# THREE-DIMENSIONAL KINETIC ANALYSIS OF LONGITUDINAL SPACE-CHARGE INTERACTIONS IN A RELATIVISTIC ELECTRON BEAM

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# OUTLINE OF THE TALK

- Introduction and Motivation.
- 3-D dispersion relation with emittance, energy spread, focusing and edge-effects.
- Edge effects and betatron motion for fundamental mode.
- Thermal effects (emittance/enegy spread)
- Application to Longitudinal Space-Charge Amplifier
- Higher order modes

# MOTIVATION FOR THIS WORK

PHYSICAL REVIEW SPECIAL TOPICS - ACCELERATORS AND BEAMS **13,** 110701 (2010)

**Using the longitudinal space charge instability for generation of vacuum ultraviolet and x-ray radiation**

E. A. Schneidmiller and M. V. Yurkov
*Deutsches Elektronen-Synchrotron (DESY), Notkestrasse 85, D-22607 Hamburg, Germany*
(Received 1 April 2010; published 13 November 2010)

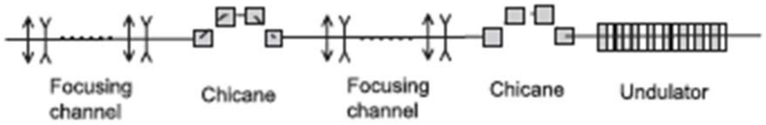
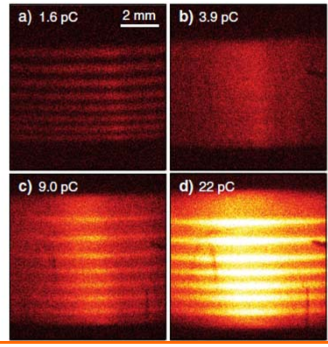


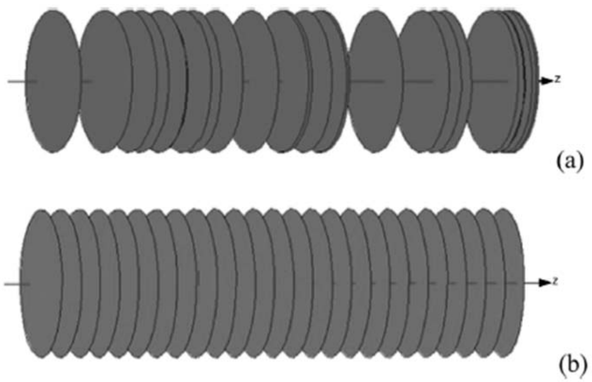

FIG. 1. Conceptual scheme of an LSC amplifier.

PRL **106,** 184801 (2011) PHYSICAL REVIEW LETTERS week ending 6 MAY 2011

**Nonlinear Longitudinal Space Charge Oscillations in Relativistic Electron Beams**

P. Musumeci, R. K. Li, and A. Marinelli
*Department of Physics and Astronomy, UCLA, Los Angeles, California, 90095, USA*
(Received 16 February 2011; published 4 May 2011)




PRL **102,** 154801 (2009) PHYSICAL REVIEW LETTERS week ending 17 APRIL 2009

**Collective-Interaction Control and Reduction of Optical Frequency Shot Noise in Charged-Particle Beams**

A. Gover and E. Dyunin
*Tel-Aviv University, Faculty of Engineering, Department of Physical-Electronics, Tel-Aviv, Israel*
(Received 15 December 2008; published 14 April 2009)

PHYSICAL REVIEW SPECIAL TOPICS - ACCELERATORS AND BEAMS **14,** 060710 (2011)

**Analysis of shot noise suppression for electron beams**

Daniel Ratner
*Department of Applied Physics, Stanford University, Stanford, California 94305, USA*

Zhirong Huang and Gennady Stupakov
*SLAC, Menlo Park, California 94309, USA*
(Received 14 August 2010; published 24 June 2011)

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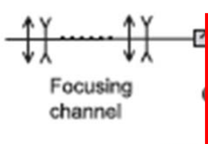


FIG.

PRL **106**, 184801 (2011) PHYSICAL REVIEW LETTERS week ending 6 MAY 2011

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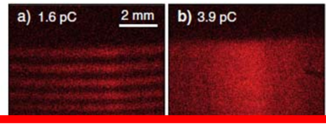
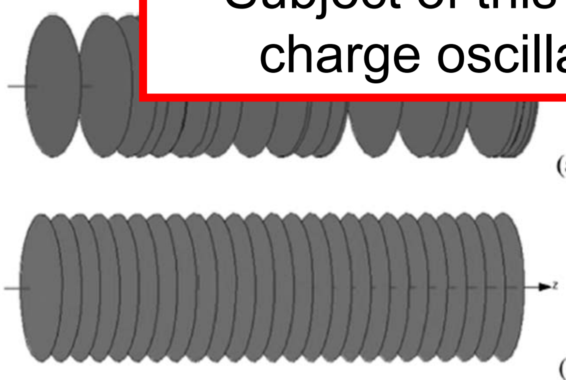



In all of these applications knowledge of the evolution of micro-bunching under the effects of Longitudinal Space-Charge is crucial!

Subject of this work is the theory longitudinal space-charge oscillations in a relativistic electron beam.

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# PREVIOUS WORK IN THE CONTEXT OF HIGH BRIGHTNESS BEAMS

PHYSICAL REVIEW SPECIAL TOPICS - ACCELERATORS AND BEAMS, VOLUME 7, 074401 (2004)

**Suppression of microbunching instability in the linac coherent light source**

Z. Huang,[1,*] M. Borland,[2] P. Emma,[1] J. Wu,[1] C. Limborg,[1] G. Stupakov,[1] and J. Welch[1]

Theory of space-charge waves on gradient-profile relativistic electron beam: An analysis in propagating eigenmodes

Gianluca Geloni[a,b,*], Evgeni Saldin[a], Evgeni Schneidmiller[a], Mikhail Yurkov[a]

NUCLEAR INSTRUMENTS & METHODS IN PHYSICS RESEARCH Section A

Cold laminar beam approximation, 3-D effects due to finite size of the beam included

$$\omega = r_p \sqrt{\frac{e^2 n_b}{\epsilon_0 m \gamma^3}}$$

$r_p$ is a correction factor <1 that tends to 1 if $r_{beam} >> \gamma\lambda$

Novelty of this work is the fully kinetic treatment in 6-D phase space with inclusion of:
- -Emittance
- -Betatron Motion
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- -Edge Effects

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Despite the radically different physical scenario, we used the mathematical methods developed for 3-D FEL theory by Kim, Yu, Xie et al.

Particularly indebted to the work of Ming Xie for the solution techniques (variational and matrix methods) and IVP.

# 6-DIMENSIONAL DISPERSION RELATION FOR SPACE-CHARGE MODES

-Gaussian beam matched to a uniform focusing channel with $k_\beta/c$.

-Work in the longitudinal spatial frequency domain.

-Evolution of a small phase-space perturbation described in terms of space-charge waves:

$$\mathcal{E}_z = E_z(\vec{x})e^{ik_z z - i\frac{\omega\tau}{c}}$$

$$f_0 = n_0 e^{-\frac{\vec{x}^2}{2\sigma_x^2}-\frac{\vec{\beta}_\perp^2}{2\sigma_x^2 k_\beta^2}-\frac{\eta^2}{2\sigma_\eta^2}}/(2\pi)^{3/2}\sigma_x^2 k_\beta^2\sigma_\eta$$

$$\partial_\tau f_1 + \vec{\beta}\cdot\vec{\nabla}_{\vec{x}} f_1 - k_\beta^2\vec{x}\cdot\vec{\nabla}_{\vec{\beta}_\perp} f_1 + ik_z\dot{z}f_1 + \frac{e\mathcal{E}_z}{\gamma mc^2}\partial_\eta f_0 = 0$$

$$(\nabla_\perp^2 - \frac{k_z^2}{\gamma^2})\frac{\mathcal{E}_z}{-\frac{ik_z}{\gamma}} = -\frac{e}{\gamma\epsilon_0}\int f_1 d\eta d^2\vec{\beta}_\perp.$$

Coupled Poisson/Vlasov Equations:

$$\left(\frac{1}{D^2}\nabla_\perp^2 - 1\right)E_z = -\int E_z(\vec{X}')\Pi(\vec{X},\vec{X}')d^2\vec{X}'$$

$$\Pi(\vec{X},\vec{X}') = \int_{-\infty}^{0}\frac{Te^{-\frac{(K_\gamma T)^2}{2}-i\Omega T}e^{-\left(\vec{X}^2+\vec{X}'^2-2\vec{X}\cdot\vec{X}'\cos K_\beta T\right)\frac{(1+iK_\epsilon T)}{2\sin^2 K_\beta T}}}{2\pi\sin^2 K_\beta T}dT.$$

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Dispersion relation has to be solved in $E_z$ and $\Omega$ with:

$$\Omega = \omega/\omega_{p,1-d}$$

$$\omega_{p,1-d}^2 = n_0 e^2/\gamma^3 m\epsilon_0$$

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Dispersion relation expressed in terms of 4 scaling parameters

$$\Pi(\vec{X},\vec{X}') = \int_{-\infty}^{0} \frac{T e^{-\frac{(K_\gamma T)^2}{2} - i\Omega T} e^{-\left(\vec{X}^2 + \vec{X}'^2 - 2\vec{X}\cdot\vec{X}'\cos K_\beta T\right)\frac{(1+iK_\epsilon T)}{2\sin^2 K_\beta T}}}{2\pi \sin^2 K_\beta T} dT.$$

# DIMENSIONLESS SCALING PARAMETERS

**Energy spread parameter**

$$K_\gamma = k_z c \sigma_\eta / \omega_p \gamma^2 = \frac{\sigma_{v_z} \tau_p}{\lambda}$$

Longitudinal displacement due to energy spread in a 1-d plasma period / microbunching wavelength.

**Emittance parameter**

$$K_\epsilon = k_z c (k_\beta \sigma_x)^2 / 2\omega_p = \frac{\sigma^\epsilon_{v_z} \tau_p}{\lambda}$$

Longitudinal displacement due to transverse emittance in a 1-D plasma period / microbunching wavelength.

Analogous to energy spread and emittance parameters in 3-D FEL theory:

Cold beam limit: $K_\gamma$, $K_\varepsilon$ << 1

**3-D parameter**

$$D = k_z \sigma_x / \gamma$$

Transverse beam size/ microbunching wavelength in the rest frame.

**Focusing parameter**

$$K_\beta = k_\beta c / \omega_p$$

Betatron frequency/1-D plasma frequency

Analogous to diffraction parameter in 3-D FEL theory: edge effects are negligible if $D>>1$

Unlike 3-D FEL theory, the normalized betatron frequency is independent of the other scaling parameters.

Transverse motion negligible if: $K_\beta<<1$ (laminar beam limit)

# ONE DIMENSIONAL LIMIT

The one dimensional limit is approached by taking:

*D*>>*1* , $K_\beta$ << 1, $K_\varepsilon$ << 1

Modes are fully degenerate (all eigenmodes have the same eigenvalue) and the dispersion relation reduces to the well known 1-D plasma oscillation dispersion relation for a warm plasma (Landau/Jackson):

$$1 - \frac{1}{2K_\gamma^2} Z' \left( \frac{\Omega}{\sqrt{2} K_\gamma} \right) = 0$$

$$Z(\zeta) = 2ie^{-\zeta^2} \int_{-\infty}^{i\zeta} e^{-x^2} dx$$

In the 1-D limit for a cold beam (*D*>>*1* , $K_\beta$ << 1, $K_\varepsilon$ << 1 and $K_\gamma$ << 1) we get the well established result:

$$\Omega^2 = 1$$

or:

$$\omega = \pm \omega_p$$

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Note that transverse focusing breaks the degeneracy of the plasma eigenmodes in the D>>1 limit (infinite beam limit).

Novel result from kinetic analysis in 6-D phase-space.

In the 1-D limit for a cold beam ($D >> 1$ , $K_\beta << 1$, $K_\varepsilon << 1$ and $K_\gamma << 1$) we get the well established result:

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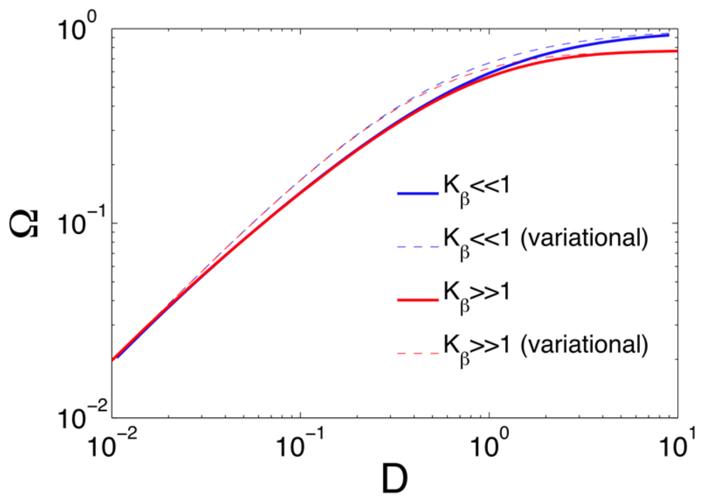
# GEOMETRICAL EFFECTS: PLASMA REDUCTION FACTOR FOR A COLD BEAM




Handy variational formula for laminar beam limit:

$\Omega = 2D/(1+2D)$

Assume Cold beam limit: $K_\gamma$, $K_\varepsilon$ << 1.

In the "infinite beam limit" (or short wavelength limit) $D \gg 1$

$\Omega=1$ for a laminar beam ($K_\beta \ll 1$)
$\Omega \approx 0.756$ for high betatron frequency($K_\beta \gg 1$)

In the "small beam limit" (or long wavelength limit) $D \ll 1$

$\Omega \approx 2D$ regardless of betatron motion

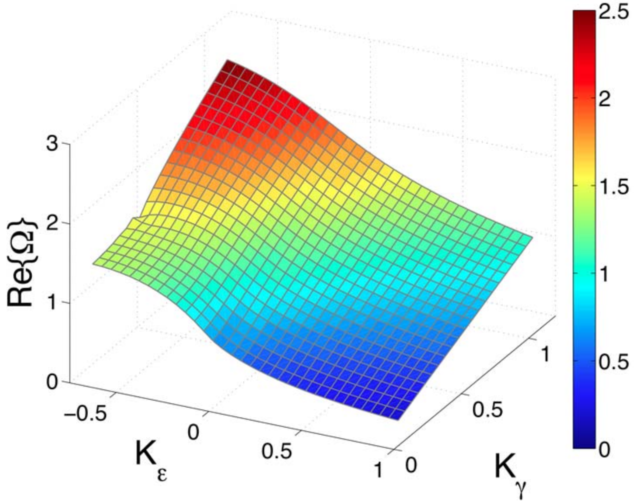
Reduction due to geometry of the microbunched beam in the rest frame. For for small $D$ you have: $\gamma\lambda \gg \sigma_x$

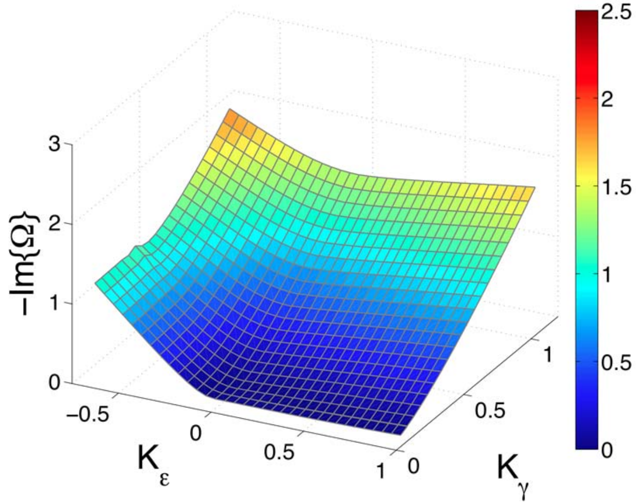
Also: for small $D$ an electron oscillating transversely samples no field variation since the field extends well outside of the beam thus betatron motion makes no difference!

# EMITTANCE AND ENERGY SPREAD EFFECTS

Example:

$D=1\ K_\beta >>1$




Longitudinal thermal motion induced by energy spread and transverse emittance gives rise to an exponential damping process (Landau damping).

Response is different for forward/backward propagating waves (in beam coordinate system).

Emittance induced velocity spread is always negative resulting in a stronger landau damping of backward propagating modes!

Emittance induced Landau damping sets the optimum beam size for space-charge experiments since:

$\omega_p \sim 1/\sigma_x$ and $K_\varepsilon \sim 1/\sigma_x$

Increasing the density by focusing comes at the expense of increasing longitudinal velocity spread!

Important for longitudinal space-charge amplification!

# LONGITUDINAL SPACE-CHARGE AMPLIFIER

Initial value problem in 6-D phase space solved in terms of bi-orthogonal mode expansion

$$f_1(\tau) = \sum_n f_n e^{-i\frac{\omega_n \tau}{c}} \frac{\langle f_n^\dagger, f_{1,0}\rangle}{\langle f_n^\dagger, f_n\rangle}.$$

This formula gives the evolution of the initial perturbation under the effect of LSC.
Dispersion in magnetic chicane results in a factor Exp(-ik$R_{56}$)

$f_n$ and $f_n^\dagger$ are the eigenmodes and the adjoint eigenmodes of the 6-D phase space operator:

$$f_n = -\frac{e}{\gamma m c^2}\partial_\eta f_0 \int_{-\infty}^{0} e^{-i\frac{\omega_n \tau}{c}+i\frac{k_z \eta \tau}{\gamma^2}} E_n\left(\vec{x}\cos(k_\beta \tau) + \vec{\beta}_\perp \frac{1}{k_\beta}\sin k_\beta \tau\right) d\tau$$

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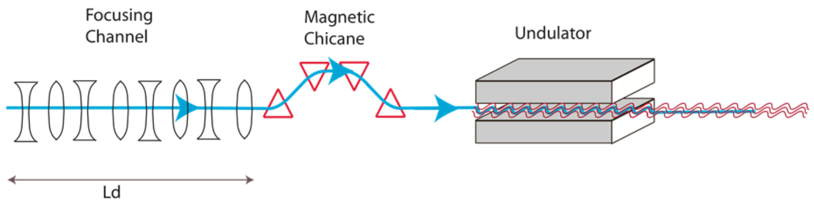


For a cold beam the gain due to space-charge starting from a density modulation is

$$\tilde{n} = -\tilde{n}_0 \gamma^2 R_{56} e^{-\frac{(k_z \sigma_\eta R_{56})^2}{2}} \Gamma \frac{\omega_p}{c}\Omega_+ \sin \omega_p \Omega_+ \frac{L_d}{c}$$

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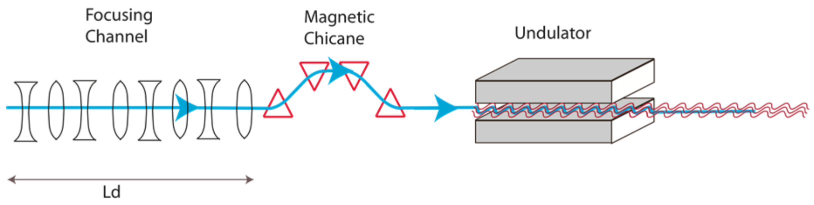


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Time derivative of the density modulation at the drift exit!

Up to the coupling coefficient $\Gamma$~1 the formula can be interpreted as:
-a fraction of plasma oscillation of length $L_d$
-followed by a space-charge free drift of length $\gamma^2 R_{56}$

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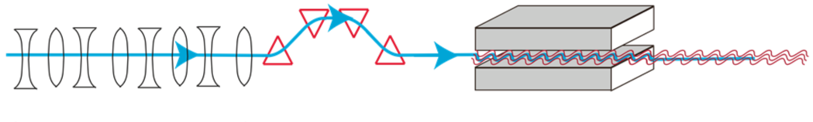


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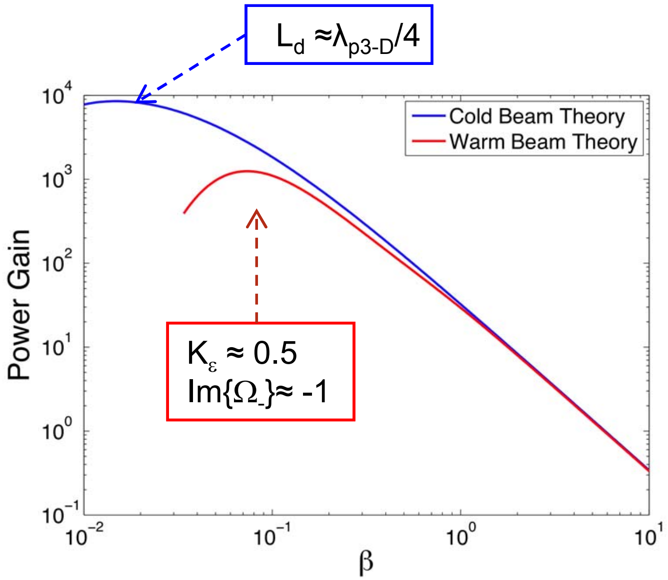
Gain Maximized at ¼ plasma oscillation!

# 3-D AND THERMAL EFFECTS IN THE LONGITUDINAL SPACE-CHARGE AMPLIFIER




Example: compressed NLCTA beam

- $I_{peak}$ = 500 A
- $\varepsilon$ = 3 × $10^{-6}$ mm mrad (slice emittance)
- $\gamma$ = 240 (E = 120 MeV)
- $\lambda$ = 800 nm
- $E_{spread}$ = 5 × $10^{-5}$
- $L_d$ = 1 m

1-D Plasma frequency scales like:

$$\omega_p \sim 1/\sigma_x \sim 1/\beta^{1/2}$$

Emittance parameter scales like:

$$K_\varepsilon \sim 1/\sigma_x \sim 1/\beta^{1/2}$$

Stronger focusing enhances the collective response but increases thermal effects due to emittance!
Gain has an optimum when the two effects balance each other!

For the cold beam case, gain increases with stronger focusing until the drift length is equal to ¼ plasma period.

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# HIGHER ORDER MODES AND PLASMA-BETATRON BEATWAVES

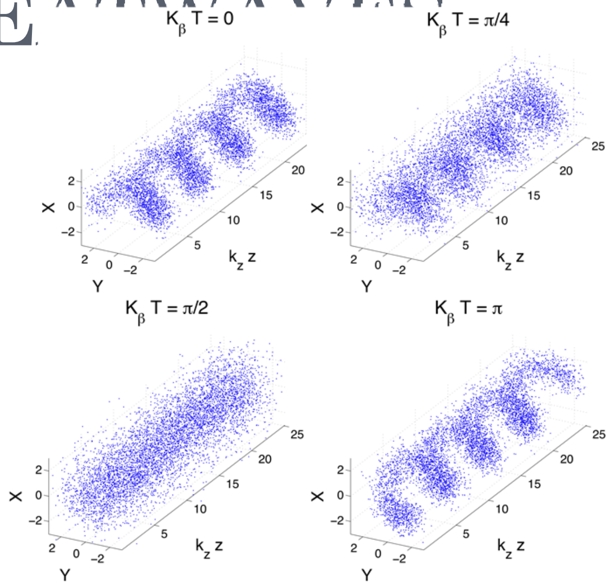
Azimuthal mode expansion of dispersion relation

$$E_z = E_m(R)e^{im\theta}$$

+ cold beam limit:

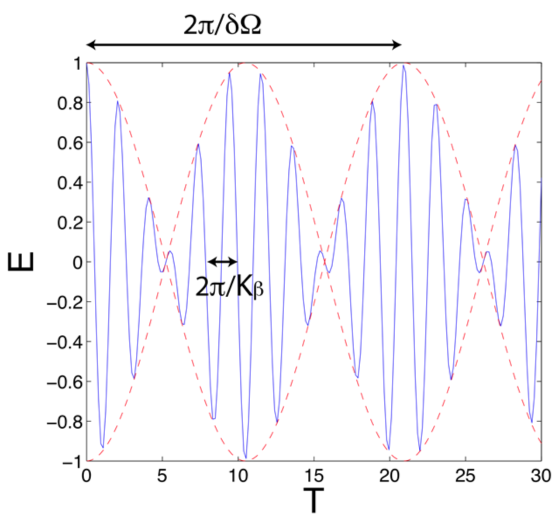
$$\hat{E}_{m,r}(Q) = \frac{1}{\Omega^2(1+\frac{Q^2}{D^2})}\int_0^\infty I_m\left(QQ'\right)e^{-\frac{Q^2+Q'^2}{2}}\hat{E}_{m,r}(Q')Q'dQ'.$$

$$T_m(Q,Q') = \frac{e^{-\frac{Q^2+Q'^2}{2}}}{\left(1+\frac{Q^2}{D^2}\right)}\sum_n I_{\frac{m+n}{2}}\left(\frac{QQ'}{2}\right)I_{\frac{m-n}{2}}\left(\frac{QQ'}{2}\right)\frac{1}{(\Omega-nK_\beta)^2}$$



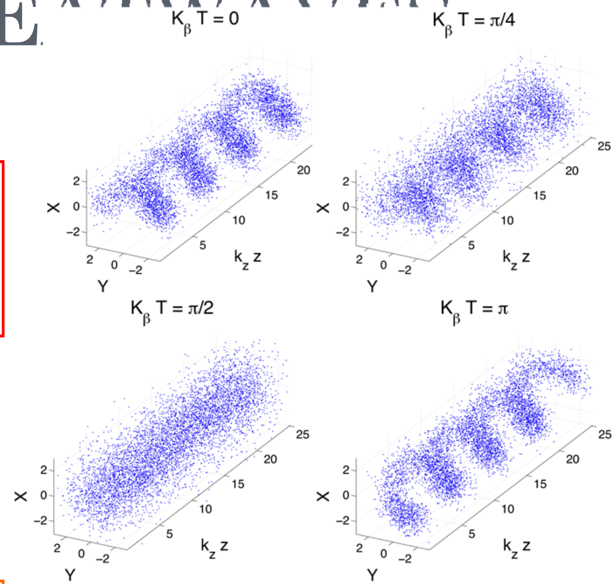

Evolution of an even/odd m charge perturbation under transverse focusing composed of even/odd harmonics of betatron oscillation. (example: m=1)

For $K_\beta$>>1 we look for solutions in the form: $\Omega=hK_\beta \pm\delta\Omega$ with $\delta\Omega<<K_\beta$
For h≠0 response is a beat between betatron and plasma oscillation.




Note:
for an emittance dominated beam odd m modes only exist as beatwaves…

# HIGHER ORDER MODES AND PLASMA-BETATRON BEATWAVES




Azimuthal mode expansion of dispersion relation

$$E_z = E_m(R)e^{im\theta}$$

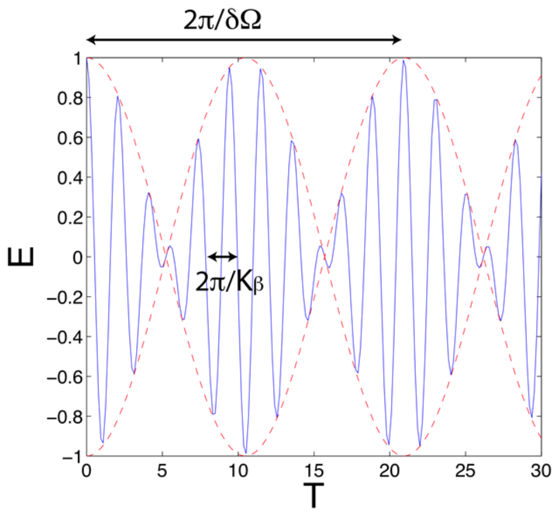
+ cold beam limit:

$$\hat{E}_{m,r}(Q) = \frac{1}{\Omega^2(1+\frac{Q^2}{D^2})}\int_0^\infty I_m(QQ')\, e^{-\frac{Q^2+Q'^2}{2}}\, \hat{E}_{m,r}(Q')Q'dQ'$$

$$T_m(Q,Q') = \frac{e^{-\frac{Q^2+Q'^2}{2}}}{\left(1+\frac{Q^2}{D^2}\right)}\sum_n I_{\frac{m+n}{2}}\left(\frac{QQ'}{2}\right) I_{\frac{m-n}{2}}\left(\frac{QQ'}{2}\right) \frac{1}{(\Omega - nK_\beta)^2}$$

Sum performed over even/odd harmonics for even/odd m
Dispersion relation highly peaked around $\Omega \approx nK_\beta$ (cold beam limit)

For $K_\beta >> 1$ we look for solutions in the form: $\Omega = hK_\beta \pm \delta\Omega$ with $\delta\Omega << K_\beta$
For $h \neq 0$ response is a beat between betatron and plasma oscillation.

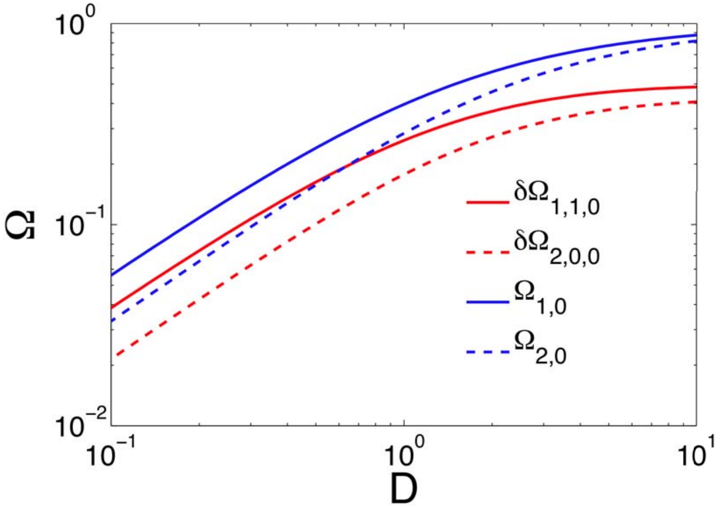



Note:
for an emittance dominated beam odd m modes only exist as beatwaves…

Evolution of an even/odd m charge perturbation under transverse focusing composed of even/odd harmonics of betatron oscillation. (example: m=1)

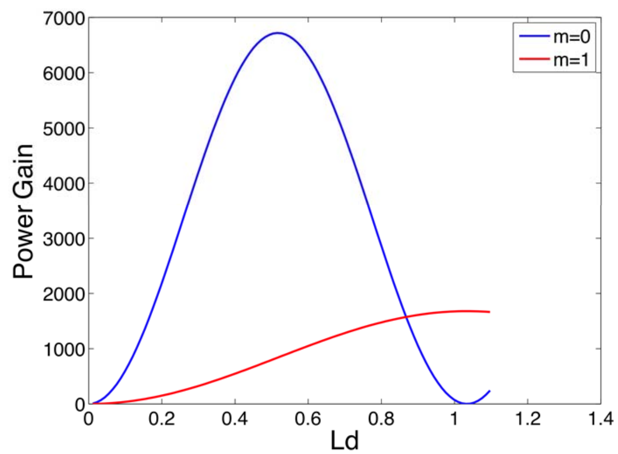
# HIGHER ORDER MODES IN LSCA

CONSEQUENCE:
suppression of higher order modes due to transverse focusing.




The reduced eigenvalue $\delta\Omega$ describes the collective physics of the system and it's independent of $K_\beta$.

This effect can be empolyed to either suppress or amplify higher order modes in a LSCA

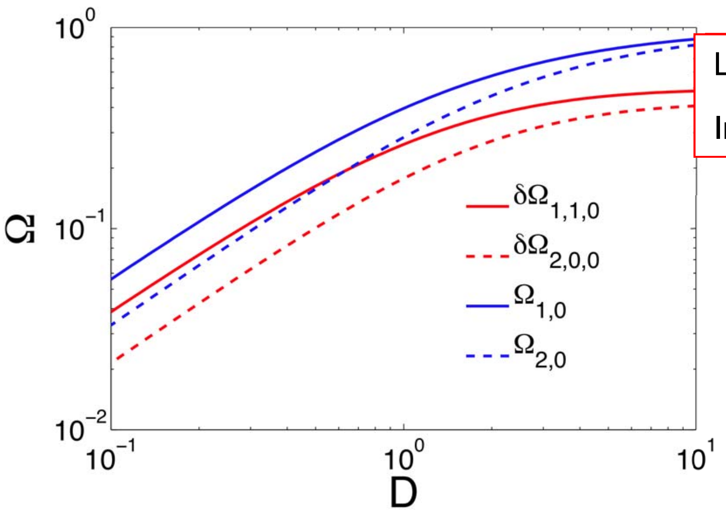



$$G_{m=0} = \left(\frac{\omega_p}{c}\gamma^2 R_{56} e^{-\frac{K_\phi^2}{2}} \Gamma_0 \Omega_0 \sin\left(\Omega_0 \frac{\omega_p}{c} L_d\right)\right)^2$$

$$G_{m=1} = \left(\frac{\omega_p}{c}\gamma^2 R_{56} e^{-\frac{K_\phi^2}{2}} \Gamma_1 \delta\Omega_1 \sin\left(\delta\Omega_1 \frac{\omega_p}{c} L_d\right) \cos\left(K_\beta \frac{\omega_p}{c} L_d\right)\right)^2$$

# HIGHER ORDER MODES IN LSCA

CONSEQUENCE:
suppression of higher order modes due to transverse focusing.

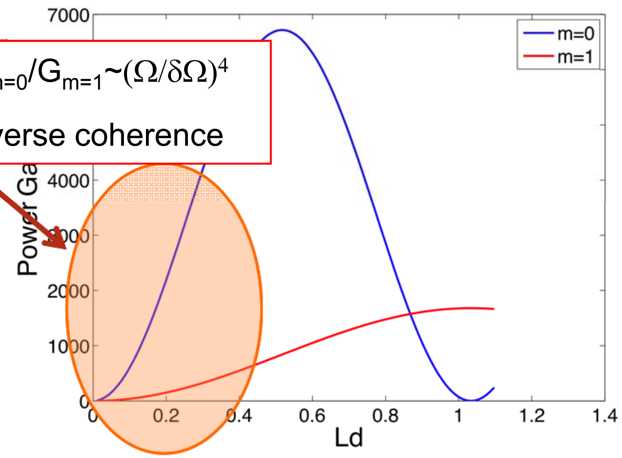



The reduced eigenvalue $\delta\Omega$ describes the collective physics of the system and it's independent of $K_\beta$.

This effect can be employed to either suppress or amplify higher order modes in a LSCA

$Ld \ll \lambda_p/4$   $G_{m=0}/G_{m=1} \sim (\Omega/\delta\Omega)^4$

Increased transverse coherence

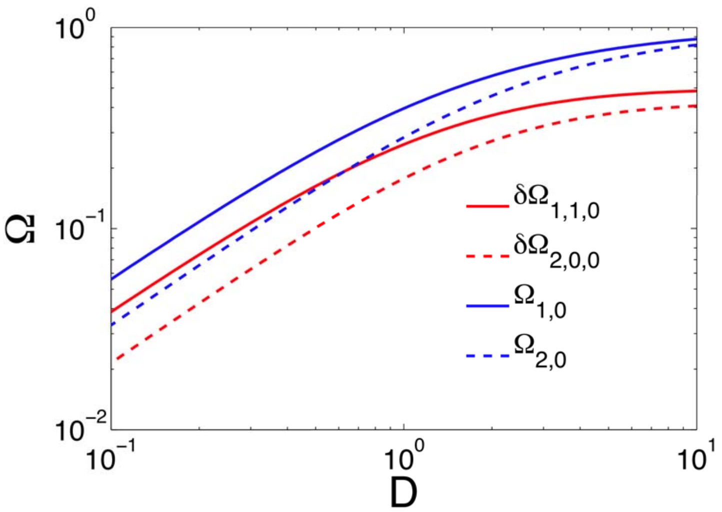



$$G_{m=0} = \left(\frac{\omega_p}{c}\gamma^2 R_{56} e^{-\frac{K_\phi^2}{2}} \Gamma_0 \Omega_0 \sin\left(\Omega_0 \frac{\omega_p}{c} L_d\right)\right)^2$$

$$G_{m=1} = \left(\frac{\omega_p}{c}\gamma^2 R_{56} e^{-\frac{K_\phi^2}{2}} \Gamma_1 \delta\Omega_1 \sin\left(\delta\Omega_1 \frac{\omega_p}{c} L_d\right) \cos\left(K_\beta \frac{\omega_p}{c} L_d\right)\right)^2$$

# HIGHER ORDER MODES IN LSCA

CONSEQUENCE:
suppression of higher order modes due to transverse focusing.

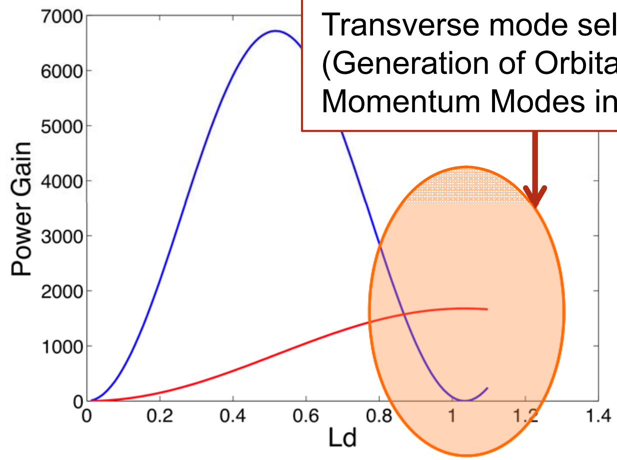



The reduced eigenvalue $\delta\Omega$ describes the collective physics of the system and it's independent of $K_\beta$.

This effect can be employed to either suppress or amplify higher order modes in a LSCA

$Ld \approx \lambda_p/2$ $G_{m=0} << G_{m=1}$

Transverse mode selection!
(Generation of Orbital Angular Momentum Modes in FELs)




$$G_{m=0} = \left(\frac{\omega_p}{c}\gamma^2 R_{56} e^{-\frac{K_\phi^2}{2}} \Gamma_0 \Omega_0 \sin\left(\Omega_0 \frac{\omega_p}{c} L_d\right)\right)^2$$

$$G_{m=1} = \left(\frac{\omega_p}{c}\gamma^2 R_{56} e^{-\frac{K_\phi^2}{2}} \Gamma_1 \delta\Omega_1 \sin\left(\delta\Omega_1 \frac{\omega_p}{c} L_d\right) \cos\left(K_\beta \frac{\omega_p}{c} L_d\right)\right)^2$$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**Generating Optical Orbital Angular Momentum in a High-Gain Free-Electron Laser at the First Harmonic**


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(Received 6 December 2010; published 22 April 2011)

# CONCLUSIONS

- An analytical model for longitudinal space-charge waves was developed which includes e-spread, emittance, transverse focusing and edge effects (corresponding to 4 dimensionless scaling parameters).
- Our analysis is based on a modal expansion in 6-D phase space and employs techniques developed for 3-D FEL theory.
- The kinetic analysis reveals interesting physical effects like:

  1) emittance induced anisotropy of LSC modes

  2) focusing induced degeneracy breaking

  3) plasma-betatron beatwaves.
- Solution of the initial value problem allows for a 3-D kinetic description of the concept of LSCA proposed by Schneidmiller et al.
- This model has interesting applications in the optimization of space-charge based microbunching experiments.

- Details of this derivation to be published on Physics of Plasmas "Three dimensional analysis of longitudinal plasma oscillations in a thermal relativistic electron beam" (tentatively scheduled for publication in the september issue) and two subsequent papers yet to be submitted.

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