAL CORRELATION FUNCTION

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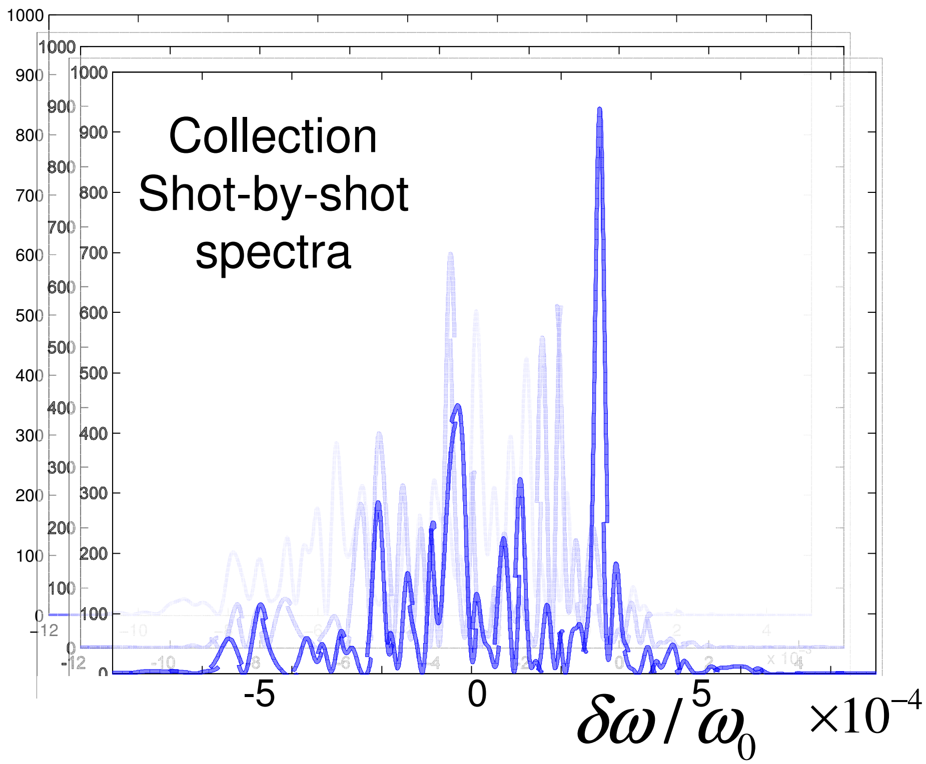
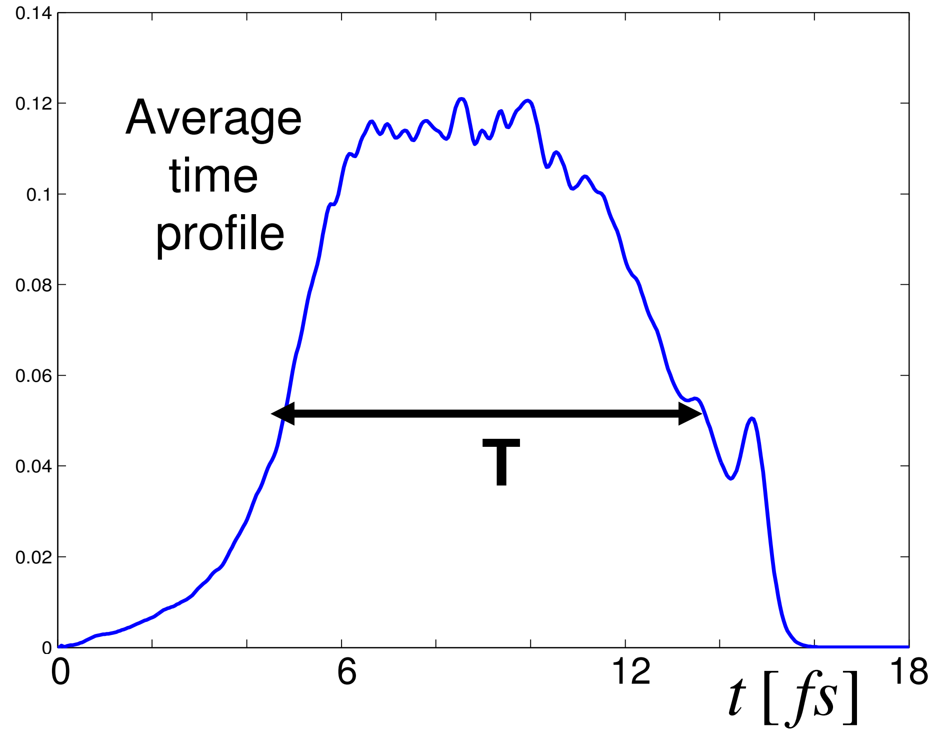
X-ray pulse duration measurement





We recover the average x-ray time profile length **T** from a collection of single shot spectra

Model description

Electron beam current:

$$I(t) = (-e)\sum_{k=1}^{N} \delta(t - t_k)$$

t_k are independent random variables with probability density $\mathrm{f}(t)$

SASE FEL Amplifier in linear regime:

$$E(t) = \int_{-\infty}^{+\infty} h(t,\tau) I(\tau) d\tau = \int_{-\infty}^{+\infty} h_{ti}(t-\tau) h_{td}(\tau) I(\tau) d\tau$$

$$h_{ti}(t-\tau) = A_0(z) e^{i(k_0 z - \omega_0 (t-\tau))} e^{-\frac{(t-\tau-z/v_g)^2}{4\sigma_t^2}\left(1+\frac{i}{\sqrt{3}}\right)}$$ time independent

$h_{td}(\tau)$ time dependent

Model description

Electric field spectrum:

$$\tilde{E}(\omega) = (-e)\tilde{H}_{ti}(\omega)\sum_{k=1}^{N} h_{td}(t_k)e^{i\omega t_k}$$

Average x-ray profile:

$$X(t) = \left|h_{td}(t)\right|^2 \mathrm{f}(t)$$

First and second order correlations

$$\left\langle \tilde{E}(\omega')\tilde{E}^*(\omega'')\right\rangle = e^2 N\tilde{H}_{ti}(\omega')\tilde{H}_{ti}(\omega'')\tilde{X}(\omega'-\omega'')$$

$$\left\langle \left|\tilde{E}(\omega')\right|^2 \left|\tilde{E}(\omega'')\right|^2 \right\rangle = e^4 N^2 \left|\tilde{H}_{ti}(\omega')\right|^2 \left|\tilde{H}_{ti}(\omega'')\right|^2 \left(\tilde{X}(0) + \left|\tilde{X}(\omega'-\omega'')\right|^2\right)$$

Model description

First and second order correlations functions

$$g_1(\omega',\omega'') = \frac{\left\langle \tilde{E}(\omega')\tilde{E}^*(\omega'')\right\rangle}{\sqrt{\left\langle\left|\tilde{E}(\omega')\right|^2\right\rangle\left\langle\left|\tilde{E}(\omega'')\right|^2\right\rangle}} \qquad g_2(\omega',\omega'') = \frac{\left\langle\left|\tilde{E}(\omega')\right|^2\left|\tilde{E}(\omega'')\right|^2\right\rangle}{\left\langle\left|\tilde{E}(\omega')\right|^2\right\rangle\left\langle\left|\tilde{E}(\omega'')\right|^2\right\rangle}$$

$$\left|g_1(\omega',\omega'')\right| = \left|\tilde{X}(\omega'-\omega'')\right| \Big/ \tilde{X}(0)$$

Valid in the linear regime

$$g_2(\omega',\omega'') = 1 + \left|g_1(\omega',\omega'')\right|^2$$

For non-linear regime, we run numerical simulations

Model description

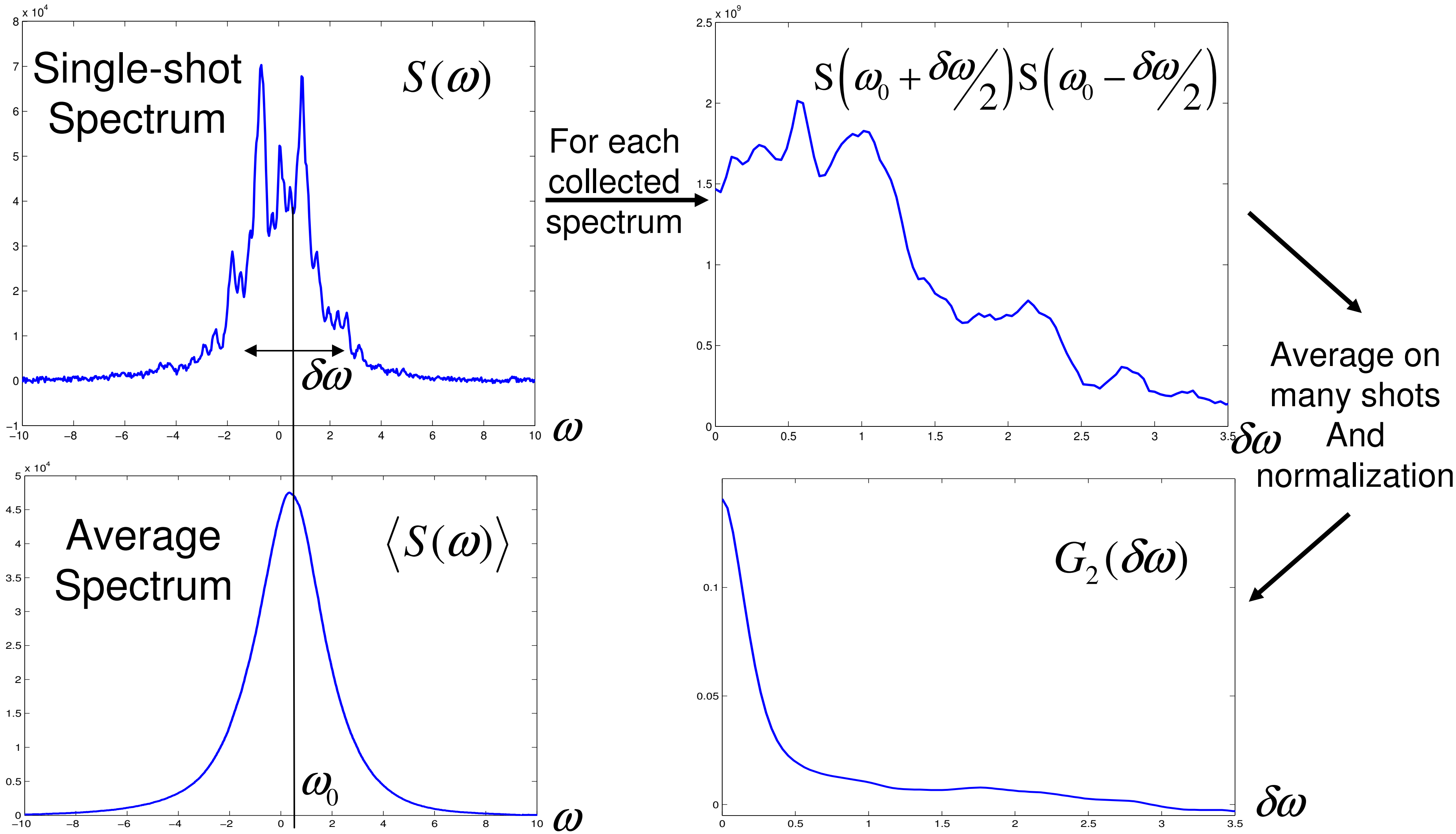
Correlation of the intensity at the exit of the spectrometer

$$G_2(\delta\omega) = \frac{\left\langle S\left(\omega_0 + \delta\omega/2\right) S\left(\omega_0 - \delta\omega/2\right)\right\rangle}{\left\langle S\left(\omega_0 + \delta\omega/2\right)\right\rangle \left\langle S\left(\omega_0 - \delta\omega/2\right)\right\rangle} - 1$$

$S(\omega)$ Intensity spectrum at frequency ω

ω_0 Central frequency of amplification

Procedure to calculate G_2



G_2 function for different X(t) profiles

$$\left|\tilde{H}_{ti}(\omega)\right|^2 \propto e^{-\frac{(\omega-\omega_0)^2}{2\sigma_a^2\omega_0^2}}$$

σ_a relative FEL bandwidth

We assume that the spectrometer has Gaussian resolution function

σ_m relative rms spectrometer resolution

$$G_2(\delta\omega) = \int_{-\infty}^{+\infty} \frac{e^{-\frac{(\zeta-\delta\omega\xi_0)^2}{2\sigma^2}}\left|\tilde{X}(\zeta,T)\right|^2}{\sqrt{2\pi}\sigma} d\zeta$$

$$\xi_0 = \frac{\sigma_a^2}{\sigma_m^2+\sigma_a^2}\omega_0$$

$$\sigma = \sqrt{2}\frac{\sigma_a\sigma_m}{\sqrt{\sigma_m^2+\sigma_a^2}}\omega_0$$

To find an analytical expression for G_2 we need just to plug in the *X*(*t*) average profile

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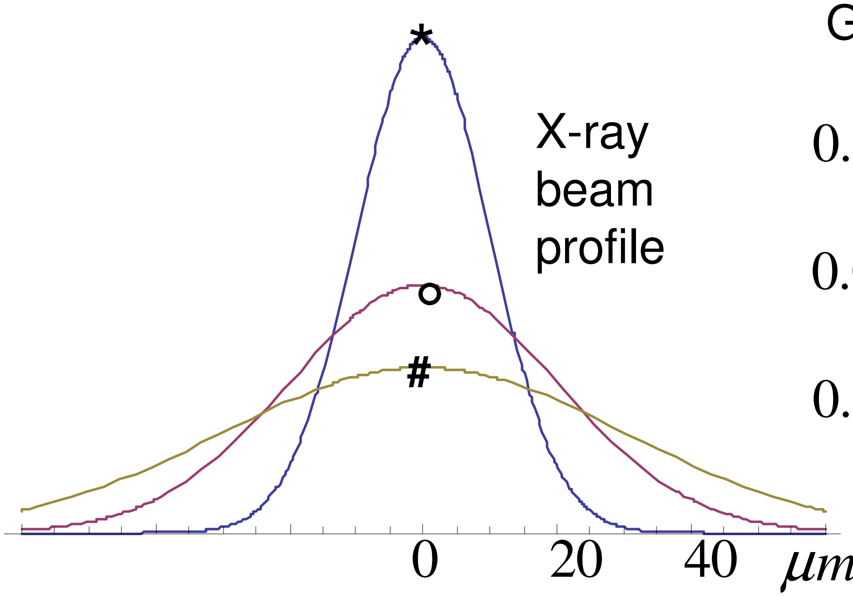
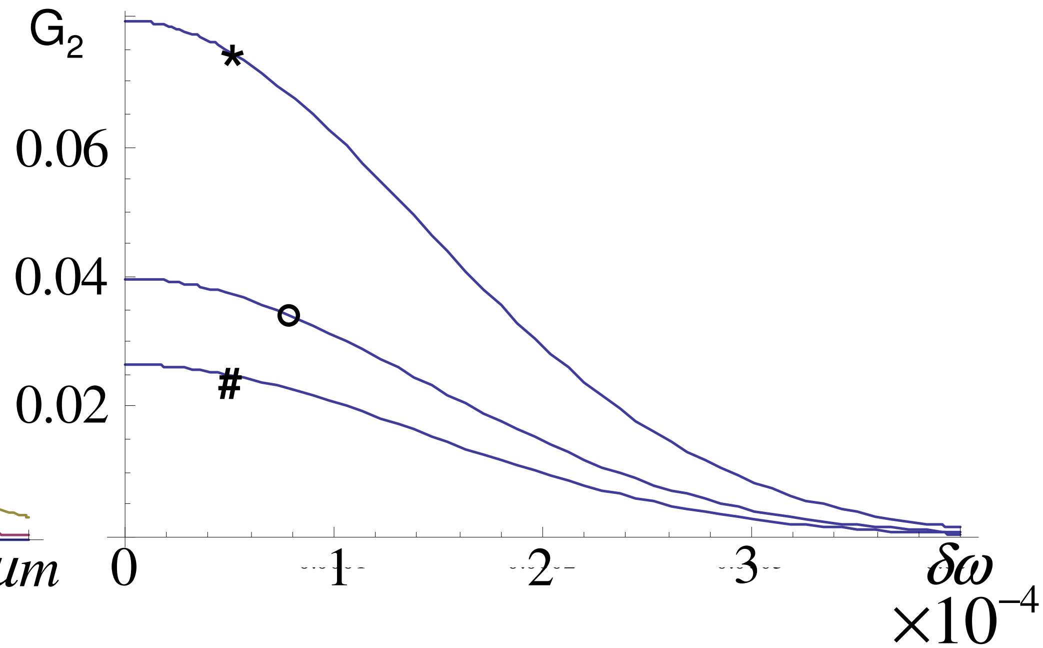
G_2 with Gaussian profile

$$X(t,\sigma_T) = \frac{e^{-\frac{t^2}{2\sigma_T{}^2}}}{\sqrt{2\pi}\sigma_T}$$

$$G_2(\delta\omega) = \frac{e^{-\frac{\delta\omega^2\xi_0^2\sigma_t^2}{1+2\sigma^2\sigma_t^2}}}{\sqrt{1+2\sigma^2\sigma_t^2}}$$

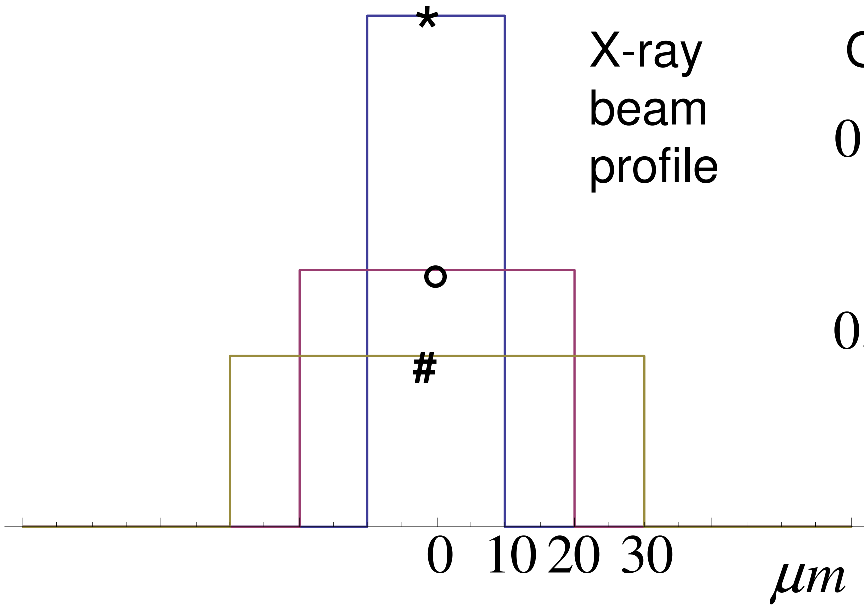


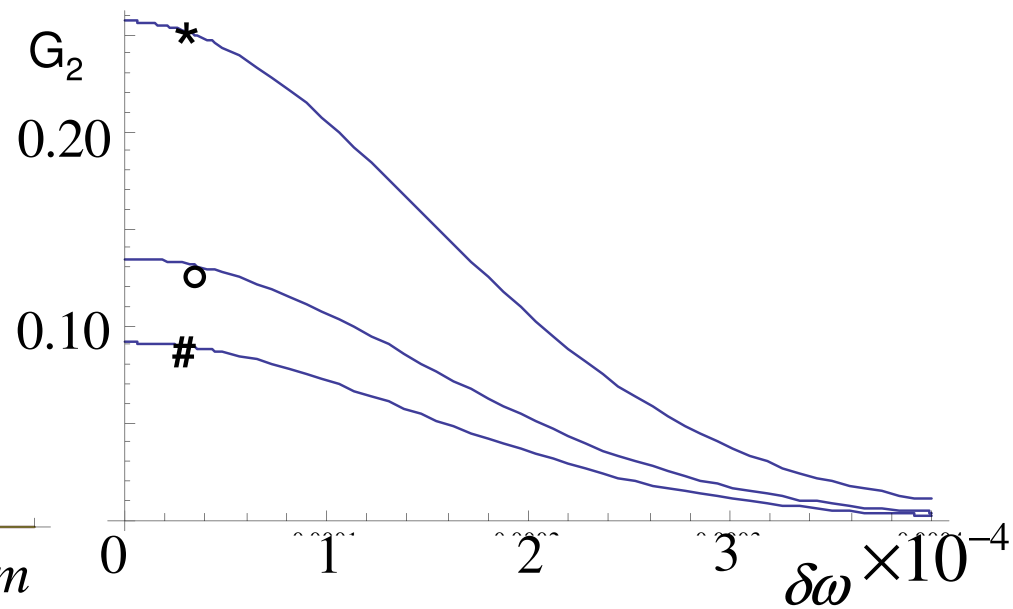


G_2 with flat top profile

$$X(t,T)=\begin{cases} 1/T & |t|\le T/2 \\ 0 & |t|>T/2 \end{cases}$$

$$G_2(\delta\omega)=2\int_0^1 e^{-\frac{\zeta^2\sigma^2T^2}{2}}(1-\zeta)\cos(\delta\omega\xi_0 T\zeta)\,d\zeta$$





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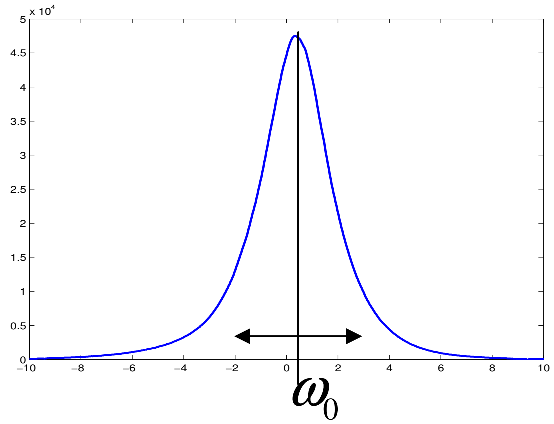
Non stationary ω_0 and FEL gain jitter



Central frequency jitters with Gaussian law with rms $\sigma_\omega \omega_0$

$$G_2(\delta\omega) = K(\delta\omega)\int_{-\infty}^{+\infty} \frac{e^{-\frac{(\zeta-\delta\omega\xi_0)^2}{2\sigma^2}}\left|\tilde{X}(\zeta,T)\right|^2}{\sqrt{2\pi}\sigma}d\zeta$$

$$K(\delta\omega) = \frac{\left(\sigma_a^2+\sigma_m^2+\sigma_\omega^2\right)e^{-\frac{\delta\omega^2\sigma_\omega^2}{4(\sigma_a^2+\sigma_m^2)(\sigma_a^2+\sigma_m^2+\sigma_\omega^2)}}}{\sqrt{\left(\sigma_a^2+\sigma_m^2\right)\left(\sigma_a^2+\sigma_m^2+\sigma_\omega^2\right)}}$$

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Non stationary ω_0 and FEL gain jitter

Shot to shot gain as random variable with average $\bar{G}$

correlate spectral intensities

$$I' = (\bar{G} + \Delta G)(\bar{S}' + \Delta S') \text{ at } \omega'$$

$$I'' = (\bar{G} + \Delta G)(\bar{S}'' + \Delta S'') \text{ at } \omega''$$

$$\frac{\langle I' I'' \rangle}{\langle I' \rangle \langle I'' \rangle} = \left(1 + \frac{\langle \Delta S' \Delta S'' \rangle}{\bar{S}' \bar{S}''}\right)\left(1 + \frac{\langle \Delta G^2 \rangle}{\bar{G}^2}\right)$$

$$\mathrm{G}_2(\omega', \omega'') = \frac{\dfrac{\langle I' I'' \rangle}{\langle I' \rangle \langle I'' \rangle}}{1 + \dfrac{\langle \Delta G^2 \rangle}{\bar{G}^2}} - 1$$

Numerical simulations

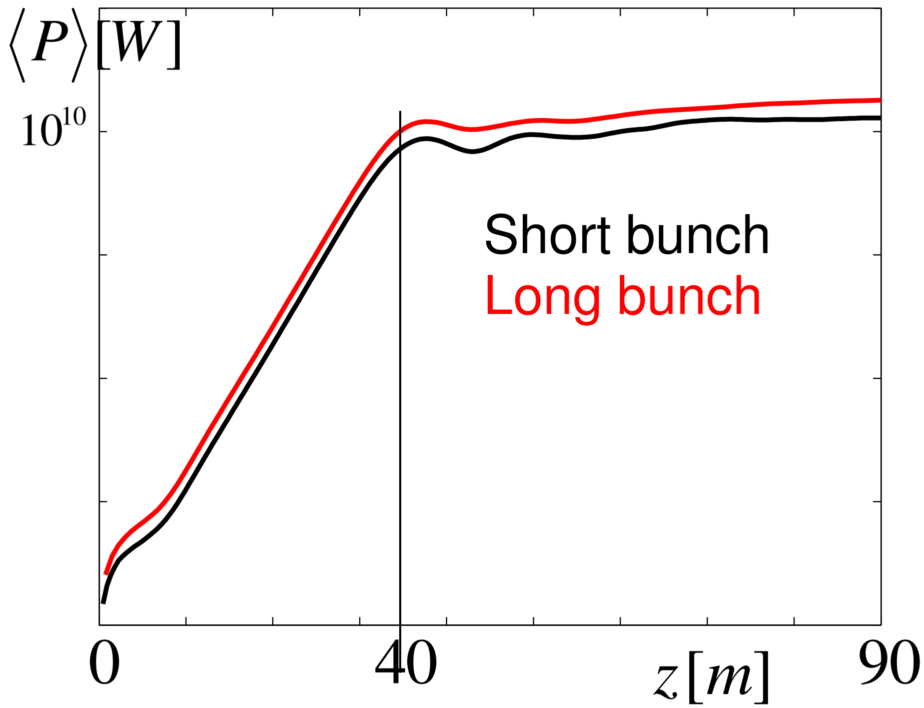
1) Verify that relations hold well enough in saturation

2) Recover bunch length and spectrometer resolution

- $30\mu m$ electron bunch
- $3\mu m$ electron bunch

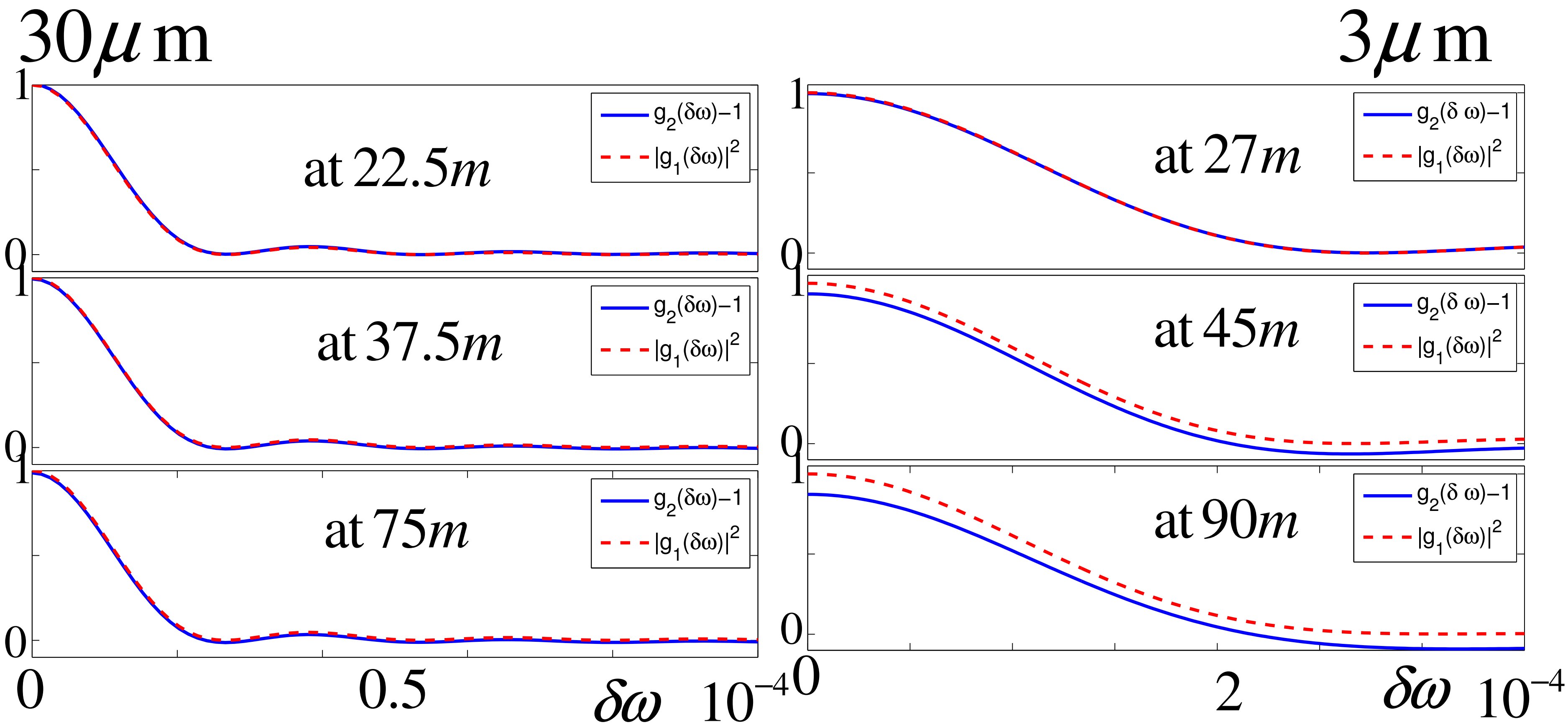
Wavelength 0.8 nm
Undulator period 3 cm

- $\sigma_m = 1\times10^{-4}$
- $\sigma_m = 2\times10^{-4}$



Verify relation

$$g_2(\omega', \omega'') - 1 = \left| g_1(\omega', \omega'') \right|^2$$



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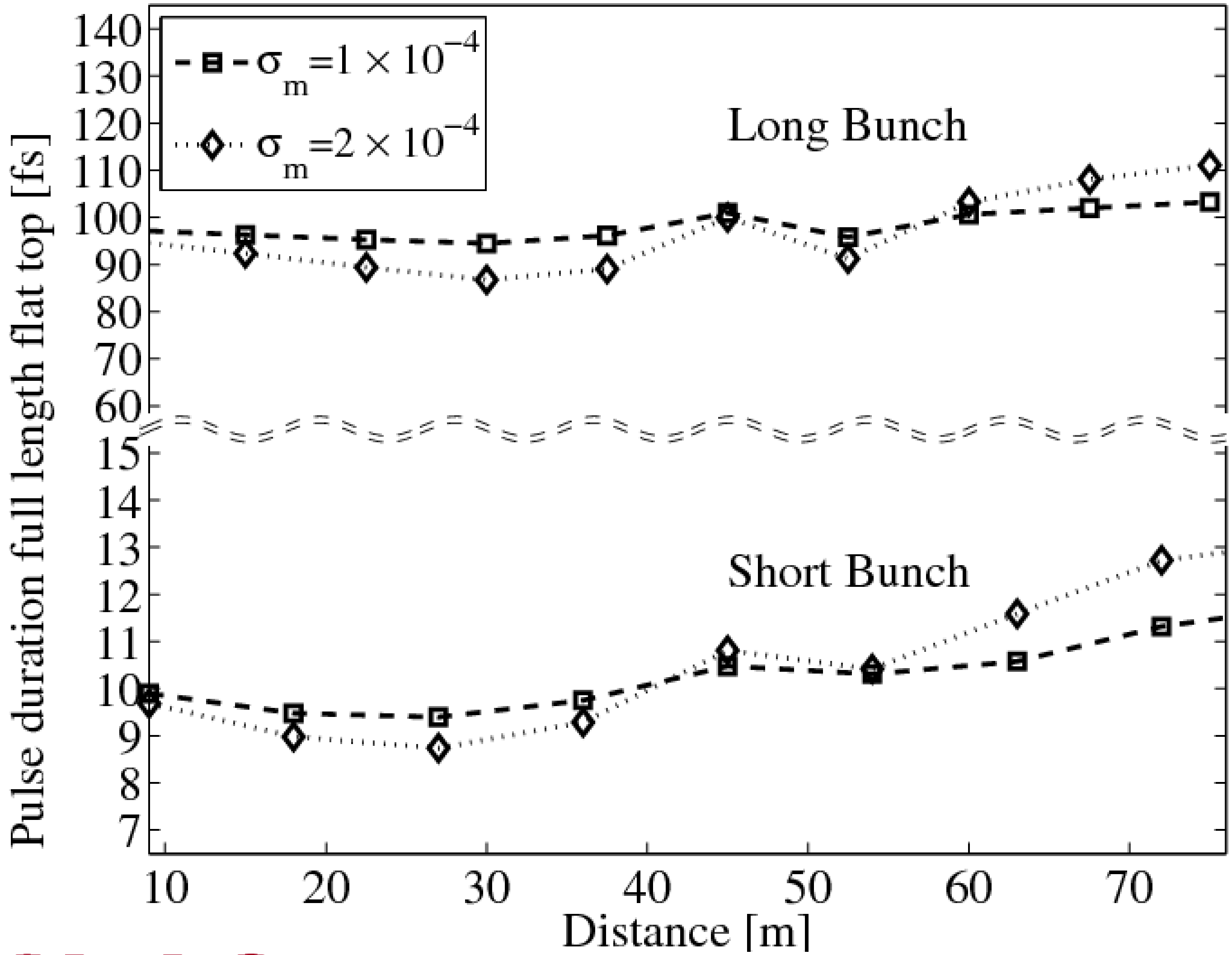
Simulated measurements results



Retrieved spectrometer resolutions

Long bunch

$(1.01\pm0.01)\times10^{-4}$

$(2.05\pm0.05)\times10^{-4}$

Short bunch

$(0.98\pm0.03)\times10^{-4}$

$(1.98\pm0.09)\times10^{-4}$

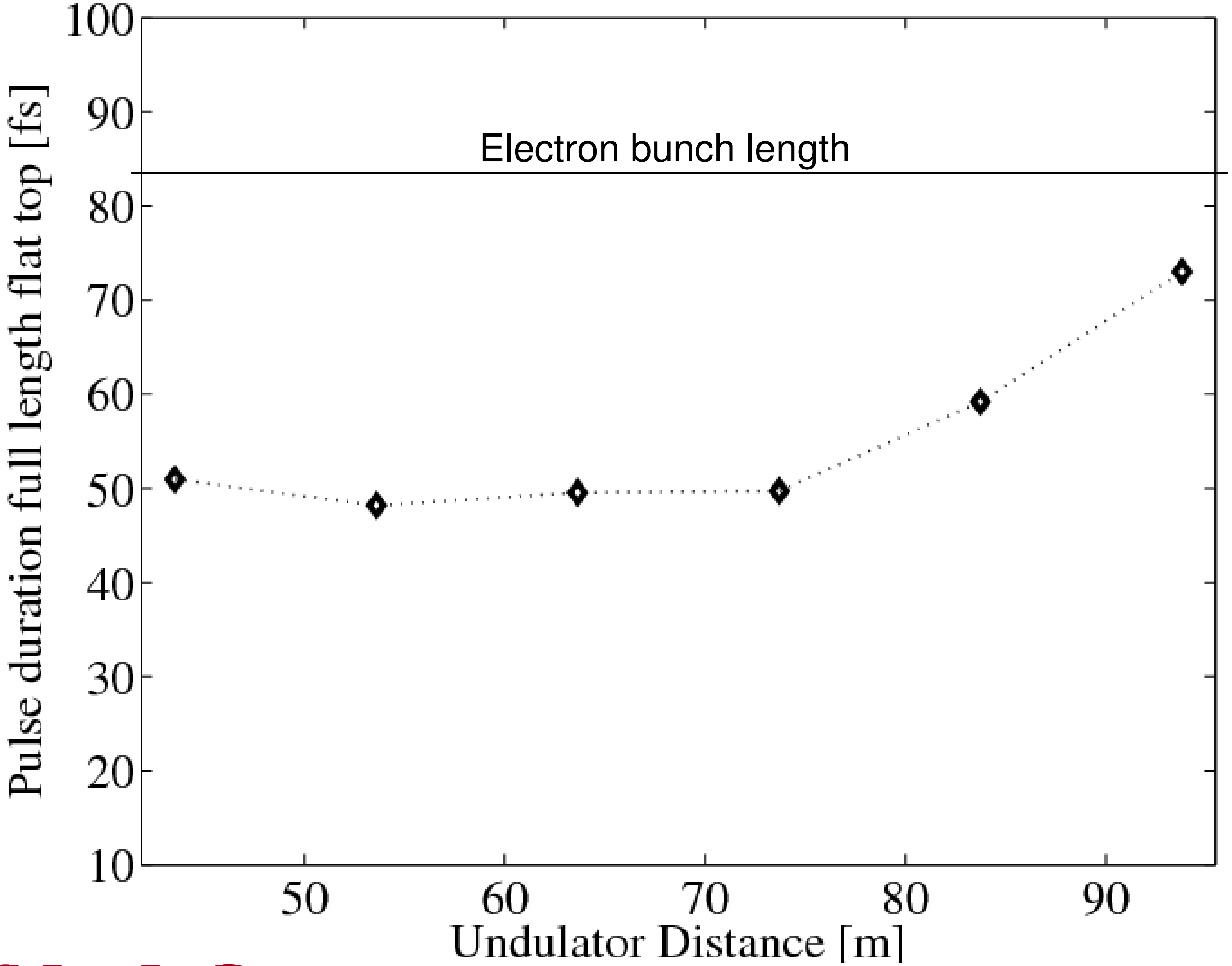
Experimental results

Experimental demonstration performed at LCLS

Photon energy 1.5 keV
Electron charge 250 pC

Different undulator length
Different peak current
Controlling lasing part of electron bunch with slotted foil

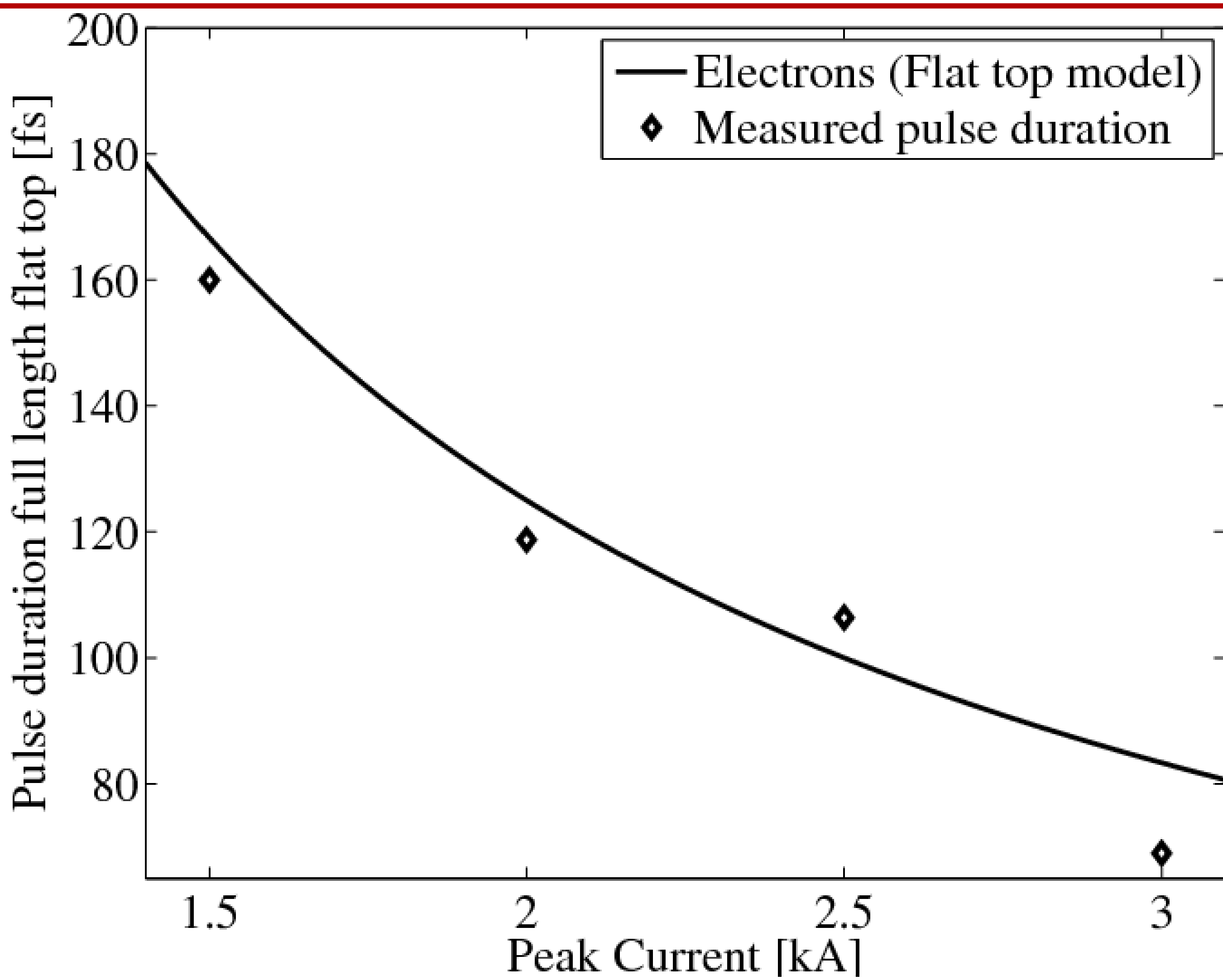
Different undulators distance



Measured spectrometer relative resolution σ_m $(1.00 \pm 0.04) \times 10^{-4}$

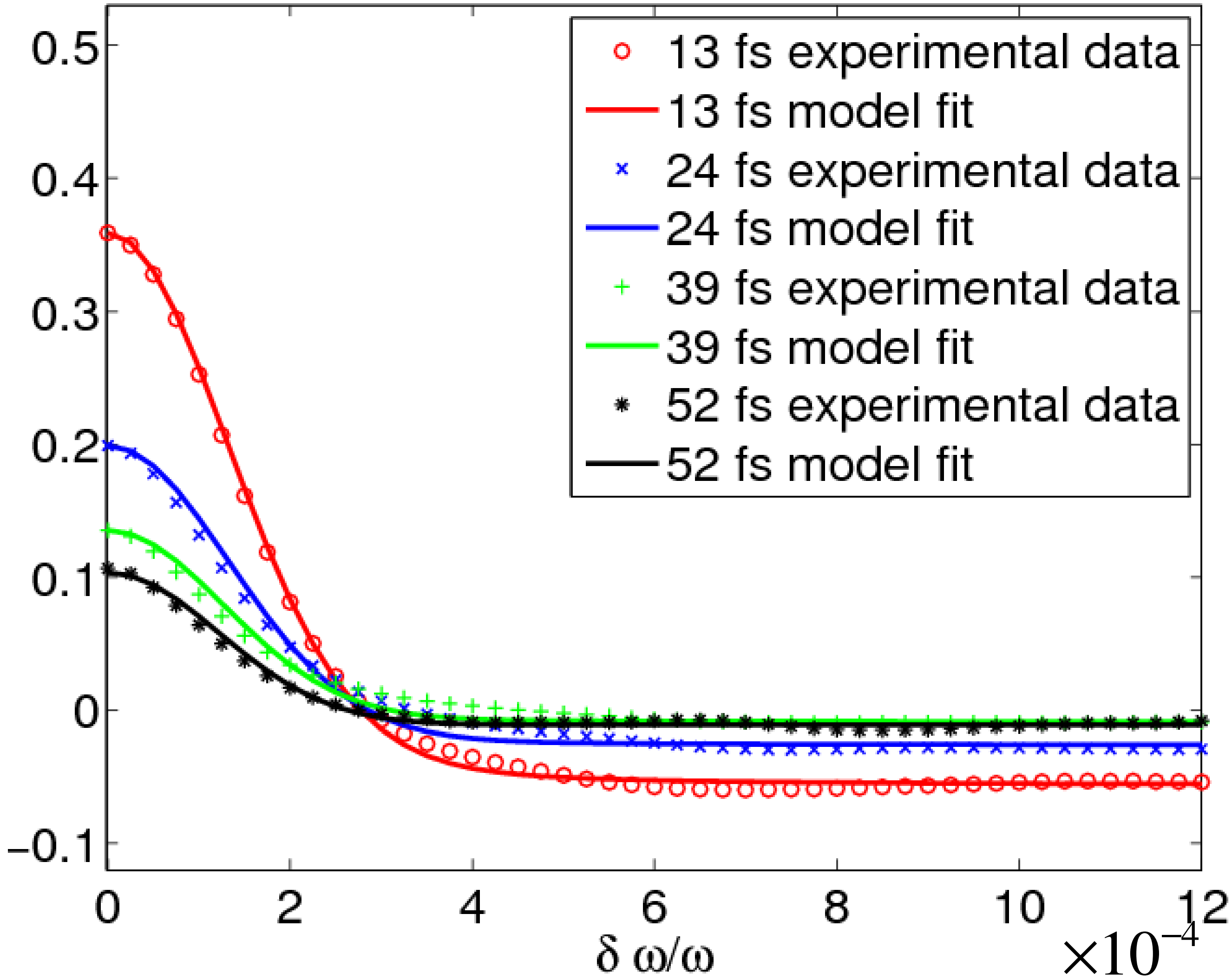
Measured x-rays are shorter than the electron bunch

Different peak currents



Measured x-rays pulse durations are consistent with electron bunch length change.

Slotted foil measurements



Electron bunch FWHM	x-ray FWHM
10 fs	13 fs
18 fs	24 fs
27 fs	39 fs
56 fs	52 fs

Thank you for your attention