Simple Physics for Marvelous Light: FEL Theory Tutorial

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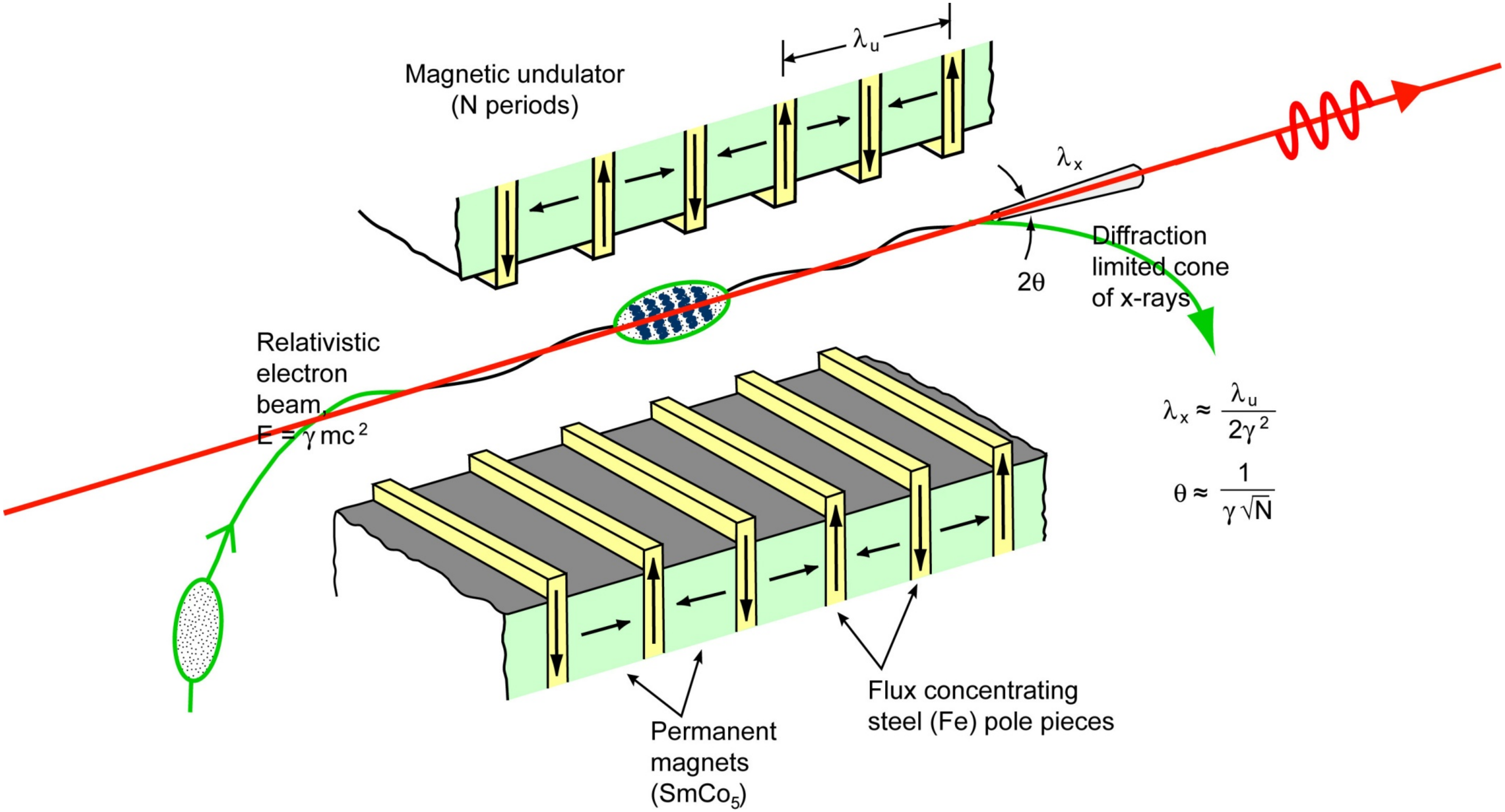
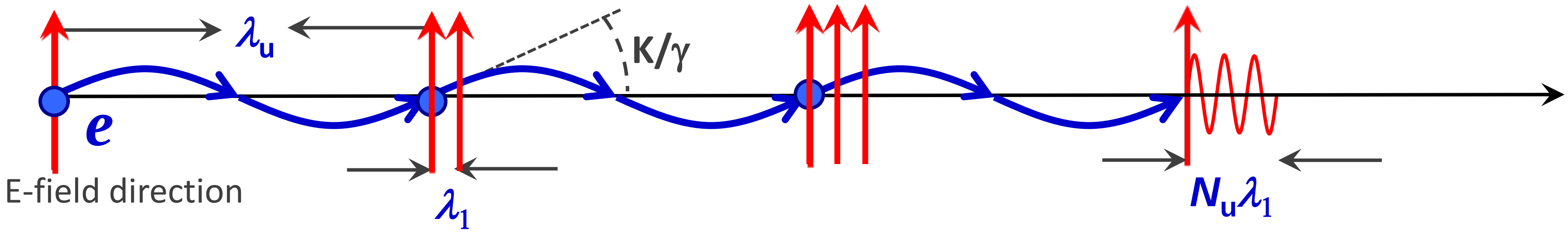
Undulators and Free Electron Lasers



Undulator **FEL Oscillator** **High-gain, single-pass FEL**

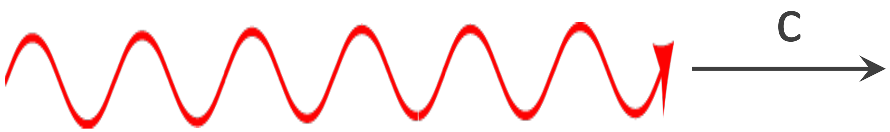
Spontaneous emission by one e^- in undulator

- The e^- emits EM wave in the forward direction due to its x-acceleration. Consider the wave fronts from successive undulator periods:



- The e^- is slower since (1) $c > v \simeq c(1-1/2\gamma^2)$, and (2) its trajectory is curved
- The distance the EM wave slips ahead of the e^- in one undulator period is the wavelength of the spontaneous emission:
  - $\lambda_1 = \lambda_u(1+K^2/2)/2\gamma^2$
- The length of the spontaneous emission for an N_u period undulator is
  - $\Delta z_{rad} = N_u \lambda_1$

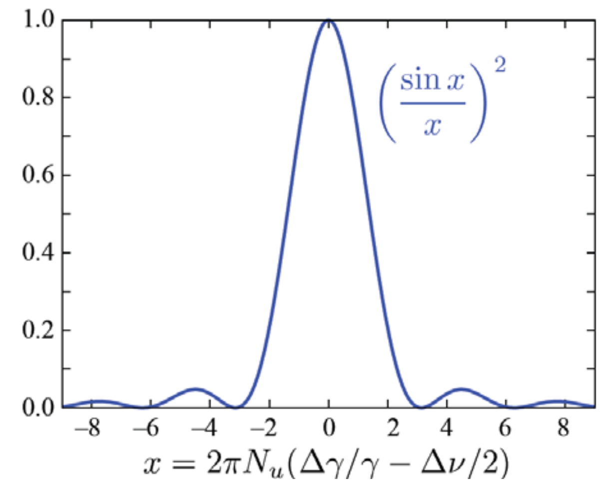
A coherent wavetrain of length $\Delta z = N_1\lambda_1$



- *At z* = 0*:* $E(t) = E_0 \exp(i\omega_1 t)$, $\omega_1 = 2\pi c/\lambda_1$, $-\Delta z/2c < t < \Delta z/2c$

- Frequency domain: $\tilde{E}(\omega) = \int dt\, exp(-i\omega t) E_0(t)$

$$\tilde{E}(\omega) = const \times N_1 \frac{\sin \pi N_1 \Delta \nu}{\pi N_1 \Delta \nu}, \quad \Delta \nu = \frac{\omega - \omega_1}{\omega_1}$$

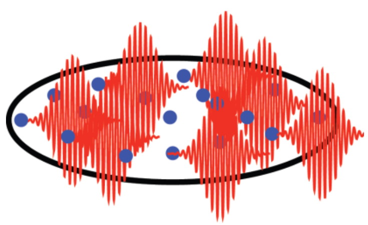


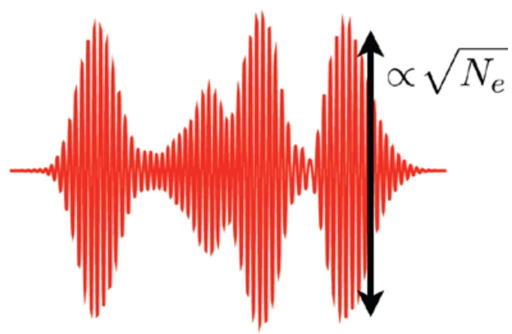
- Spectral intensity $dW(\omega)/d\omega \sim \left|\tilde{E}(\omega)\right|^2 \sim (\sin x/x)^2$ peaked around with relative bandwidth $\Delta\nu = \Delta\omega/\omega \sim 1/N_1$

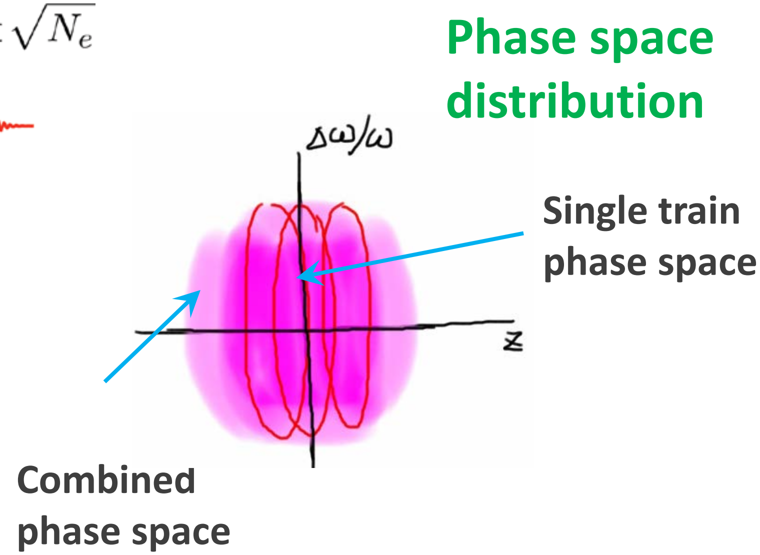
- For undulator radiation, the electron energy may be off by $\Delta\gamma$. Then $x = \pi N_u(\Delta\nu - 2\Delta\gamma/\gamma)$

Undulator radiation from a collection of electrons—a “bunch”

- The wave trains from N_e electrons in a bunch of length Δz_{el} combine to “chaotic light” of length Δz consisting of coherent “spikes” of length Δz_{rad}







$$\Delta z = \sqrt{\Delta z_{el}{}^2 + \Delta z_{rad}{}^2} = \sqrt{\Delta z_{el}{}^2 + (\lambda_1 N_u)^2}$$

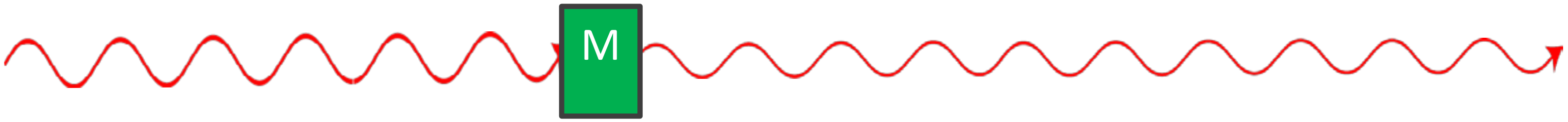
- Phase space area :

$$\Delta\Omega_t = \Delta z \times \Delta\omega / \omega = \sqrt{\lambda^2 + (\Delta z_{el} / N_u)^2} \geq \lambda_1$$

- **Temporal coherence: if $\Delta\Omega_t \sim \lambda_1$, then the front and back of the radiation pulse can be brought together for interference.**

A monochromator increases temporal coherence

A monochromator extends a wavetrain: $\Delta\omega \rightarrow \Delta\omega_M << \Delta\omega,\ \omega_1 / \Delta\omega_M = N_M$



- A collection of wavetrains becomes coherent $\Delta\Omega \rightarrow \lambda_1$ if $\Delta z_{el} / N_M << \lambda_1$

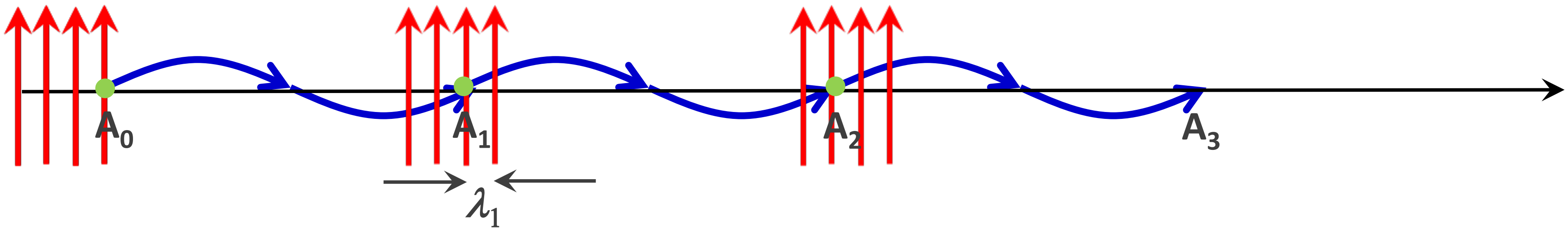
$$\Delta\Omega_{t,M} = \sqrt{\lambda_1^2 + (\Delta z_{el} / N_M)^2}$$

- However, the intensity of wavetrains in general add incoherently
- **The amplitudes add in phase if $\Delta z_{el} << \lambda_1$ *or if* electrons are concentrated at positions z $= n\lambda_1$, n=1,2,..**

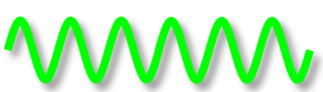
- ***This is what FELs are about!***

Energy exchange between e⁻ and coherent radiation

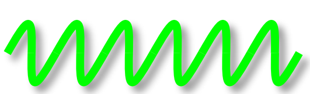
- An e⁻ and a coherent EM wave travel through the undulator. They can exchange energy due to the electron's x-velocity. However, the net exchange over many periods vanishes because of the velocity mismatch



- However, when the EM wavelength is λ_1 electrons see the same EM field in the successive period and the energy exchange can accumulate
- An e⁻ arriving at $\mathbf{A_0}$ loses energy to the field ($e\boldsymbol{v}\cdot\boldsymbol{E}<0$). Similarly for e⁻ s nearby and at distances $n\lambda_1$, n=1,2,… also lose energy. However, those at $\lambda_1/2$ away gain energy.
- The electron beam develops energy modulation (period length λ_1).



- Higher energy electrons are faster → density modulation develops



- Coherent EM of wavelength λ_1 is generated→ FEL gain

Equations for electron motion

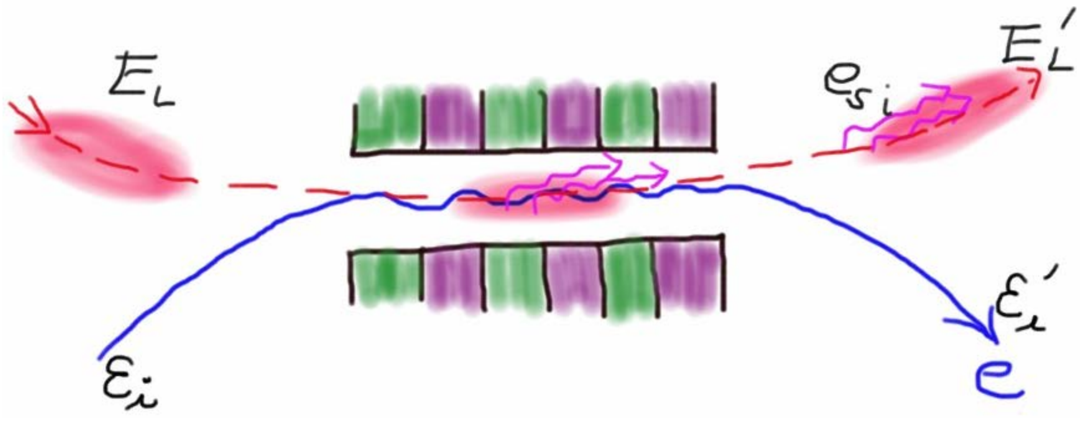
- Variables
  - z = distance along the undulator axis
  - θ = electron position in the moving bunch in units of $2\pi/\lambda$
  - η =electron relative energy: $(\gamma - \gamma_0)\ /\gamma_0$
- EM wave: $E_x = \hat{E}\cos(kz - \omega t)$
- **Pendulum equations**

$$\frac{d\theta_i}{dz} = (4\pi/\lambda_u)\eta_i$$: higher energy e moves faster

$$\frac{d\eta_i}{dz} = \frac{eK}{2\gamma^2 mc^2}\hat{E}\sin\theta_i$$: θ-dependent energy gain

- The electrons' energy loss averaged over initial θ becomes the gain in the EM field

Spontaneous & stimulated emission, and gain



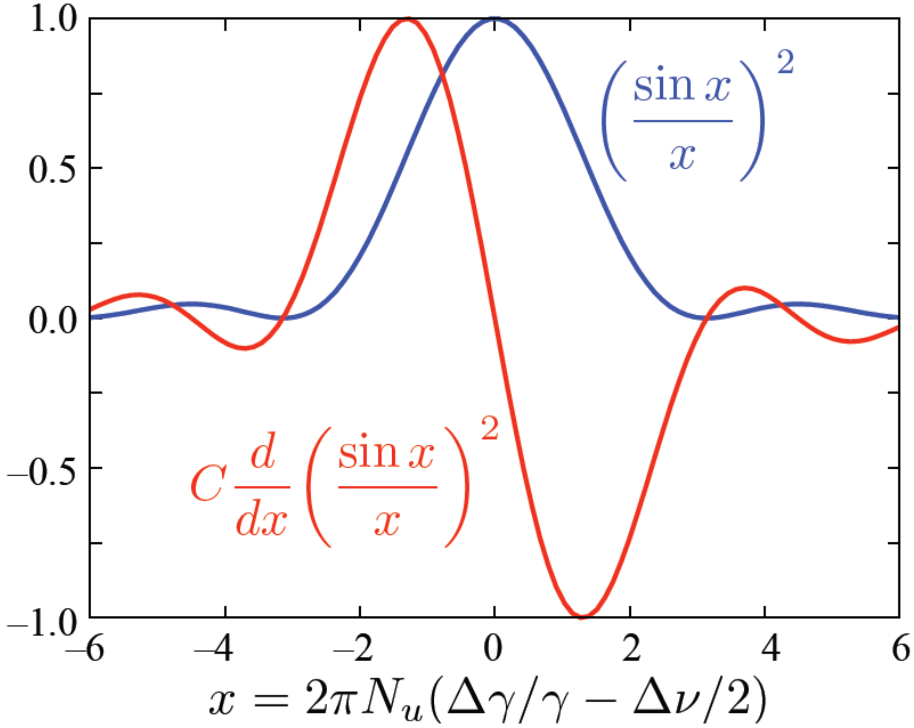
- Total energy conservation: $\mathcal{E}_i + \int |E_L|^2 = \mathcal{E}'_i + \int \left|E_L' + e_{s,i}\right|^2$
- Laser energy conservation : $\int |E_L|^2 = \int |E'_L|^2$
- $\rightarrow \mathcal{E}'_i = \mathcal{E}_i - 2\, Re \int e_{s,i} * E_L' - \int \left|e_{s,i}\right|^2$: $2\, Re \int e_{s,i} {}^* E_L' = W_{stim}$
- The amplitude $e_{s,i}$ depends on electron energy. Since, $\mathcal{E}_i \neq \mathcal{E}'_i$, we guess that it depends on the average: $\bar{\mathcal{E}}_i = \mathcal{E}_i - 0.5 \times W_{stim}$
- Thus $e_{s,i}(\mathcal{E}_i) \rightarrow e_{s,i}(\mathcal{E}_i - 0.5\, W_{stim}) = e_{s,i}(\mathcal{E}_i) - (\int e_{s,i} * E_L')(\partial e_{s,i} / \partial \mathcal{E}_i)$
- The last term adds to E_L' : $E_L' \rightarrow (1+g) E_L'$, $g = -\frac{1}{2}\frac{\partial}{\partial \mathcal{E}} \sum_i \left|e_{s,i}\right|^2$
- ***Gain is the derivative of the spontaneous emission spectrum***

Madey's theorem for power gain

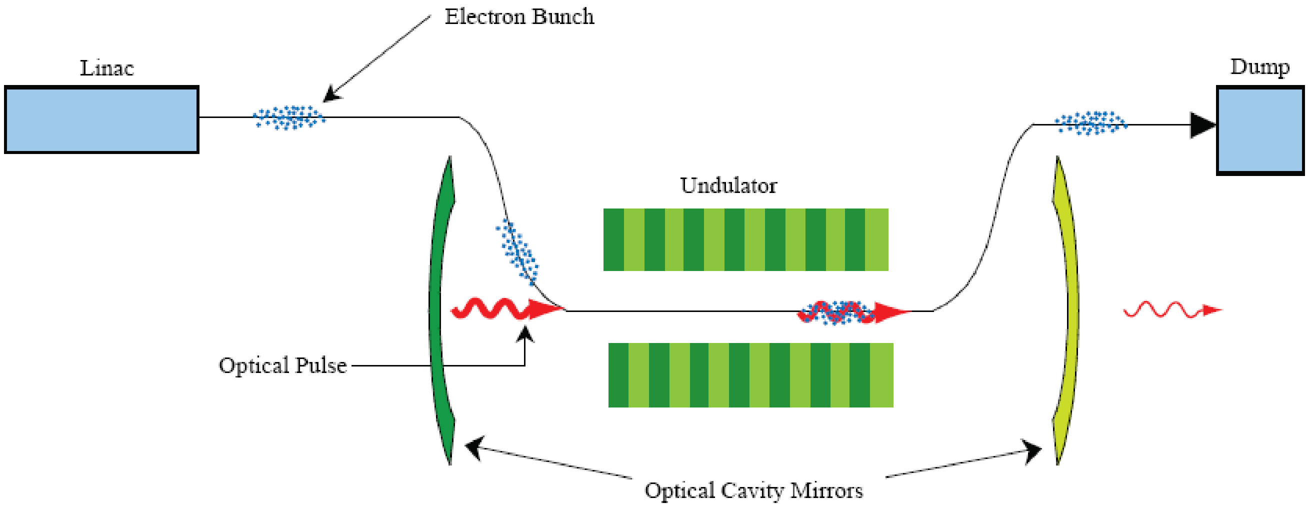
$$G = -\pi \frac{I}{emc^2} \frac{\partial}{\partial \gamma} \left(\frac{dW_{spon}(\omega, \gamma)}{d\omega} \right)$$

- $dW_{spon}(\omega, \varepsilon)/d\omega$ = the spectral density of spontaneous emission, *I* = the electron current)

The FEL gain BW is about half of the spontaneous emission BW



Low-gain FEL oscillator principles



- Synchronism: The spacing between e bunches =2L/n (L=cavity length)
- Lasing starts if: $(1+G_0)\ R_1\ R_2 > 1$ ($R_{1,2}$: mirror reflectivity), G_0= initial gain

→ Need high-reflectivity, normal-incidence mirrors

→ Could be difficult for soft x-ray and shorter wavelengths?

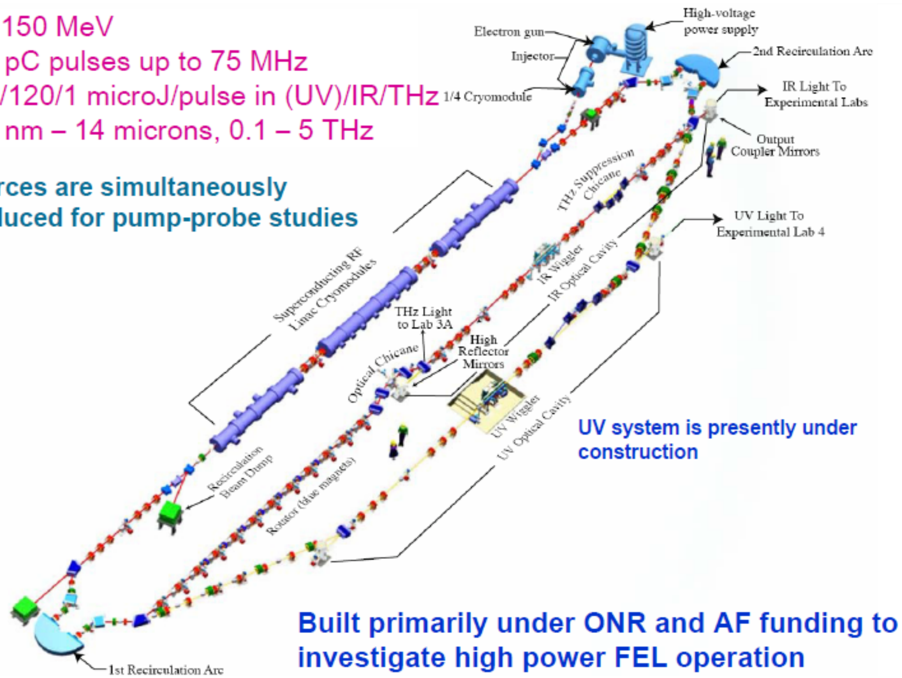
- The gain G decreases as the intracavity power increases. A steady state (“saturation”) is reached when $(1+G_{sat})\ R_1\ R_2 = 1$
- Output power = $(1-R_{1,2})$ intracavity power –loss in mirrors
- FELOs have been built for IR, visible, and UV wavelengths

Existing and future FEL oscillators

Jlab IR-UV User Facility

E = 150 MeV
135 pC pulses up to 75 MHz
(20)/120/1 microJ/pulse in (UV)/IR/THz
250 nm – 14 microns, 0.1 – 5 THz

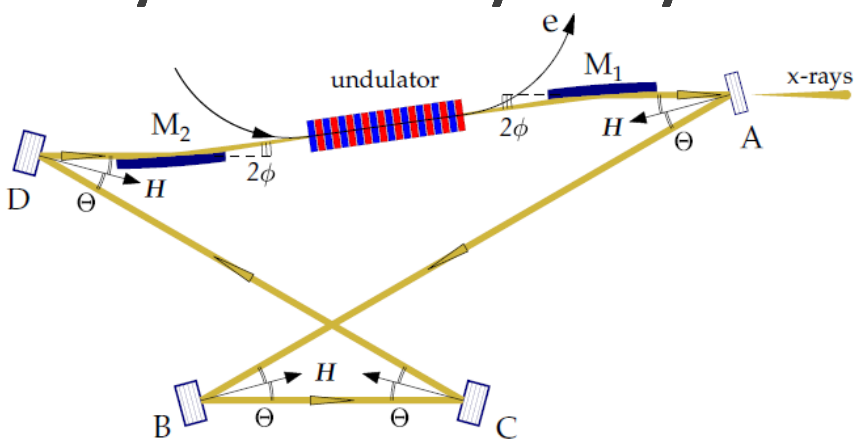
Sources are simultaneously produced for pump-probe studies



UV system is presently under construction

Built primarily under ONR and AF funding to investigate high power FEL operation

Possible future hard x-ray FEL oscillator using diamond crystals for x-ray cavity



By varying the incidence angle Θ, one can obtain a wide range of photon energies that satisfy Bragg's law $E = E_H \cos\Theta$

Tunability allows one to pick a single crystal for all wavelengths of interest

Low gain FEL oscillator performance: Intensity

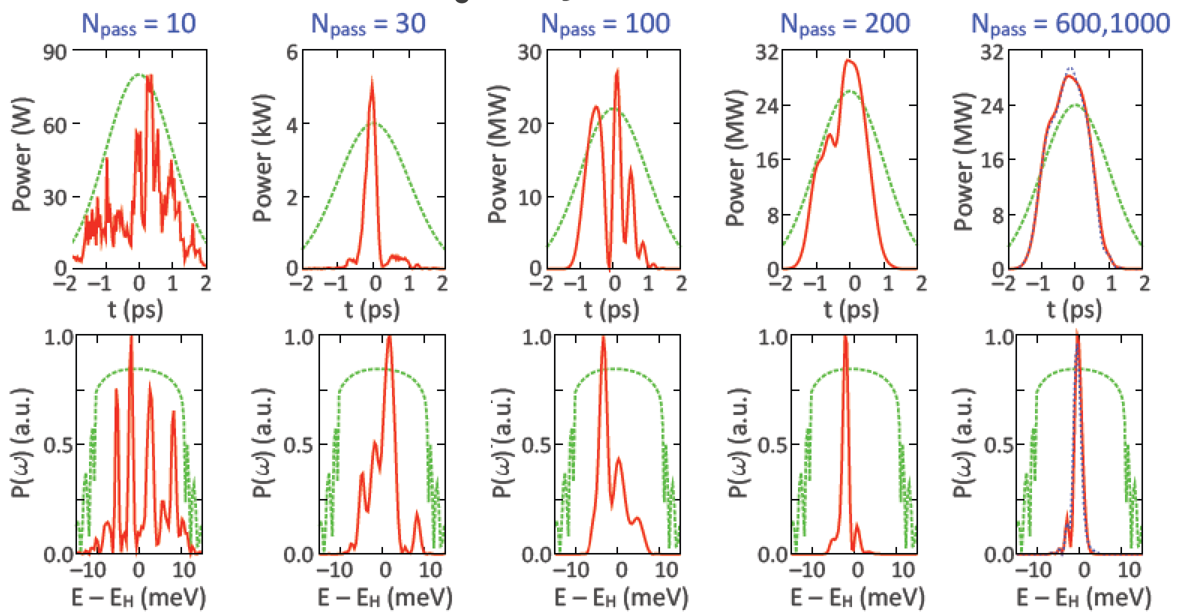
- FEL efficiency:
  - Spontaneous radiation spectral width $\Delta\omega/\omega = 2\Delta\gamma/\gamma \sim 1/N_u$
  - We have seen : $\Delta\gamma/\gamma|_{FEL} = {}^1/_2\, \Delta\gamma/\gamma|_{spon} \approx \frac{1}{4N_u}$
- **FEL output power** $\approx (\mathbf{1/4}\boldsymbol{N_u}) \times$ **electron beam power**
- Electron energy spread requirement: $\Delta\gamma/\gamma|_{spread} \leq 1/4N_u$

Temporal and spectral evolution

- As the roundtrip pass number *n* increases
  - The spectral width decreases: $\Delta\boldsymbol{\omega}/\boldsymbol{\omega} \propto \mathbf{1}/\sqrt{\boldsymbol{n}}$
  - The pulse width decreases: $\Delta\boldsymbol{z} \propto \mathbf{1}/\sqrt{\boldsymbol{n}}$
- Evolution stops when $\Delta\boldsymbol{z} \times \Delta\boldsymbol{\omega}/\boldsymbol{\omega} \rightarrow \boldsymbol{\lambda}$
- → The limiting spectral width (the super-mode theory)

$$\frac{\Delta\omega}{\omega} \rightarrow \sqrt{\frac{1}{2N_u}\frac{\lambda}{\Delta z|_0}}$$ =(gain BW □ "transform limited BW")$^{1/2}$

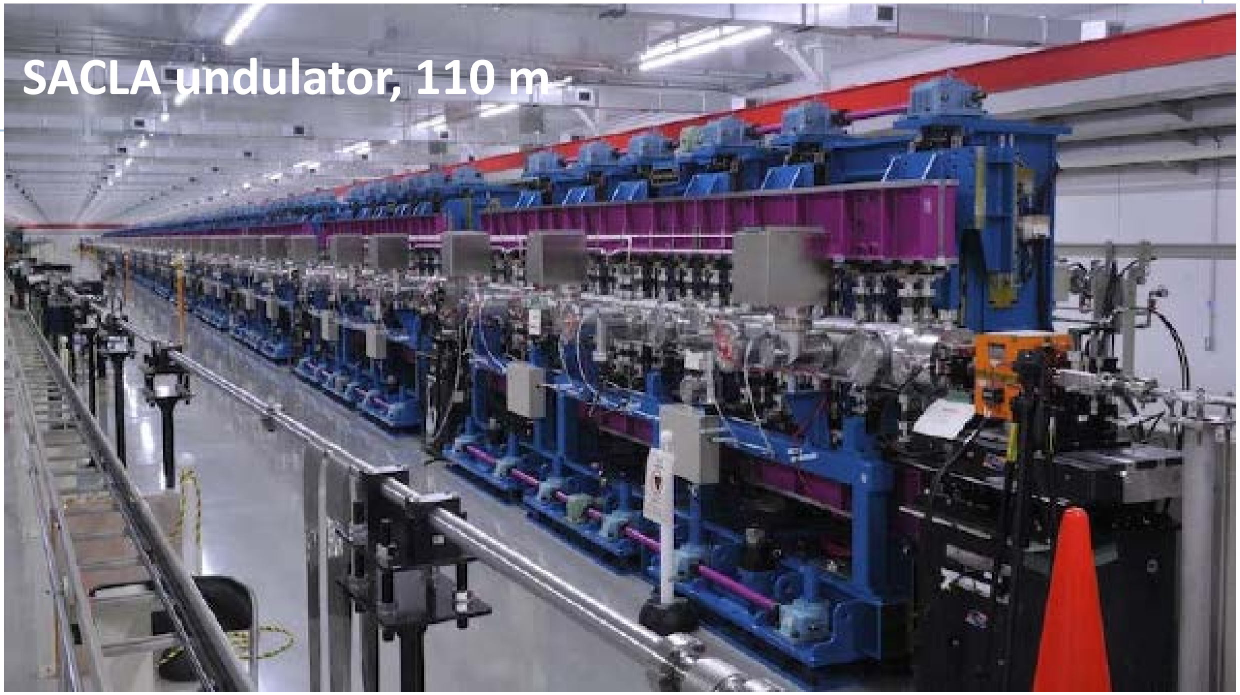
- However, the full transform limit $\Delta\omega/\omega=\lambda/\Delta z_0$ may be achieved with nonlinear saturation:



XFELO: $\lambda/\Delta z$ ~ 10^{-7} for λ=1Å and Δz=1 ps

High-gain, single-pass FELs

- With high electron beam qualities and **a long undulator**, the single pass gain can be made very high
- If coherent seed is available, then amplifier mode with harmonic generation produces intense, coherent, short wavelength output
- Without a seed, SASE (self-amplified-spontaneous–emission) produce quasi-coherent output



Exponential growth in high-gain FEL

- Three quantities characterizing the amplification process
  - ***E*** :EM amplitude:
  - $\boldsymbol{b}$=< exp(-iθ) > :Density modulation:
  - $\boldsymbol{P}$=<η exp(-iθ)> :Energy modulation:
- Evolution as a function of *z*
  - EM field induces energy modulation: $\mathbf{d}P/\mathbf{dz}= 2k_u C_1 E$
  - Energy modulation induces density modulation: $\mathbf{d}b/\mathbf{dz}= -i2k_u P$
  - Density modulation generates EM field: $\mathbf{d}E/\mathbf{dz}= 2k_u C_2 b$

→ $d^3E/dz^3= -i(2k_u)^3 C_1 C_2 E$; $\rho^3 = C_1 C_2 = \frac{1}{8\pi}\frac{I}{I_A}\frac{K^{*2}}{1+K^2}\frac{\gamma\lambda^2}{\Sigma_A}$

- The solution is $E=\Sigma\, a_i \exp(-2i\mu_i\rho\, k_u z)$:

□ μ_i are solutions of $\mu^3=1$ $\mu_1=1, \mu_2 = \frac{-1-\sqrt{3}i}{2}$, $\mu_3 = \frac{-1+\sqrt{3}i}{2}$

– **The root μ_3 gives rise to an exponential growth**

General 1-D solution in linear regime

- Slowly varying amplitude in frequency domain $E_\nu(z)$, $\nu = \Delta\omega/\omega_1$

- The solution of the initial value problem for an arbitrary momentum distribution V(η) is:

$$E_\nu(z) = \sum_j \frac{e^{-2i\mu_j \rho k_u z}}{D'(\mu_j)} \left[E_\nu(0) + i \frac{eK^* n}{8\varepsilon_0 \gamma \rho k_u N_\lambda} \sum_{j=1}^{N_e} \frac{e^{-i\nu\theta_j(0)}}{\eta_j(0)/\rho - \mu_j} \right], \quad D'(\mu) = \frac{dD(\mu)}{d\mu}$$

- The first term is coherent amplification and the second SASE
- Here μ_j is the solution of ($\Delta\nu$ = "detuning"= $\nu - 1 << 1$):

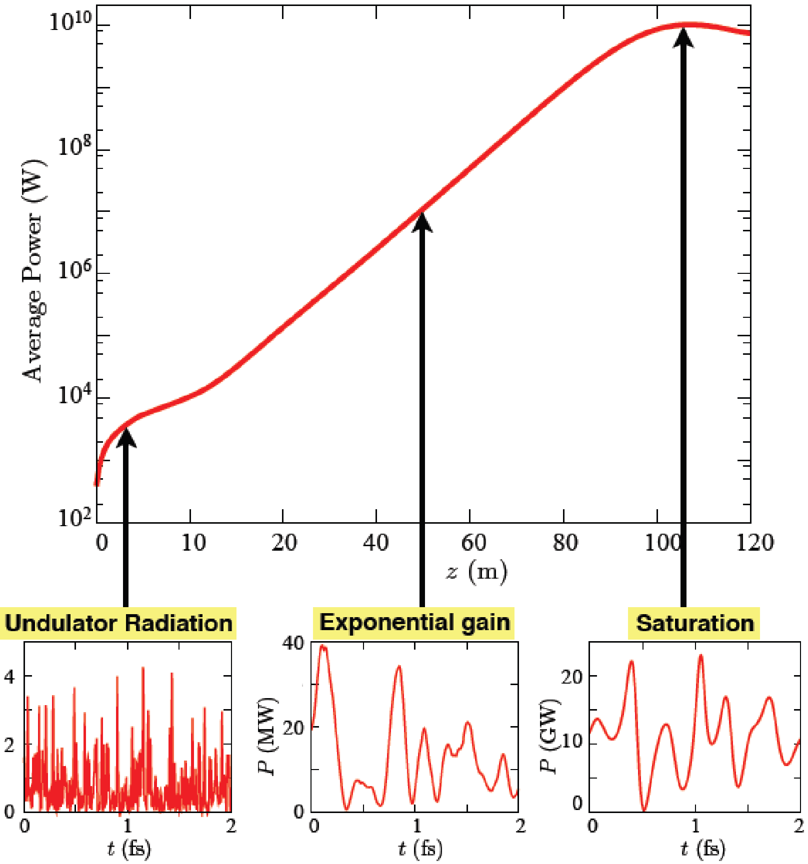
$$D(\mu) = \mu - \frac{\Delta\nu}{2\rho} - \int d\eta \frac{V(\eta)}{(\eta/\rho - \mu)^2} = 0$$

- **For cold beam at $\Delta\nu = 0$: $\mu - 1/\mu^2 = 0$ (reproduce the cubic equation)**

The main characteristics of high-gain FEL in terms of ρ

- For a typical x-ray FEL, $\rho \sim 10^{-3}$
- Power gain length: $L_G \sim \lambda_u/(4\pi\rho)$
- Undulator periods for saturation: $\sim 1/\rho$
- Spectral bandwidth: $\Delta\omega/\omega \sim \rho$
- Coherence length: $\ell_c \sim \lambda/\rho \ll \Delta z_e$

  → "Chaotic" light
- Saturation power: $\sim \rho \times$ beam power
- Maximum amplification factor:
  ~ # of electrons in ℓ_c



Transverse properties of propagating EM waves

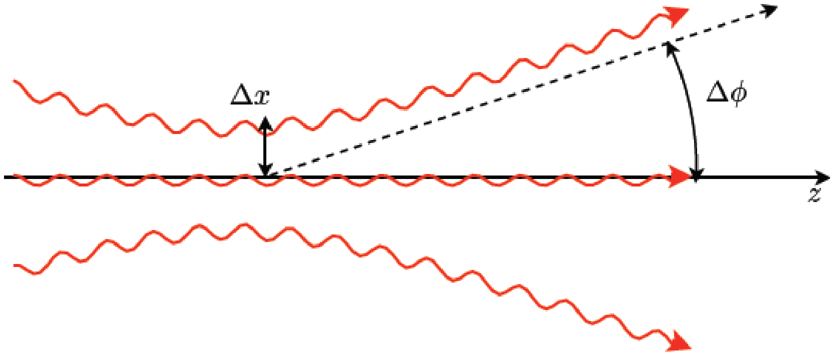
- **Coherent EM wave diffracts if transversely restricted by Δx:**

  → $\Delta\phi = \lambda/\Delta x$ → $\Delta\Omega_x = \Delta x\, \Delta\phi = \lambda$ -----(1)

- **An extended source of length L has a depth-of-field blur**

  → $\Delta x = \Delta\phi\, L$ ----------(2)

  → $\Delta x = \sqrt{\lambda L}$ **and** $\Delta\phi = \sqrt{\lambda / L}$



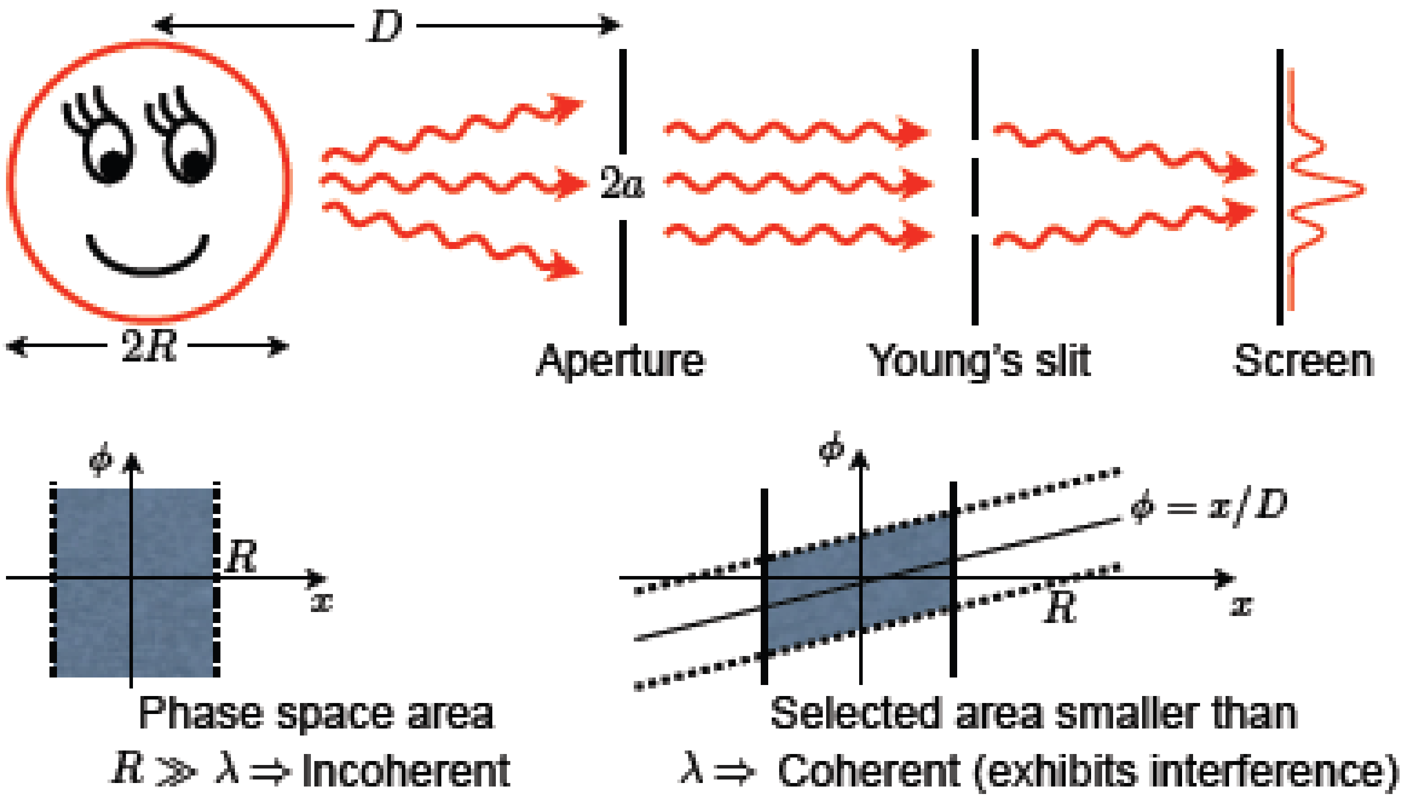
- For fundamental Gaussian mode:

$$\sigma_r = \sqrt{\frac{\lambda}{4\pi} Z_R} \quad \sigma_{r'} = \sqrt{\frac{\lambda}{4\pi}\frac{1}{Z_R}} \quad \sigma_r \sigma_{r'} = \frac{\lambda}{4\pi}$$

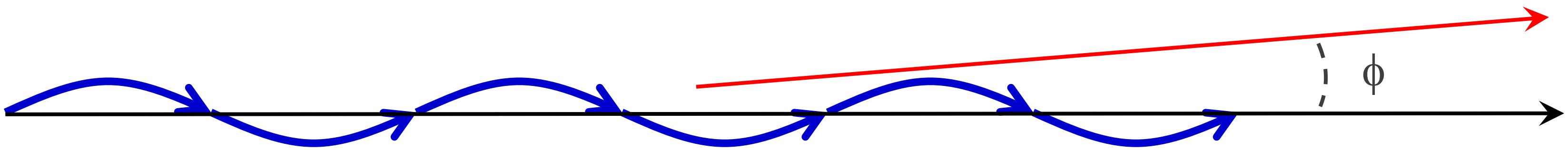
- **Note the correspondence between the light and e-beam optics:**
  - **$\lambda/4\pi$ ↔ ε_x (rms emittance)**

    Z_R (Rayleigh length) ↔ β– function

An incoherent light can be made coherent by selecting a phase space area of λ. Transversely coherent light exhibits interference.

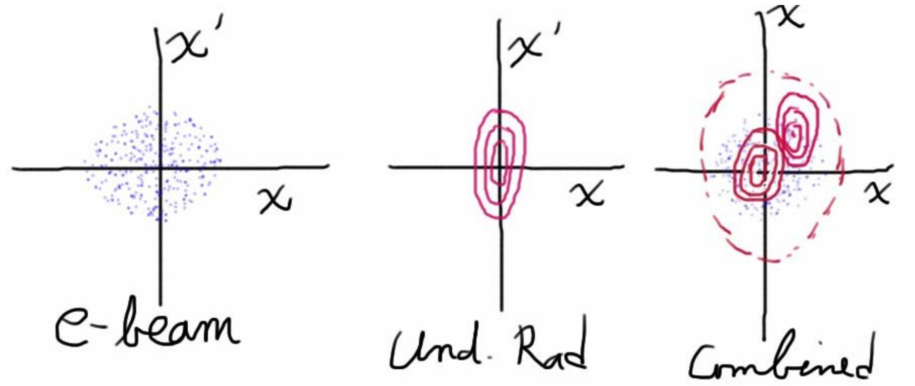


Transverse properties of undulator radiation



- Red shift at an angle ϕ, $\lambda_1(\phi)=\lambda_u(1+\phi^2\gamma^2+K^2/2)/2\gamma^2$
- Angular divergence of central cone $\sigma_{\Delta\phi}=\sqrt{\lambda_1/2L_u}=\sqrt{\left(\frac{\lambda_1}{4\pi}\right)\left(\frac{2\pi}{L_u}\right)}$
- The undulator radiation by one-electron is approximately the transversely coherent Gaussian mode with $Z_R \sim L_u/2\pi$
- Undulator radiation from a beam of electrons is a convolution of the one-electron radiation and the electron beam distribution

→Transversely coherent if $\varepsilon_x \leq \lambda$

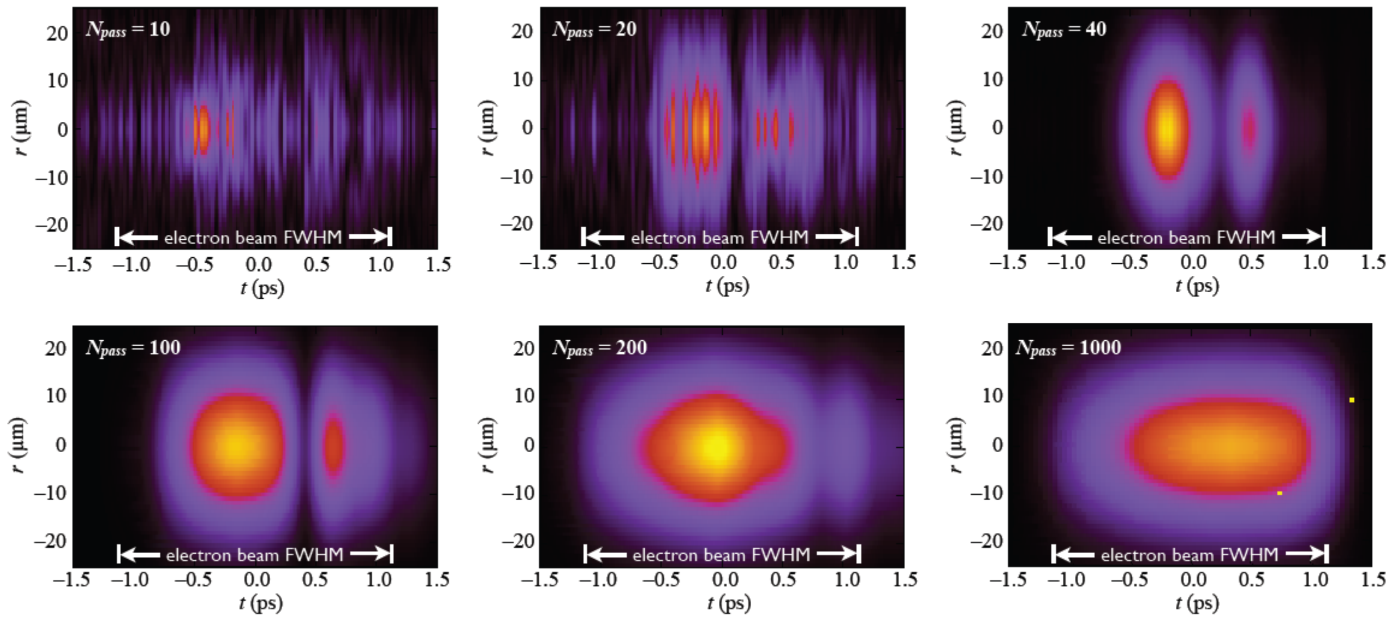


Temporal bunching implies transverse coherence → FELs are transversely coherent

- **In low gain oscillator, the mode is similar to free-propagating Gaussian laser mode**
- **In high-gain FEL the mode is guided**

Low gain FEL oscillator is transversely coherent

- The eigenmode is an approximately free-propagating Gaussian mode with $\boldsymbol{Z_R \sim L_u/2\pi}$ and the waist in the middle of the undulator
- Temporally coherent along the full length of the pulse→ transversely coherent
- ***FELO is coherent transversely as well as temporally***

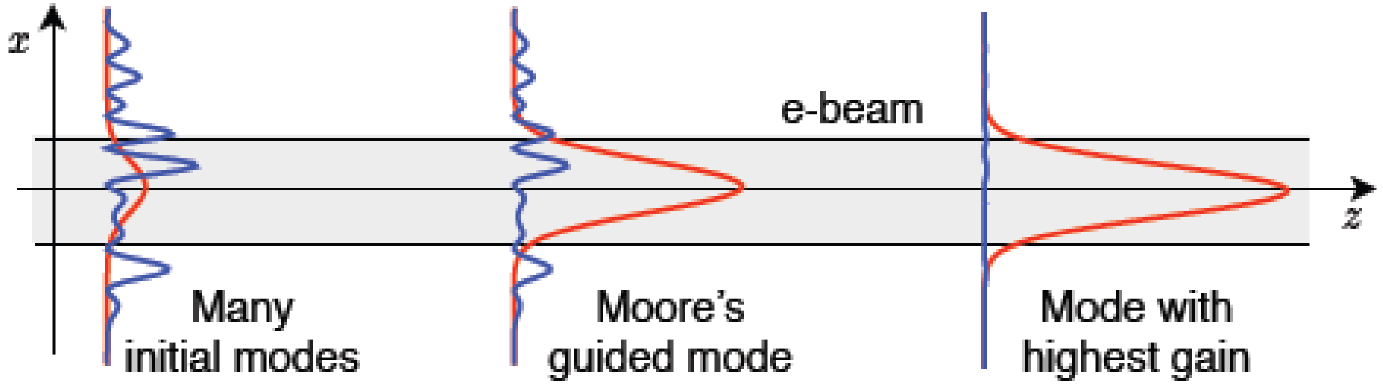


Transverse behavior in high-gain FELs

- The 1D dispersion relation becomes the eigenmode equation for the complex growth rate μ and the associated transverse mode shape. For parallel electron beam with the density profile $U(x)$:

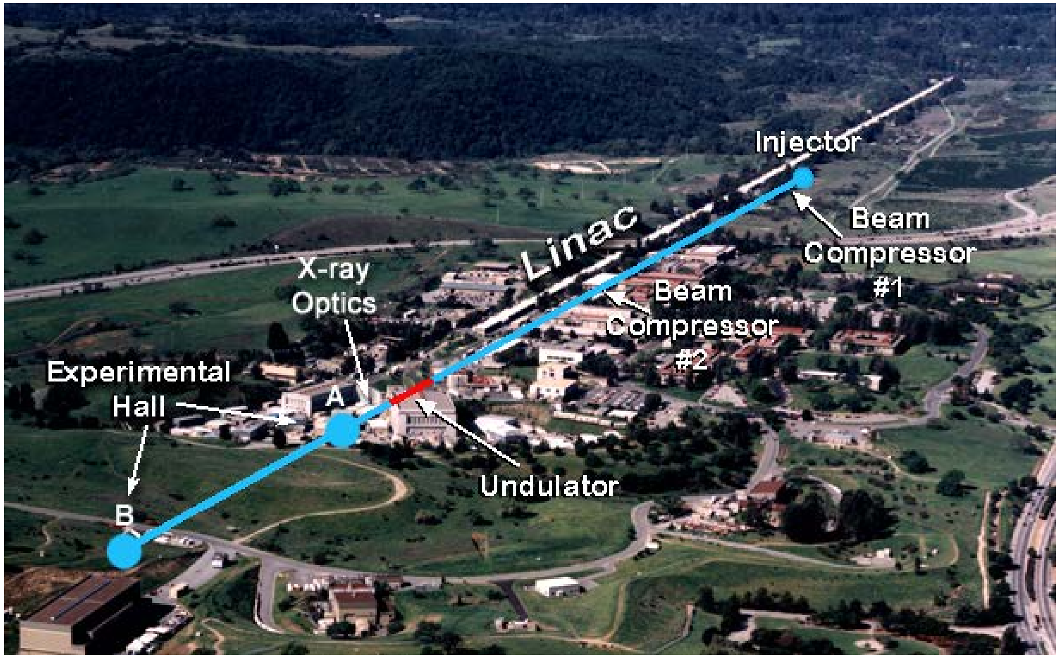
$$\left(\mu - \frac{\Delta\nu}{2\rho} + \Lambda^2\nabla_T^2 - U(x)\int d\eta \frac{V(\eta)}{(\mu - \eta/\rho)^2}\right)A(x) = 0$$

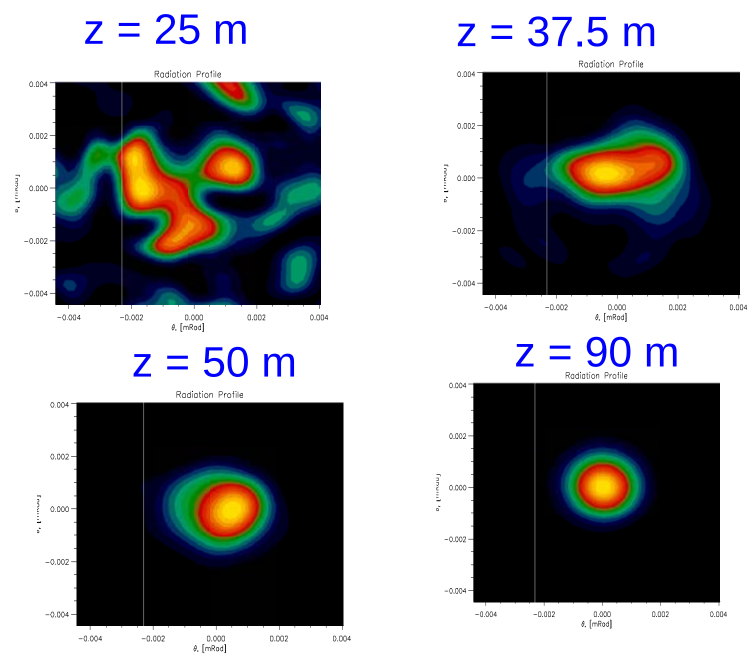
- The electron's betatron motion can be included.
- The initial value problem can be solved as a sum of eigenmodes
- **A single mode dominates in the exponential growth regime → Frozen (or guided) transverse profile of mode size ~$\sqrt{(\lambda/4\pi)L_G}$**

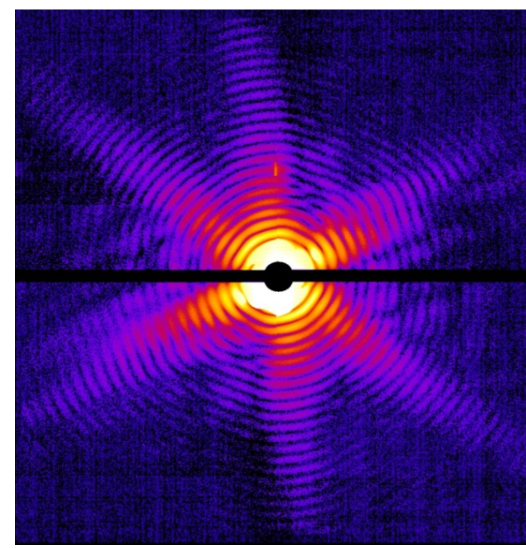


LCLS

Transverse mode development

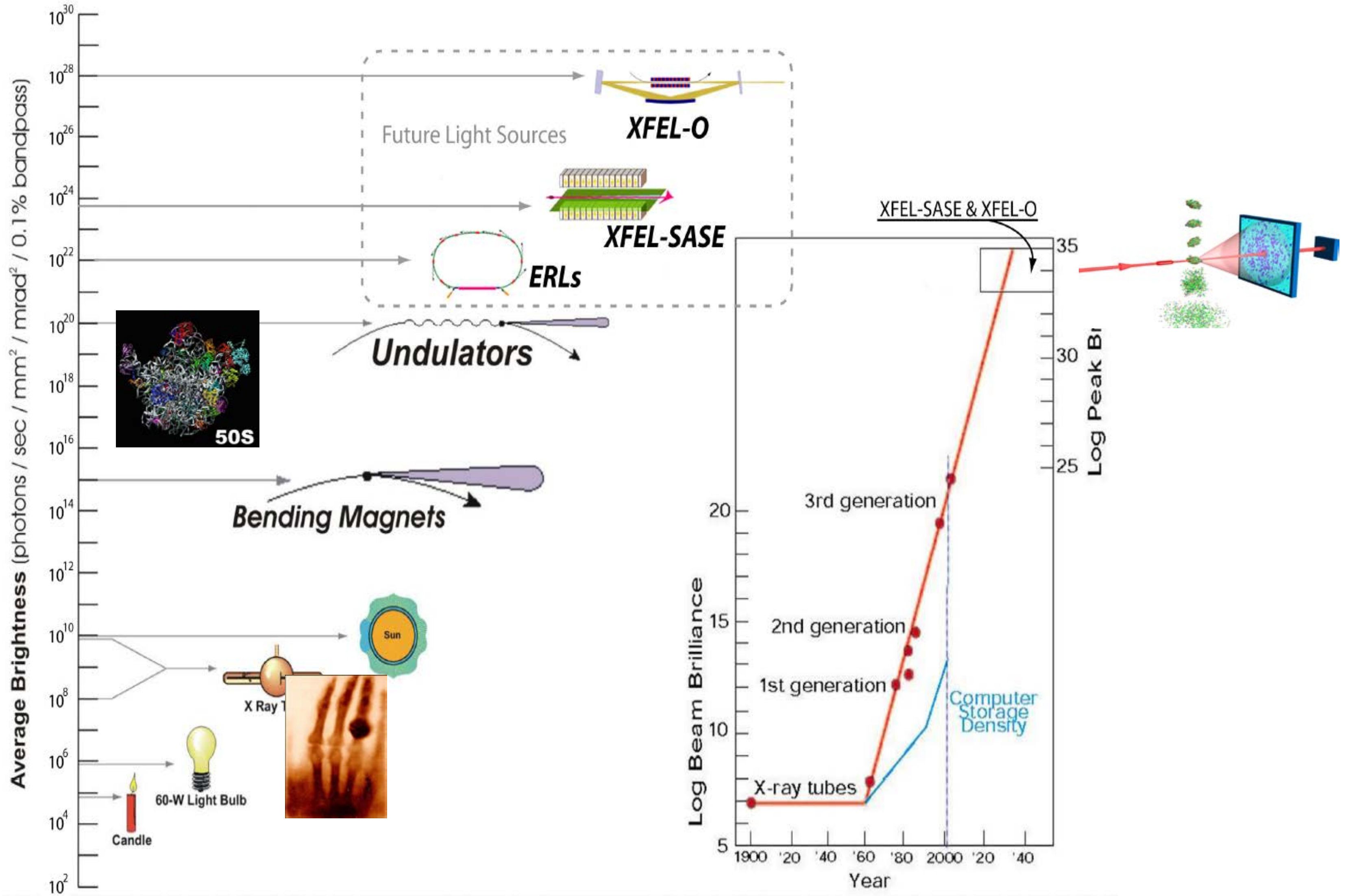






X-ray diffraction pattern of a single Mimivirus particle imaged at the LCLS. (Image courtesy of Tomas Ekeberg, Uppsala University.)

The progression of marvelous x-ray sources from spectral brightness point of view



Additional topics (in x-ray FEL)

- **Harmonic generation for coherent soft x-ray**
  – Echo-enabled scheme for high harmonics
- **Improving the temporal coherence of high-gain x-ray FEL**
  – Use input coherence radiation via harmonic generation (FEL, laser, or both)
  – Filter SASE by a monochromator and send through another high gain FEL—self seeding scheme
- **Ultra short pulse (~100 atto-second) generation**
  – single-cycle energy modulation→ compression→ subfemto-second current spike → enhanced FEL growth rate
- **Other bright ideas YOU might invent, triggered by this tutorial!**

Thank you for the Program Committee for the invitation, and Those who helped me to prepare this talk

- **Ryan Lindberg**
- **Sven Reiche**
- **Max Zolotorev**
- **…..**

Thank you!