

## TWO-DIMENSIONAL EFFECTS ON THE BEHAVIOR OF THE CSR FORCE IN A BUNCH COMPRESSION CHICANE\*

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### Abstract

The endeavour to reach higher brightness of electron beams in designs of future FEL is often challenged by the coherent synchrotron radiation (CSR) effect in bunch compression chicanes. Extensive studies on the CSR effects have shown that the 1D approximation of the CSR force is valid for a wide parameter regime. However, as the bunch gets increasingly compressed in the chicane, the behaviour of the CSR interaction force will be influenced by the evolution of the 2D bunch charge-density distribution in the bending plane. Here we explore this 2D effect for over-compression cases using the previously developed semi-analytical study of the effective longitudinal CSR force for a bunch with evolving 4D Gaussian phase space distribution and an initial linear energy chirp. We will display the dependence of the 2D CSR force on the initial horizontal emittance and uncorrelated energy spread for path lengths around that of the minimum bunch length. We will also present and discuss the comparison of these results with their counterparts in the 1D rigid-line bunch model.

### INTRODUCTION

The high-peak-current electron beams required by the short-wavelength high-gain FEL are often obtained by compressing the high-charge electron bunches with low-transverse emittance using one or more bunch compression chicanes. Inside a magnetic chicane, the electrons are transported through portions of circular orbits in the magnetic bends. The curvature of these circular orbits will induce additional electromagnetic interaction among the electrons via the acceleration term in the Lienard-Wiechert fields, which is closely related to the coherent synchrotron radiation (CSR) emitted by the bunch. Hence the curvature-induced additional EM interaction is often called the CSR interaction, and the corresponding forces are called the CSR forces. The CSR interaction in a bunch compression chicane can potentially cause degradation of the bunch phase space quality, and pose serious challenge to designs of future FEL.

The bunch compression by a magnetic chicane is realized by imposing an initial energy chirp on the bunch prior to its entrance to the chicane and utilizing the correlation between path-length and energy in regions of non-zero dispersion. As a result, the bunch length changes during the bunch compression process, which is closely correlated to the evolution of bunch transverse size due to the non-zero dispersion in the chicane. The evolving 2D

bunch distribution in the bending plane is the foundation for the calculation of retarded potentials and the collective Lienard-Wiechert fields experienced by the bunch. As the first-order approximation, many of previous analyses [1,2] and simulations [3] of the CSR interaction used 1D model for the calculation of the longitudinal CSR force, wherein the bunch is treated as a rigid-line bunch with the bunch length at all retarded times being fixed at the same value as the one when the CSR force is evaluated. The validity of the rigid-line bunch model was first discussed in Ref. [4], where it is stated that the effect of the transverse bunch size on the CSR force can be ignored when

$$\kappa = \sigma_x / (R\sigma_z^2)^{1/3} \ll 1. \quad (1)$$

Here  $R$  is the bend radius of the magnet, and  $\sigma_x$  and  $\sigma_z$  are the horizontal and longitudinal RMS bunch sizes. Even though this condition does not take the dependence of  $\sigma_x$  and  $\sigma_z$  on path length into account, it can still serve as an indicator of the occurrence of 2D CSR effect.

The effects of the evolution of the 2D bunch distribution in the bending plane on the behaviour of CSR force, such as the delayed response of the longitudinal CSR force to the bunch-length variation and the dependence of longitudinal CSR force on the transverse coordinates of the particles, were studied extensively in Ref. [5] for path length around that of the minimum bunch length. In this paper, we apply the previously developed semi-analytical approach to further study the dependence of the 2D CSR force on the initial uncorrelated transverse and longitudinal phase space area, as well as on the initial energy chirp of the bunch. We will first describe the parameters used for our case study, including the parameters of the model chicane and the initial beam phase-space parameters for a bunch sent through the chicane. We then examine the results of the CSR force on the bunch at various path lengths along the chicane. Here we assume the bunch propagates through the chicane only following the linear optics (unperturbed by CSR). Comparison of the 2D results with their counterparts in the 1D rigid-line bunch model will be presented and discussed.

### BUNCH COMPRESSION IN THE MODEL CHICANE

The model chicane we employed is illustrated in Fig. 1 [5]. The chicane consists of three rectangular dipole magnets with bending radius  $R = 1$  m. The length for the two side dipoles is  $L_b = 0.3$  m and the length for the centre dipole is  $L_B = 0.6$  m. The drift length between adjacent dipoles is  $L_d = 0.4$  m. We now let an electron

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bunch with initial 4D Gaussian phase space distribution to be sent through the chicane. The initial bunch parameters are chosen as  $\beta_{x0} = 5$  m,  $\alpha_{x0} = 1$ ,  $E = 70$  MeV, and  $\sigma_{z0} = 0.5$  mm.

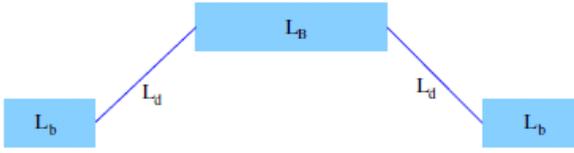


Figure 1: Layout of the model chicane

At the entrance of the chicane, the energy of a particle  $\delta_0$  is correlated to the longitudinal position  $z_0$  of the particle by

$$\delta_0 = u z_0 + \delta_{un}, \quad (2)$$

with  $u$  the initial energy chirp and  $\delta_{un}$  the uncorrelated relative energy deviation from the design energy. In this paper, we will study bunch compression and CSR force behavior for two different values of initial energy chirp,

$$(a) u = -10.56 / m, \quad (b) u = -18.52 / m \quad (3)$$

For uncorrelated rms energy spread  $\sigma_{un} = 10^{-4}$ , we plot the initial longitudinal phase space distributions in Fig. 2.

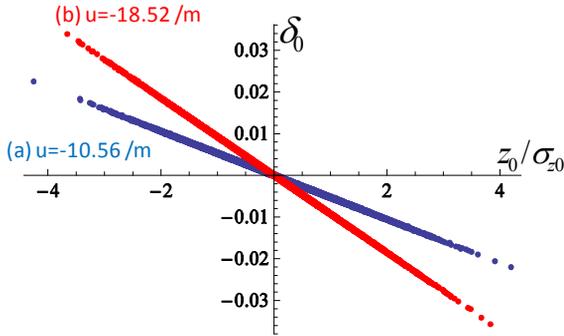


Figure 2: Initial longitudinal phase space at  $s=0$ ,  $\delta_0$  vs.  $z_0$ , for the cases in Eq. (3).

Following linear optics in the chicane, the longitudinal coordinate of a particle at path-length  $s$  depends on its initial phase-space coordinates at the entrance of the chicane:

$$\begin{aligned} z &= R_{51}(s)x_0 + R_{52}(s)x'_0 + z_0 + R_{56}(s)\delta_0 \\ &= \bar{R}_{51}(s)x_0 + R_{52}(s)x'_{p0} + \bar{R}_{55}(s)z_0 + R_{56}(s)\delta_{un}, \end{aligned} \quad (4)$$

Here  $\bar{R}_{51}(s) = R_{51}(s) - (\alpha_{x0} / \beta_{x0})R_{52}(s)$ , and also  $x'_0 = x'_{p0} - (\alpha_{x0} / \beta_{x0})x_0$ . The degree of bunch compression is described by the bunch compression factor

$$\bar{R}_{55}(s) = 1 + uR_{56}(s) \quad (5)$$

which depends on  $u$ . In Fig. 3, we illustrate the bunch compression factor as a function of path length along the

chicane for the two initial chirps in Fig. 2. Let  $s = s_c$  be the path length where the bunch compression factor reaches zero. Fig. 3 shows that  $s_c = 1.0$  m for case (a) and  $s_c = 1.2$  m for case (b), respectively.

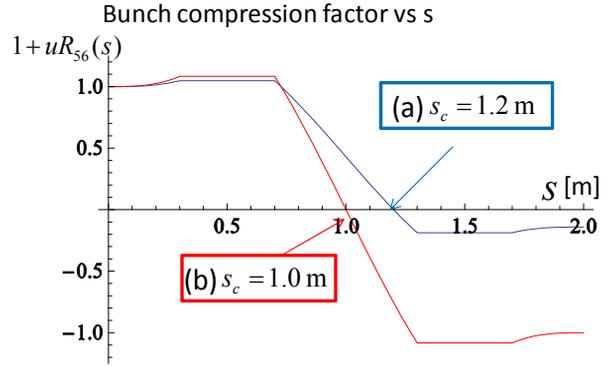


Figure 3:  $\bar{R}_{55}(s)$  vs.  $s$  for the two cases in Fig. 2.

The width of the bunch around  $s = s_c$  is characterized by the minimum bunch length  $\sigma_z$  at  $s = s_c$ , which depends on the initial horizontal emittance and uncorrelated energy spread since

$$\begin{aligned} \sigma_z(s) &= [(\bar{R}_{51}(s)\sigma_{x0})^2 + (R_{52}(s)\sigma_{xp0})^2 \\ &+ (\bar{R}_{55}(s)\sigma_{z0})^2 + (R_{56}(s)\sigma_{un})^2]^{1/2} \end{aligned} \quad (6)$$

For the two over-compression cases (Fig. 2) selected for investigation, and for the normalized horizontal emittance  $\epsilon_{x0} = 1 \mu\text{m}$ , we plot the bunch-length variation along the chicane in Fig. 4, where the bunch length reaches its minimum value when  $s = s_c$  for  $s_c = 1.0$  m and 1.2 m respectively.

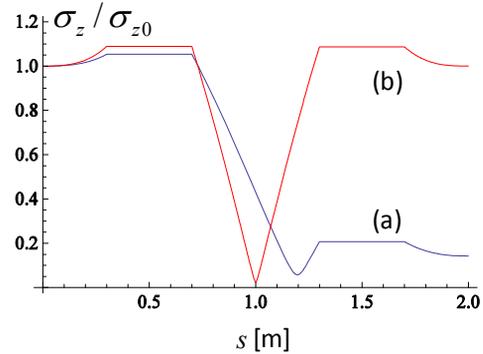


Figure 4: Bunch length variation for the cases in Fig. 2.

Since we are using the semi-analytical approach [5] for the CSR force study, we are constrained by the scope of this approach which considers CSR interaction taking place only within the same circular arc (no straight included). Here our choice for the values of initial energy chirp is such that the CSR interactions among particles in the bunch all occur in the second bend, which ranges from  $s = 0.7$  m to  $s = 1.3$  m. Note that the bunch self-

interaction via the longitudinal space charge force is not included in this study.

## DEPENDENCE OF THE CSR FORCE ON THE INITIAL BEAM PARAMETERS

The longitudinal effective CSR force depends on the 2D distribution of the bunch in the bending plane. This 2D distribution, on the other hand, rotates as the path length crosses over  $s = s_c$ . For the case  $s_c = 1.0$  m, we illustrate this bunch rotation in Fig. 5, where the green dots are the bunch  $x$ - $z$  distribution represented by macro particles. We can see that the bunch deflection ( $x$ - $z$  slope) in the bending plane changes from positive value to negative value as the bunch moves across  $s = s_c$ .

For our study we let the intrinsic spread of the Gaussian bunch to vary among five sets of initial normalized horizontal emittance  $\mathcal{E}_{x0}$  and uncorrelated energy spread  $\sigma_{un}$ , as listed in Table 1. The other bunch parameters are the same as described in the previous section.

Table 1: Initial Bunch Parameters

Cases	Normalized $\mathcal{E}_{x0}$ ( $\mu\text{m}$ )	$\sigma_{un}$
Thinner	0.1	$10^{-5}$
Thin	0.1	$10^{-4}$
Regular	1.0	$10^{-4}$
Thick	1.0	$10^{-3}$
Thicker	10	$10^{-3}$

For various path lengths around  $s = s_c$ , we apply the semi-analytical approach [5] to calculate the normalized effective longitudinal CSR force on the bunch

$$\tilde{F} \equiv F_z^{\text{eff}}(x, z, s) / F_0,$$

Detailed description of  $F_z^{\text{eff}}$  and  $F_0$  can be found in Ref. [5], where  $F_0$  is independent of  $\mathcal{E}_{x0}$  or  $\sigma_{un}$ . Here we assume the bunch distribution evolves from its initial Gaussian distribution following nominal optics unperturbed by the CSR force. As shown earlier [5], when the bunch reaches deep compression, its distribution in the bending plane gets further and further deflected. Meanwhile the longitudinal CSR force on the bunch shows more dependence on the particles' transverse coordinates. Some example plots of the CSR force on the bunch around path length for the minimum bunch length is shown in Fig. 6. Here the gray meshes are the CSR force in the vicinity of the bunch, the red dots are the CSR

force on the bunch represented by  $N = 2000$  macro-particles, and the green dots represent the bunch  $x$ - $z$  distribution.

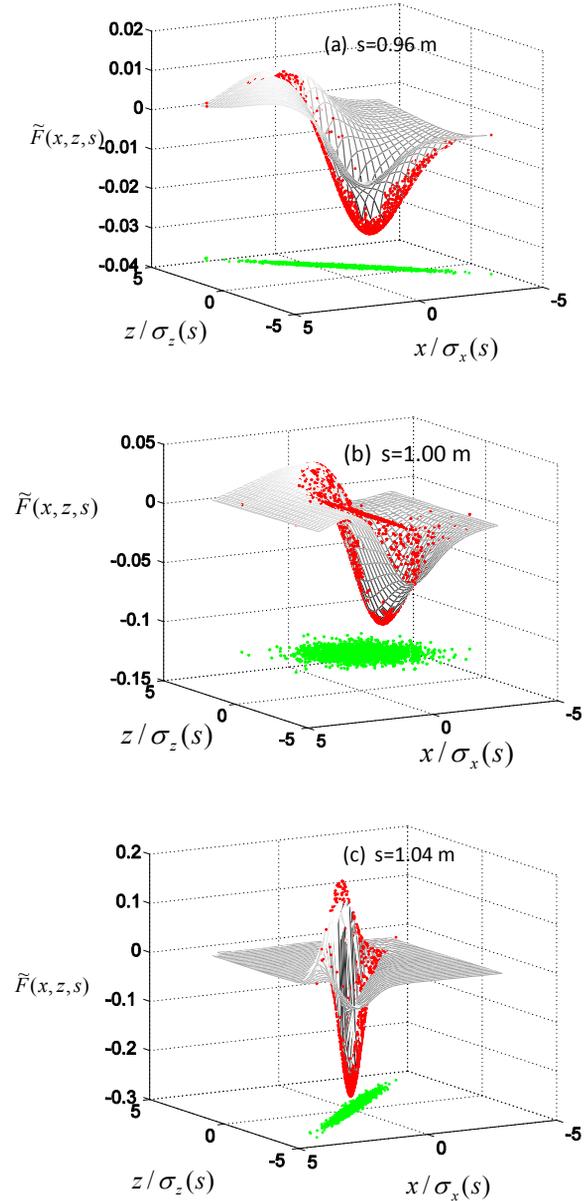


Figure 5: The effective longitudinal CSR force (red) on the bunch at (a)  $s = 0.96$  m (b)  $s = 1.00$  m and (c)  $s = 1.04$  m, for the regular case with  $u = -18.52/\text{m}$ . The green dots are the bunch  $x$ - $z$  distribution in the bending plane.

We now study how different values of the initial energy chirp impact the behavior of the CSR forces on the bunch in the over-compression process. For each of the two initial energy-chirp values in Eq. (3), we allow the bunch to assume each set of the initial uncorrelated parameters as listed in Table 1. We then calculate the CSR forces on the bunch, as exemplified in Fig. 5, for an array of path lengths around  $s = s_c$ . For each path length, one

can calculate the average of the CSR force over the 2D Gaussian bunch distribution, and also find the magnitude of the CSR force over the bunch by getting the maximum and minimum of the CSR force on the bunch represented by  $N = 2000$  particles. The two-dimensional effect on the amplitude of the CSR force can then be exhibited by comparing these quantities with their rigid-line-bunch counterparts, as calculated using the analytical formula [1] where the bunch length  $\sigma_z(s)$  in Eq. (4) is used for each  $s$ . These comparisons are shown in Fig. 6 for  $s_c = 1.2$  m and in Fig. 7 for  $s_c = 1.0$  m, where we plot (a)  $\sigma_z(s)/\sigma_{z0}$  vs. path length  $s$ , (b)  $\kappa(s)$  vs.  $s$  for  $\kappa$  defined in Eq. (1), (c) minimum and maximum of  $\tilde{F}(x, z, s)$  for the  $N = 2000$  representative particles of the Gaussian bunch, as a function of path length, and (d) average of  $\tilde{F}(x, z, s)$  over the bunch  $x-z$  distribution vs.  $s$ .

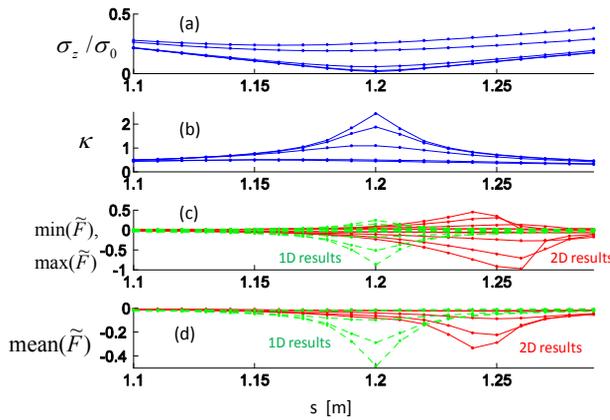


Figure 6: Results for all the five cases in Table 1 for path lengths around  $s_c = 1.2$  m for  $u = -10.56/\text{m}$ .

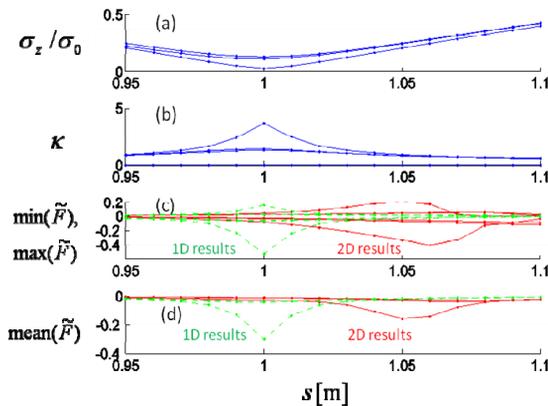


Figure 7: Results for the last three cases in Table 1 for path lengths around  $s_c = 1.0$  m for  $u = -18.52/\text{m}$ .

### DISCUSSIONS

For the energy chirp values listed in Eq. (3), Figs. 6 and 7 show that the criteria for 1D approximation in Eq. (1),  $\kappa \ll 1$ , is not satisfied for path length around  $s = s_c$ .

This is the regime where the 2D effect takes place in the

CSR interaction. One can see that when the bunch length reaches its minimum value, the 1D result shows maximum amplitude of the CSR force, whereas the 2D CSR forces reach their maximum at a delayed path length [5], as a result of retarded response to the longitudinal charge-density variation via the bunch-length change. The distance of path-length delay depends on the minimum bunch length  $\sigma_z(s_c)$  in Eq. (6), and it is usually much smaller than the length of the magnet within which the CSR interaction takes place. In the FEL designs, one often cares about the degradation of the bunch longitudinal and transverse phase space after the beam moves through the whole chicane. Such degradation is the integrated effect of CSR force on the bunch along the whole chicane convoluted with linear optics. Hence the slight delay of the peak CSR interaction should not make significant differences on the bunch dynamics from that of the 1D model. However, unlike the 1D CSR interactions, the 2D CSR force around  $s = s_c$  shows strong dependence on the particles' transverse coordinates (as depicted in Fig. 5). Therefore the 2D CSR interaction around  $s = s_c$ , which acts as an impulse to the beam, can potentially have more adverse effect on the transverse slice emittance than its 1D counterpart. Note that Figs. 6 and 7 show similar delayed response of the 2D results. However, the thin and thinner cases are not included in Fig. 7, because the CSR force peaks at very narrow range of path length, which requires more detailed studies.

In summary, we studied the simple case of the 2D CSR interaction for a perfect Gaussian bunch with a linear initial energy chirp. The results exhibit the delayed response of the magnitude of the CSR force to bunch length variation, as well as the transverse dependence of the CSR force on the bunch in the vicinity of path length for the minimum bunch length. This study needs to be extended to investigate the 2D effect on the microbunching instability in a magnetic bending system. Comparison with simulations and experiments are to be further pursued.

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