Collective versus Individual Aspects in FEL Electron Beam Fluctuations

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August 22, 26, 2011

International FEL Conference

Shanghai, China

Overview

- A. Pines and D. Bohm (Phys. Rev., 85, 338 (1952)) showed that fluctuations in a non-relativistic plasma exhibit both individual particle behavior as well as collective plasma oscillations, and the collective behavior dominates for fluctuations of wavenumber k, if $k\lambda_D$ **<<1,** where λ_D = "Debye length" = mean particle velocity/ plasma frequency

→ Particles separated more than the Debye length interact only via collective forces.

- In microwave devices, the collective plasma oscillation has been utilized for shot noise reduction
- We study the validity limits of a similar noise reduction technique proposed for high-gain FELs*, finding
  - ***The presence of the individual particle behavior not subject to collective control***
  - ***$1/k\lambda_D$ scaling of the momentum noise***

* A.Gover and E. Dyunin**,** Phys. Rev. Letts., 102, 154801 (2009); IEEE-JQE, **46**, 1511, 2010

Variables and Equations

- **Klimontovich distribution function:**

$$f(\zeta,\Delta\beta,z)=\sum_i \delta(\zeta-\zeta_i(z))\delta(\Delta\beta-\Delta\beta_i(z))$$

- **Variables:**
  - "Time" variable: z
  - Position: $\zeta = z - v_0 t$
  - Momentum: $\Delta\beta \equiv d\zeta / dz = 1-\beta_0/\beta = (\Delta\gamma/\gamma)(1/\beta\gamma^2)$
- **Klimontovich (Vlasov) equation:**

$$\frac{\partial f}{\partial z}+\Delta\beta\frac{\partial f}{\partial \zeta}+\frac{eE}{mc\beta\gamma^3}\frac{\partial f}{\partial \Delta\beta}=0$$

- **Gauss (Poisson) equation for electric field *E***

$$\frac{\partial E}{\partial z}+\frac{\partial E}{\partial \zeta}=\frac{e}{\varepsilon_0\Sigma_A}\int d\Delta\beta\, f$$

Our approach*

- **Decompose *f* into smooth background and the rest:**
  - $f = f_0 + \hat{f}$
  - f_0: smooth background, zeroth order part

    $$f_0(\Delta\beta) = n_0 g(\Delta\beta) \qquad g(\Delta\beta) = \exp(-\Delta\beta^2 / 2\sigma_{\Delta\beta}^2) / \sqrt{2\pi}\sigma_{\Delta\beta}$$
  - $\hat{f}$ **:** high frequency part, including particle discreteness, regarded to be small, first order term
- Smooth background does not produce electric field→ treat *E* as the first order quantity
- Introduce Fourier transform

$$\hat{f}_k(\Delta\beta; z) = \int_{-\infty}^{\infty} d\zeta e^{-ik\zeta} \hat{f}(\zeta, \Delta\beta, z), \quad E_k(z) = \int_{-\infty}^{\infty} d\zeta e^{-ik\zeta} E(\zeta, z)$$

- Do Laplace transform in z, turn to algebraic equation containing the initial function $\hat{f}_k(\Delta\beta; z = 0)$, solve the equation and perform the inverse Laplace transform*.

* Similar to the SASE study, KJK, Nucl. Instr. Methods, A250, 396 (1986)

The solution for the bunching factor:

- Bunching factor: $b_k(z) = \frac{1}{N_e}\int d\zeta d\Delta\beta e^{-ik\zeta}\hat{f}\,(\zeta,\Delta\beta;z) = \frac{1}{N_e}\sum_j\ e^{-ik\zeta_j(z)}$
- Solution: $b_k(z) = \frac{i}{2\pi N_e}\int_{\boldsymbol{L}}\ d\omega\,\frac{e^{-i\omega z}}{\varepsilon(k,\omega)}\sum_j\ \frac{e^{-ik\zeta^0{}_j}}{\omega - k\Delta\beta^0{}_j}$

where the "dielectric function"

$$\varepsilon(k,\omega) = 1 + \Omega_P^2 \int d\Delta\beta \frac{g'(\Delta\beta)}{\omega - k\Delta\beta}$$

$$\Omega_P = \sqrt{e^2 n_0 / \varepsilon_0 m\beta\gamma^3}\quad \text{(Plasma frequency)}$$

- The Landau-contour ***L*** is above all the singularities in the integrand
- ***There are two classes of poles***
  - Collective: $\boldsymbol{\omega = \omega_q;\ \varepsilon(k,\omega_q) = 0}$
  - Individual: $\boldsymbol{\omega = k\Delta\beta^0{}_i}$ **, *i* = 1,2,…N_e**

The collective and individual parts

- Accordingly we have the decomposition: $b_k(\mathrm{z}) = b_k{}^C(z) + b_k{}^I(z)$,
- Collective part: $\boldsymbol{b_k{}^C(z) = \sum_q e^{-i\omega_q z} \frac{1}{\varepsilon'(k,\omega_q)} \frac{1}{N_e} \sum_i \frac{e^{-ik\zeta_i^0}}{\omega_q - k\Delta\beta^0{}_i}}$
- Individual part: $\boldsymbol{b_k{}^I(z) = \frac{1}{N_e} \sum_i \frac{e^{-ik(\zeta_i^0 + \Delta\beta^0{}_i z)}}{\varepsilon(k,k\Delta\beta^0{}_i)}}$
- ***This is, as far as we know, the first precise formulation of the decomposition of plasma fluctuations, first introduced by Bohm and Pines in 1952***

Incoherent fluctuations due to individual motion

- $$b_k{}^I(z) = \frac{1}{N_e}\sum_i \frac{e^{-ik\left(\zeta_i^0+\Delta\beta^0{}_i z\right)}}{\varepsilon(k,k\Delta\beta^0{}_i)}$$
- This corresponds to free motion of Debye-shielded particles (See, for example, Nicholson)
- The magnitude of the incoherent term:
- $$\langle |b_k^I|^2\rangle = \left\langle \frac{1}{N_e^2}\sum_j \frac{1}{|\varepsilon(k,k\Delta\beta_j^0|^2} \right\rangle + \left\langle \frac{1}{N_e^2}\sum_{j\neq m} \frac{e^{-ik\left(\zeta_j^0+\Delta\beta_j^0 z-\zeta_j^0-\Delta\beta_m^0 z\right)}}{\varepsilon(k,k\Delta\beta_j^0)\varepsilon^*(k,k\Delta\beta_m)} \right\rangle$$
- The second term vanishes invoking random phase approximation and the first term can be calculated exactly* for a Gaussian $g(\Delta\beta)$:

$$\langle |b_k^I|^2\rangle = \frac{1}{N_e}\int d\Delta\beta \,\frac{g(\Delta\beta)}{|\varepsilon(k,k\Delta\beta)|^2} = \frac{1}{N_e}\frac{(k\lambda_D)^2}{1+(k\lambda_D)^2}, \; \lambda_D = \sigma_{\Delta\beta}/\Omega_p$$

- ***The part is not subject to plasma oscillation, and is large when*** $k\lambda_D \geq 1$

*N. Rostoker, Nucl. Fusion, **1**, 101 (1961)

The collective part

- Assume $k\lambda_D \ll 1$ → $\varepsilon(k,\omega) \approx 1 - {\Omega_P}^2/\omega^2$. Then fluctuation is determined by the collective plasma oscillation. Otherwise, the plasma oscillation is poorly defined due to Landau damping*

$$\rightarrow b_k^C(z) = \frac{1}{2N_e}\sum_j e^{-ik\zeta_j^0}\left(e^{-i\Omega_P z}\frac{\Omega_P}{\Omega_P - k\Delta\beta_j} + e^{i\Omega_P z}\frac{\Omega_P}{\Omega_P + k\Delta\beta_j}\right)$$

$$\approx b_k(0)\cos(\Omega_P z) - i\frac{k}{\Omega_P}p_k(0)\sin(\Omega_P z\Big)$$

- We also find the "collective momentum"

$$p_k^C(z) = \frac{1}{N_e}\int d\Delta\beta\,\hat{f}_k\,(\Delta\beta, z) = \frac{1}{N_e}\sum_j \Delta\beta_j\, e^{-ik\zeta_j(z)}$$

$$\approx -i\frac{\Omega_P}{k}b_k(0)\sin(\Omega_P z) + p_k(0)\cos(\Omega_P z\Big)$$

These equations describe collective plasma oscillation

See, for example, S. Ichimaru, *Basic Principles of Plasma Physics*, The Benjamin/Cummins Pub. Co. (1973)

The values of bunching factor and the collective momentum

- For $z = 0$

$$|b_k^C(0)| \sim 1/\sqrt{N_e}\,, \quad |b_k^C(0)| \sim \sigma_{\Delta\beta}/\sqrt{N_e}$$

- For $z = \Lambda_p/4 = \pi/2\Omega_p$ (one quarter of the plasma oscillation period)

$$|b_k^C(\Lambda_p/4)| \sim k\,\lambda_D/\sqrt{N_e}\,, \quad |p_k^C(\Lambda_p/4)| \sim \Omega_p/k\,\sqrt{N_e}$$

- Note that
  - $\dfrac{|b_k(\Lambda_p/4)|}{|b_k(0)|} = k\lambda_D$: The "shot" noise decreases as $k\lambda_D$
  - $\dfrac{|p_k(\Lambda_p/4)|}{|p_k(0)|} = \dfrac{1}{k\lambda_D}$: The momentum noise increases as 1/ $k\lambda_D$

Effective input noise for high-gain FELs

- 1-D formula for effective input noise (KJK, 1986) is proportional to

$$s_k(z_0) = \left| \sum_{j=1}^{N_e} \frac{e^{-ik\zeta_j(z_0)}}{(\mu - \gamma^2 \Delta\beta_j/\rho)} \right|^2$$

- Which may be expanded by assuming $\frac{\gamma^2\Delta\beta}{\rho} \equiv \frac{\Delta\gamma}{\gamma}\frac{1}{\rho} < 1$:

$$s_k(z_0) \approx N_e{}^2 |b_k(z_0) + (\mu/\rho)\gamma^2 p_k(z_0)|^2$$

  - Here μ is the solution of the dispersion relation($|\mu| = 1$ for the cold beam case)
  - ρ is the FEL strength parameter

Reducing the input noise for high-gain FELs via collective plasma oscillation, when $k\lambda_D$ <<1

- Using the values of the b_k and p_k, we obtain
  - $s_k(0) \sim N_e\left(1 + |\frac{\mu\Delta\gamma/\gamma}{\rho}|^2\right)$
  - $s_k(\Lambda_p/4) \sim N_e\left((k\lambda_D)^2 + |\mu\frac{\Delta\gamma/\gamma}{\rho}\frac{1}{k\lambda_D}|^2\right)$
- The red terms are from the "shot" noise which is reduced after a quarter plasma period
- The "momentum" noise term for $z = \Lambda_p/4$ scales as $1/k\lambda_D$. Since FELs work well if $\frac{\Delta\gamma}{\rho\gamma} \sim 0.1$, this term could become larger than 1 if $k\lambda_D < 0.1$
- **It seems that the input SASE noise reduction via plasma oscillation is difficult even for the case** $k\lambda_D < 0.1$

Numerical example for the LCLS case*

	LCLS	LCLS injector
Energy [GeV]	14.35 GeV ($\gamma = 28 \times 10^3$)	135×10^{-3}
Peak current [A]	3.4 kA	40
Energy spread (rms)	1×10^{-4}	$2\ 10^{-5}$
Beam size (rms) [microns]	7.7	67.3
Modulation wavelength [Å]	1.5	1×10^{-4}
FEL parameter (ρ)	5×10^{-4}	5.5×10^{-3}
$k\lambda_D$	1.13	0.85×10^{-2}

(The FEL for the LCLS injector used K=1.41, λ_u=7.3 cm to obtain λ=1 μ with 135 MeV)