# LCLS-II UNDULATOR TOLERANCE ANALYSIS*

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*Abstract*

The SLAC National Accelerator Laboratory is building a new FEL user facility, LCLS-II, as a major upgrade to the Linear Coherent Light Source (LCLS). The upgrade will include two new Free Electron Lasers (FELs), to generate soft (SXR) and hard x-ray (HXR) SASE FEL radiation, based on planar, variable gap hybrid undulators with two different undulator periods (SXU: 55 mm, HXU: 32 mm). An algebraic FEL tolerance analysis for the undulator lines, including tuning, alignment, and phase correction tolerances has been performed. The methods and results are presented in this paper.

## INTRODUCTION

Individual parameter tolerance studies for X-ray FEL undulators have been done for many years (e.g [1]). For the LCLS undulator design a tolerance budget approach based on computer simulations [2] was performed. For LCLS-II an attempt is being made to derive algebraic expressions of the basic tolerances. This method will reduce the dependence on computer simulations, which can become quite extensive for multiple, variable gap undulator systems. This paper reports on work in progress. Parameters are likely to be added in the future.

## TOLERANCE ANALYSIS

The output power of an x-ray SASE FEL can be reduced if any of a number of parameters, $p_i$, deviates from its optimum value. To determine tolerances for these deviations, a budget approach was introduced during the LCLS construction period. The algorithm is based on the fact that the average reduction in output power, $P_i$, due to a random spread (with standard deviation $q_i = \left(f(p_i)\right)_{rms}$) of many of these parameters, $p_i$, or simple functions, $f(p_i)$, thereof, can be modeled as a Gaussian

$$\frac{P_i}{P_0} = e^{-\frac{q_i^2}{2\sigma_i^2}} \quad (1)$$

for $q_i < \sigma_i$. Once the rms performance degradations, $\sigma_i$, are determined, the allowable performance degradations, $P_i/P_0$, for that parameter can be used to determine the tolerance values

$$t_i = f^{-1}\left(\sigma_i\sqrt{-2ln(P_i/P_0)}\right). \quad (2)$$

The individual levels of acceptable performance degradations can be fine-tuned in a performance budget such that the total performance reduction stays within a given limit

$$\frac{P}{P_0} = \prod \frac{P_i}{P_0} = e^{-\frac{1}{2}\Sigma\frac{q_i^2}{\sigma_i^2}} \stackrel{\text{def}}{=} e^{-\frac{1}{2}\Sigma r_i^2}. \quad (3)$$

For LCLS, the budget analysis was done for a set of 8 parameters, for which the $\sigma_i$ were calculated using a statistical analysis based on computer simulations using the FEL simulation code GENESIS 1.3 [3]. Some of the results were later verified with actual measurements at LCLS [4]. For the LCLS-II tolerance analysis, an attempt is presented here to establish algebraic approximations for the $\sigma_i$. The parameters used are listed in Table 1 (on the last page) and described in the next section.

## FEL PERFORMANCE DEGRADATION

The rms performance degradations, as discussed in this section, are for a SASE x-ray FEL based on a linear arrangement of a number of identical planar variable gap pure permanent magnet undulator segments that are separated by drift spaces (called breaks or break sections) and contain each, among other components, a quadrupole magnet and a phase shifter.

### Launch Angle

The electron beam should be launched into the undulator system on-axis. Finite launch angles $x'$ or $y'$ cause the electrons to execute betatron oscillations along the FEL undulator which can reduce FEL gain.

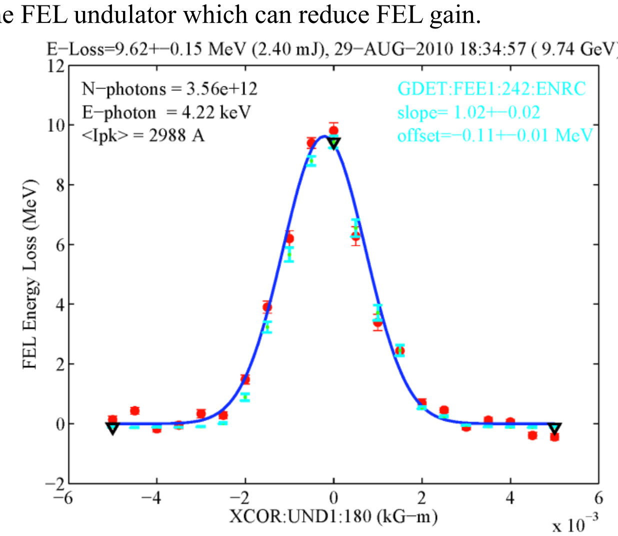


Figure 1: LCLS *eloss* [5] scan at 9.74 GeV. The x-axis is corrector strength, proportional to kick angle, $x'$.

Measurements at LCLS, using the *eloss* scan method (see example in Figure 1), have shown that the dependence of FEL power on launch angle is Gaussian with rms width $\sigma_{x'}$. The dependence of $\sigma_{x'}$ on electron energy was measured as $\sigma_{x'} = A/\sqrt{\gamma^3}$, with $A = (9.8\pm0.3)$ rad. The

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scaling found, is consistent with the scaling of the critical angle, $\Theta_c$, as defined in [6]

$$\Theta_c = \sqrt{\frac{\lambda_r}{L_G}} = \sqrt{\frac{2\pi\sqrt{3}\varrho}{\gamma^2}\left(1+\frac{1}{2}K^2\right)} \stackrel{\text{def}}{=} \frac{B}{\sqrt{\gamma^3}}, \qquad (4)$$

using the well known FEL relations

$$\lambda_r = \frac{\lambda_u}{2\gamma^2}\left(1+\frac{1}{2}K^2\right),$$
$$L_G = \frac{\lambda_u}{4\pi\sqrt{3}\varrho}, \qquad (5)$$
$$\varrho = \frac{1}{\gamma}\left[\frac{1}{16}\frac{I_e}{I_A}\frac{K^2[JJ]^2}{\sigma_{x,y}^2 k_u^2}\right]^{1/3}$$

and parameters: radiation wavelength, $\lambda_r$, undulator parameter, $K$, Lorentz factor, $\gamma$, power gain length, $L_G$, FEL efficiency parameter, $\varrho$, peak electron bunch current, $I_e$, Alven Current, $I_A$, the usual Bessel function argument, $JJ$, transverse rms electron beam size, $\sigma_{x,y}$, and undulator wave number, $k_u$. With the values used during the measurements, i.e, $I_e$= 3 kA, K=3.5, $JJ$=0.74, and $\sigma_{x,y}$ = 30 µm, we get $B$ = 32.7 rad. The ratio $B/A$ is about 3.3, which means

$$\sigma_{x'} = \frac{1}{3.3}\Theta_c = \frac{1}{3.3}\sqrt{\frac{\lambda_r}{L_G}}. \qquad (6)$$

For the tolerance analysis we need to know how the average performance is degraded if the parameter is randomly distributed over a given range. We base the calculation on a random flat-top distribution of the parameter of interest in the range $[-a, a]$. The standard deviation of such a distribution is $a/\sqrt{3} \stackrel{\text{def}}{=} q_i$. In the case of the launch angle, we can calculate the average performance degradation on $q_i \equiv q_{x'}$

$$\frac{P_{x'}}{P_0}(q_{x'}) = \int_{-\sqrt{3}q_{x'}}^{\sqrt{3}q_{x'}} e^{-\frac{x'^2}{2\sigma_{x'}^2}}dx' = \sqrt{\frac{\pi}{6}}\frac{\sigma_{x'}}{q_{x'}} Erf\left(\sqrt{1.5}\frac{q_{x'}}{\sigma_{x'}}\right). \qquad (7)$$

The right-hand side of Eq. (7) can be approximated by

$$e^{-\frac{q_{x'}^2}{2\sigma_{x'}^2}} \qquad (8)$$

The difference is

$$e^{-\frac{q_{x'}^2}{2\sigma_{x'}^2}} - \frac{P_{x'}}{P_0}(q_{x'}) = \frac{1}{10}\left(\frac{q_{x'}}{\sigma_{x'}}\right)^4 + O(q_{x'})^6. \qquad (9)$$

By combining launch angle errors in both planes $\sigma_{\Delta\Theta_L}^2 = \sigma_{x'}^2 + \sigma_{\Delta y'}^2$, we get a tolerance according to Eq. (2) of

$$t_{\Delta\Theta_L} = \sigma_{\Delta\Theta_L}\sqrt{2ln\left(P_0/P_{\Delta\Theta_L}\right)}. \qquad (10)$$

### *Phase Errors*

The break sections between undulator segments will cause the regular phase slippage, which occurs along regular undulator periods, to be disturbed. For variable gap undulators, this disturbance is corrected with phase shifters that are installed in each break section. In order to arrive at an estimate for the tolerance to phase errors we note that the launch error tolerance, derived in the previous section, corresponds to a fixed phase delay, $\Delta\varphi_{LG}$, per power gain length, $L_G$,

$$\Delta\varphi_{LG} = \frac{2\pi}{\lambda_r}\frac{L_G}{2}\sigma_{x'}^2 = \frac{\pi}{3.3^2}. \qquad (11)$$

Now, we make the assumption that the sensitivity to phase errors over a power gain length is constant, as well. GENESIS 1.3 simulations, done to estimate the tolerance for rms performance degradations due to break length errors for LCLS-I at 13.64 GeV, resulted in $\sigma_{\Delta L_{break}}$ = 17 mm. This can be converted to a phase tolerance

$$\sigma_{\Delta\varphi_{break}} = 2\pi\frac{\sigma_{\Delta L_{break}}}{\lambda_u\left(1+\frac{1}{2}K^2\right)}. \qquad (12)$$

With the LCLS parameters $\lambda_u$=0.03 m, and $K$=3.5 we get $\sigma_{\Delta\varphi_{break}} \sim 0.5\, L_{section}/L_G$. $L_{section}$ is the average distance between successive break sections, which, for LCLS at these parameters, is roughly equal to one power gain length. Section phase errors can be caused by different mechanisms: (1) an error in the phase tuning of the undulator segment ($\Delta\varphi_{cell}$) (2) an error in break length ($\Delta\varphi_{break}$) (3) an error in the phase shift provided by the phase shifter ($\Delta\varphi_{PS}$). The same tolerance should be applicable to each of these contributions

$$\sigma_{\Delta\varphi_{cell}} = \sigma_{\Delta\varphi_{PS}} = \sigma_{\Delta\varphi_{break}} = \frac{1}{2}\frac{L_{section}}{L_G} \qquad (13)$$

and for the actual break length error, $\Delta L_{break}$,

$$\sigma_{\Delta L_{break}} = \frac{\sigma_{\Delta\varphi_{break}}}{2\pi}\lambda_u\left(1+\frac{1}{2}K^2\right). \qquad (14)$$

The individual tolerances can be calculated by

$$t_j = \sigma_j\sqrt{2ln\left(P_0/P_j\right)}. \qquad (15)$$

They can be different, depending on achievability and will be determined below as part of the tolerance budget.

### *Relative Undulator Parameter, K*

Changing the $K$ value of one of a number of FEL undulator segments from its optimum setting reduces FEL power. The functional dependence can be fitted with a Gaussian with rms width of the order of the rms bandwidth of the fundamental radiation, i.e., $1/N_u$ (s. Figure 2). If all segments are randomly mistuned, the same dependence can be observed for random error distributions as function of width $(\Delta K/K)_{rms}$. In both cases, the rms width of the performance function is equal to the rms bandwidth of the radiation, which, in the latter case, is equal to the FEL parameter $\sigma_{\Delta K/K} = \varrho$. These dependencies have been verified by beam measurements at LCLS, for which the canted pole arrangement gives a well know relation between the undulator $K$ value and the horizontal position of the segment, the latter being remotely adjustable.

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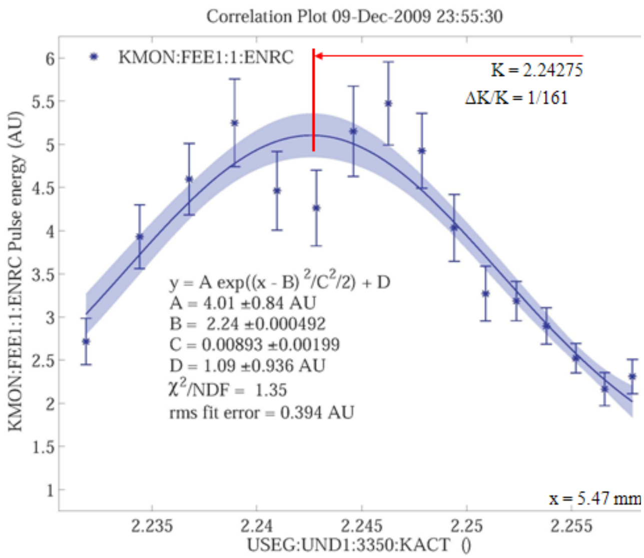


Figure 2: Measured FEL performance dependence on $K$ of a single LCLS segment can be fitted with a Gaussian.

The tolerance can be calculated by

$$t_{\Delta K/K} = \sigma_{\Delta K/K}\sqrt{2ln(P_0/P_{\Delta K/K})}. \qquad (16)$$

Temperature stability, gap errors, tuning, etc., will affect $\Delta K/K$ and can be treated in a sub-budget approach.

### *Segment Vertical Position*

The undulator parameter, $K$, depends on the vertical location in the undulator gap with a minimum in the mid-plane

$$K = K_0 \cosh(k_u \Delta y), \qquad (17)$$

or, expressed as relative change in $K$,

$$\frac{\Delta K}{K} = \frac{(K - K_0)}{K_0} = \frac{1}{2}(k_u \Delta y)^2 + O(k_u \Delta y)^4. \qquad (18)$$

The value of the rms performance degradation from a random error of $\Delta y$ can be derived from $\sigma_{\Delta K/K}$ when inserting $q_i^2 = (\Delta K/K)_{rms} = \frac{1}{2}k_u^2 \Delta y_{rms}^2$ into Eq. (1)

$$\frac{P_{\Delta y}}{P_0} = e^{-\frac{(\Delta K/K)_{rms}^2}{2\sigma_{\Delta K/K}^2}} = e^{-\frac{\left(\frac{1}{2}k_u^2 \Delta y_{rms}^2\right)^2}{2\sigma_{\Delta K/K}^2}} \overset{\text{def}}{=} e^{-\frac{f(\Delta y)_{rms}^2}{2\sigma_{f(\Delta y)}^2}}, \qquad (19)$$

where

$$f(\Delta y) = \Delta y^2$$
$$\sigma_{f(\Delta y)} = \frac{2\,\sigma_{\Delta K/K}}{k_u^2}. \qquad (20)$$

Again, the tolerance is calculated using Eq (2)

$$t_{\Delta y} = f^{-1}\left(\sigma_{f(\Delta y)}\sqrt{-2ln(P_{\Delta y}/P_0)}\right) = = \frac{1}{k_u}\sqrt{\sigma_{\Delta K/K}}\sqrt[4]{8ln(P_0/P_{\Delta y})}. \qquad (21)$$

### *Segment Horizontal Position*

For a planar undulator with pole faces parallel to each other, the undulator parameter, $K$, depends weakly on the horizontal location, $\Delta x$, in the mid-plane of the undulator gap due to a quadratic reduction in field strength towards the edges characterized by the parameter $m_s$

$$\frac{\Delta K}{K} = \frac{1}{2\,K_0}\frac{\partial K}{\partial x^2}\bigg|_{x=0} \Delta x^2 \overset{\text{def}}{=} m_s\,\Delta x^2. \qquad (22)$$

If the two jaws are not fully parallel but rotated with respect to each other around an axis parallel to the beam axis by an angle $\varphi = \partial g/\partial x$, (roll error), an additional linear dependence of the undulator parameter, $K$, on the horizontal location of the beam inside the undulator gap in the mid-plane will be present

$$\frac{\Delta K}{K} = \frac{\Delta B_u}{B_u} = \frac{1}{B_u}\frac{\partial B_u}{\partial x}\Delta x \overset{\text{def}}{=} \frac{1}{B_u}\frac{\partial B_u}{\partial g}\varphi \Delta x = -\frac{\pi}{\lambda_u}\varphi \Delta x. \qquad (23)$$

The last step assumes that $B_u$ depends on the gap, $g$, as $B_u = B_0 exp(-\pi\, g/\lambda_u)$, the Halbach formula [7]. The combination of the two effects can be written as

$$\frac{\Delta K}{K} = m_s\,\Delta x^2 - \frac{\pi}{\lambda_u}\varphi \Delta x \qquad (24)$$

If the two undulator jaws are horizontally displaced with respect to each other and parallel to the beam axis (Jaw-$\delta x$ error), the undulator parameter, $K$, will be reduced due to the quadratic field roll-off in horizontal direction. Let us assume that the on-axis field is the sum of the fields contributed from both jaws in equal proportion

$$\frac{\Delta K}{K} = \frac{\Delta K}{K}\bigg|_{up} + \frac{\Delta K}{K}\bigg|_{dn}. \qquad (25)$$

If now the $x$ position relative to each jaw center is given by a combination of the horizontal misalignment, $\Delta x$, and a horizontal jaw displacement error, $\pm\delta x$ ("+" for *up* and " − " for *down*), then we calculate a reduction in relative undulator strength using Eq. (24)

$$\frac{\Delta K}{K} = m_s\,\Delta x^2 + m_s\,\delta x^2 - \frac{\pi}{\lambda_u}\varphi \Delta x. \qquad (26)$$

In this approach $\Delta K/K$ depends on the 4 independent contributors: $m_s$, $\Delta x$, $\delta x$, and $\varphi$. We calculate $(\Delta K/K)_{rms}$ by making use of the fact that, normally, $\langle m_s \rangle^2 \gg (m_s)_{rms}^2, \langle \Delta x \rangle = \langle \delta x \rangle = 0\ m$, and $\langle \varphi \rangle = 0$ rad

$$(\Delta K/K)_{rms}^2 = \langle m_s \rangle^2((\Delta x)_{rms}^4 + (\delta x)_{rms}^4) + \left(\frac{\pi}{\lambda_u}\varphi_{rms}\Delta x_{rms}\right)^2. \qquad (27)$$



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We introduce an auxiliary variable $\Delta X$ such that $(\Delta K/K)^2_{rms} \stackrel{\text{def}}{=} \langle m_s \rangle^2 \Delta X^4$ can be handled more easily in the tolerance budget, i.e.

$$\Delta X^4_{rms} = (\Delta x)^4_{rms} + (\delta x)^4_{rms} + \left(\frac{\pi}{\lambda_u}\frac{\varphi_{rms}}{\langle m_s \rangle}\Delta x_{rms}\right)^2. \quad (28)$$

Now, we proceed as before

$$\frac{P_{\Delta X}}{P_0} = e^{-\frac{(\Delta K/K)^2_{rms}}{2\sigma^2_{\Delta K/K}}} \stackrel{\text{def}}{=} e^{-\frac{f(\Delta X)^2_{rms}}{2\sigma^2_{f(\Delta x)}}}, \quad (29)$$

here,

$$f(\Delta X) = \Delta X^2$$
$$\sigma_{f(\Delta X)} = \frac{\sigma_{\Delta K/K}}{\langle m_s \rangle}. \quad (30)$$

Once a tolerance

$$t_{\Delta X} = f^{-1}\left(\sigma_{f(\Delta X)}\sqrt{-2ln(P_{\Delta X}/P_0)}\right) = = \sqrt{\frac{\sigma_{\Delta K/K}}{\langle m_s \rangle}\sqrt[4]{2ln(P_0/P_{\Delta X})}} \quad (31)$$

has been established, allowed values for the individual contributors can be determined in a sub-budget approach.

### *Relative Yaw Pitch*

The two undulator jaws can get misaligned after gap changes and after transportation of the device. The jaw Pitch error, i.e., a rotation of the upper jaw around the x-axis at the longitudinal center of the undulator will change $K$, phase, and field integrals. The dependence of $K$ on the pitch angle has been analyzed for HXU and SXU at various gap heights using a RADIA [8] model. It turns out that the change in $\Delta K/K$ depends linearly on $\Delta\psi$, i.e, $\Delta K/K = (\partial\, \Delta K/K/\partial \boldsymbol{\psi}|_{\psi=0})\, \Delta\psi$, over a large range with

$$\left.\frac{\partial\, \Delta K/K}{\partial \psi}\right|_{\psi=0} \sim \frac{\sqrt{2}\, K^2 + e^1\, K - 1/3}{120 \times 10^4}. \quad (32)$$

Using the same approach as above, we get

$$\sigma_{\Delta\psi} = \frac{\sigma_{\Delta K/K}}{|\partial\, \Delta K/K/\partial\psi|_{\psi=0}|}. \quad (33)$$

### *Quad Transverse Position*

A horizontal error in quadrupole position, $\Delta x_Q$, will kick the electrons in horizontal direction by $x' = \frac{e}{\gamma mc}\frac{\partial B_y}{\partial x}\Delta x_Q$. A random kick from each of the $N_{seg}$ quadrupoles yields

$$\sigma_{\Delta x_Q} = \frac{1}{3.3}\frac{1}{\sqrt{N_{seg}}}\sqrt{\frac{\lambda_r}{L_G}\frac{\gamma mc}{e}\frac{1}{\partial B_y/\partial x}}. \quad (34)$$

The same applies for the vertical position errors.

## TOLERANCE BUDGET

Combining the contributions, that are discussed above, into a tolerance budget is done following Eq. (3). The $r_i$ need to be selected such that the resulting tolerances, $t_i$, are practical and that the total reduction in predicted FEL output is below an acceptable limit. For LCLS-II the target is $P/P_0 \geq 75\%$. Table 1 shows the tolerance budget for the LCLS-II HXU at $E_e$ =13 GeV and $E_{ph}$ =6.8 keV, the worst case operating point

Table 1: LCLS-II HXU tolerance budget [$P/P_0 = 75\%$]

| Parameter | $\sigma_i$ | $r_i$ | $P_i/P_0$ | $t_i$ |
|---|---|---|---|---|
| Launch Angle | 3.4 µrad | 0.186 | 98.3% | 0.45 µrad |
| Cell Phase | 44.3° | 0.136 | 99.1% | 6.0° |
| Phase Shift | 44.3° | 0.136 | 99.1% | 6.0° |
| Break Length | 31.3 mm | 0.032 | 99.9% | 1.0 mm |
| $\Delta K_{eff}/K_{eff}$ | 0.00052 | 0.489 | 88.7% | 0.00025 |
| Quad Pos Stab | 11.9 µm | 0.060 | 99.8% | 0.50 µm |
| Seg. Vert. Pos | 0.0535 mm$^2$ | 0.168 | 98.6% | 95 µm |
| Seg $\Delta X$ | 3.04 mm$^2$ | 0.081 | 99.7% | 494 µm♠ |
| Jaw Pitch | 21 µrad | 0.476 | 89.3% | 1 µrad |

° degXray;
♠ $(\varphi)_{rms}$=1 mrad; $(\Delta x)_{rms}$ = $(\delta x)_{rms}$=0.4mm; $\langle m_s \rangle$=1.7×10$^{-4}$ mm$^{-2}$

## SUMMARY

Presented is an attempt to produce a tolerance budget for the main undulator parameters of a SASE x-ray FEL based on algebraic formulae. This approach greatly reduces the need for extensive FEL simulations that would be necessary for variable gap undulators. This is work in progress. Some of the formulae used, will need to be confirmed or improved based on FEL simulations. More parameters can be added, some can be treated through sub-budgets.

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