REVERSIBLE SEEDING IN STORAGE RINGS

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Abstract

We propose to generate steady-state microbunching in a storage ring with a reversible seeding scheme. High gain harmonic generation (HGHG) and echo-enabled harmonic generation (EEHG) are two promising methods for microbunching linac electron beams. Because both schemes increase the energy spread of the seeded beam, they cannot drive a coherent radiator turn-by-turn in a storage ring. However, reversing the seeding process following the radiator minimizes the impact on the electron beam and may allow coherent radiation at or near the storage ring repetition rate. In this paper we describe the general idea and outline a proof-of-principle experiment.

INTRODUCTION

Electron storage rings can drive high average power light sources, and free-electron lasers (FELs) are now producing coherent light sources of unprecedented peak brightness [1]. While there is active research towards high repetition rate FELs (for example, using energy recovery linacs, [2, 3, 4]), at present there are still no convenient accelerator-based sources of high repetition rate, coherent radiation. As an alternative avenue, we recently proposed to establish steady-state microbunching (SSMB) in a storage ring [5, 6]. By maintaining steady-state coherent microbunching at one point in the storage ring, the beam generates coherent radiation at or close to the repetition rate of the storage ring. In this paper, we propose a method of generating a microbunched beam in a storage ring by using reversible versions of linac seeding schemes.

High gain harmonic generation (HGHG) and echo-enabled harmonic generation (EEHG) are promising methods for seeding linac-based FELs [7, 8]. In both schemes, laser-driven energy modulation and dispersive sections combine to create density modulations in the electron beam. Generally, an FEL then amplifies this seed modulation to maximize microbunching at a harmonic of the seed wavelength. In principle, both HGHG and EEHG could drive turn-by-turn microbunching in a storage ring. However, the modulation stage of the seeding increases the beam energy spread considerably, with an increase of a factor of two or more in EEHG and by the harmonic number, H, in HGHG. To recover the initial beam conditions, the beam must circulate the ring for several damping times, dropping the maximum repetition rate by three orders of magnitude or more.

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We propose to implement a modified seeding scheme that recovers the initial beam conditions by reversing the process following the radiator section. As in standard seeding schemes, HGHG or EEHG would modulate the beam density. Because the energy and dispersive effects remain imprinted in phase space, by following the radiator with opposite sign dispersion and modulations, we can recover the initial beam conditions. (The possibility of reversing energy modulations on storage beams was first noted in the proposal for femtoslicing [9].) Note that it is essential to drive the radiator directly with the seeded density modulation rather than using the irreversible FEL process to amplify the microbunching. As a result, it is imperative to maximize the level of bunching. In principle, wave form modification (e.g. a sawtooth waveform) can also be implemented in this reversible scheme to increase the bunching factor without an FEL mechanism [10].

We start by describing the conceptually simpler reversible HGHG scheme. We first described the basic reversible seeding concept. We then apply the HGHG principle to a beat wave as a scheme for generating long wavelength seeding and repeat the HGHG analysis for the EEHG scheme. Finally, we estimate the tolerances required to effectively reverse the seed and show simulation results for an example parameter set. Due to space constraints, we show tolerance and simulation examples only for the HGHG case; we will present beating and EEHG cases in a separate publication.

HGHG

In the HGHG scheme, a laser beam interacts with the electron beam to induce a sinusoidal energy modulation, and a dispersive region then converts the energy modulation into a sharply spiked density modulation. With an energy modulation at wavelength $\lambda_{\rm in}$ approximately H fold larger than the beam's initial energy spread, σ_{E0}, HGHG produces strong microbunching at wavelengths as short as $\lambda_{\rm out}{=}\lambda_{\rm in}/H$.

If HGHG is directly applied in a storage ring, the large dispersion of the ring will smear out the microbunching and leave a beam with an energy spread of $\sigma_E = H\sigma_{E0}$. In the following pass through the HGHG process, the modulation amplitude will be approximately equal to the beam's uncorrelated energy spread, and will produce microbunching only at the fundamental wavelength. While it is possible to recover the beam's initial energy spread by waiting for several damping periods, the waiting time significantly reduces the average power. Instead, we propose to reverse

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the HGHG process so as to return the beam to its initial condition immediately following the radiator, allowing for the process to repeat every pass around the storage ring.

We summarize the HGHG-HGHG^{-1} process as follows: a modulation and dispersive region produce strong microbunching at a harmonic, H, of the initial seed; a radiator extracts coherent radiation from the microbunched beam; an opposite sign dispersive region followed by a modulation with π phase shift then reverse the HGHG process; finally, the beam travels around the ring, completing a single pass (Fig. 1).

To analyze the reversible seeding process, we use a variation of the SSMB formalism [5]. We can write down the resulting longitudinal map for a particle traveling around the ring once

$$\begin{aligned} z_1 &= z_0, & E_1 &= E_0 + A\sin(k_{\rm in}\, z_0) \\ z_2 &= z_1 + R_{56}\, E_1, & E_2 &= E_1 \\ z_3 &= z_2 + \Delta z, & E_3 &= E_2 + \Delta E \\ z_4 &= z_3 - R_{56}\, E_3, & E_4 &= E_3 \\ z_5 &= z_4, & E_5 &= E_4 - A\sin(k_{\rm in}\, z_4) \\ z_6 &= z_5 - R_{56}^{(\rm ring)} E_5, & E_6 &= E_5\,, \end{aligned} \quad (1)$$

where we've defined $k_{\rm in} = 2\pi/\lambda_{\rm in}$, modulation amplitude $A = H\sigma_{E0}$, and we have added random errors $\Delta z, \Delta E$ during the radiation step to account for any smearing in the longitudinal phase space. The unwinding process uses the same HGHG steps, but implemented in reverse. (Note that stages 4 and 5 are reversed as compared to SSMB.) The dispersion of the final step has little effect on the beam's steady state distribution so long as the previous steps have erased the impact of the HGHG seeding. If the final distribution is equivalent to the initial distribution, the process can occur each pass through ring, resulting in steady-state microbunching at the radiator and producing high average power coherent radiation at $\lambda_{\rm out}$.

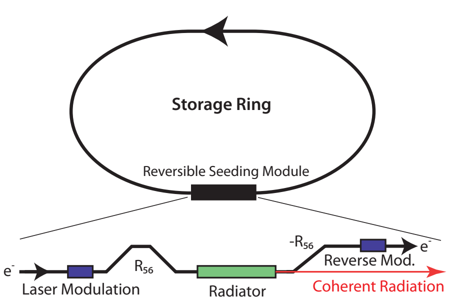

Figure 1: Schematic of the reversible HGHG scheme. A modulation and dispersive region produce microbunching according to the HGHG process. A radiator then extracts coherent radiation at a high harmonic of the seed wavelength, before an opposite sign dispersive region and second modulation reverse the HGHG process and return the beam to its original state.

LONG WAVELENGTH BEATING

It is also possible to use the principle of HGHG to seed at long wavelengths by beating together two lasers of similar frequencies [11, 12]. Analogous to the SSMB beating setup [5], a beat seed would allow steady state seeding at THz frequencies, providing an alternative path to coherent sub-mm radiation from storage rings.

Rather than a single seed wavelength of $\lambda_{\rm in}$, two seed lasers of $\lambda_2 = b/(b-1)\lambda_1$ drive a modulation that is periodic in $\lambda_{\rm out} = b\lambda_1$, with the potential to set $b \gg 1$. At the radiator, we then have a manipulation (similar to the first two lines of Eq. 1) of

$$z_2 = z_0 + R_{56}\, p_0 + R_{56}\, A\sin(k_1 z_0) + R_{56}\, A\sin(k_2 z_0)\,. \quad (2)$$

For $b \gg 1$, we find $\lambda_1 \approx \lambda_2$, and the modulation can use a single undulator, provided the bandwidth of the undulator is larger than $(\lambda_1 - \lambda_2)/\lambda_2$. A dispersive section then converts the beat energy modulation into a density modulation at $\lambda_{\rm out}$.

To evaluate the seeding efficacy, we again want to calculate the bunching factor, B_f. To find the optimal bunching, we can use the standard HGHG approach for a single pass seed (see e.g. [7]). Assuming an initially uniform longitudinal and Gaussian energy distribution, we can use the transformation of Eq. 2 to find the bunching factor at the radiator

$$B_f = J_1(k_{\rm out} R_{56}\, A)^2 e^{-(k_{\rm out} R_{56} \sigma_E)^2/2}\,, \quad (3)$$

where σ_E is the relative energy spread, and we have used $k_{\rm out} = k_1 - k_2$ to select a single term from the Jacobi-Anger expansion. For the case of $A \approx 2\sigma_E$, we find maximum bunching when $k_{\rm out} R_{56}\, A \approx 1.5$. We note that the seeding by frequency beating can be equivalently described as an EEHG manipulation with the first dispersive region set to zero, and Eq. 3 emerges as a special case of the EEHG analysis [13].

EEHG

EEHG [8] looks to be a promising method for generating microbunching at high harmonics of the seed wavelength without large modulation amplitudes. As with HGHG, we can construct a reversible version of EEHG that can be implemented each turn. We describe the reversible EEHG mechanism as a four-stage version of the SSMB mechanism [5]. The first two stages correspond to the echo mechanism; a modulation, a strong dispersive region, a modulation and a weaker dispersive region to maximize the harmonic microbunching. Following the second stage, a radiator extracts coherent radiation from the microbunched beam. After the radiator, two more stages reverse the EEHG process; an opposite sign dispersion, opposite sign modulation, opposite sign dispersion, opposite sign modulation. Finally, a dispersive region (corresponding to the unmodulated storage ring) completes a pass. The two

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EEHG energy modulation stages may have the same wavelength and amplitude, but this need not generally be the case. We have confirmed the reversible EEHG case with simulations, which will be presented elsewhere.

TOLERANCES

A major impediment to reversible seeding is the need to precisely remove the modulation without increasing the beam's equilibrium energy spread. We consider sources of energy spread from both the manipulation (mismatches of the dispersion, and amplitude and phase errors of the modulation) as well as random longitudinal beam errors (position and energy) during the seeding, reversal and radiative processes.

Due to space constraints, we show results only for the simplest case of HGHG. First we consider the manipulation tolerances on the reverse modulation and dispersion stages. The modulation reversal is designed to exactly cancel the initial sinusoidal modulation, $A\sin(k_{\rm in}\, z_0)$, where z_0 is the initial longitudinal position in the bunch (as in Eq.1). If we assume the reversal stage contains small errors ΔA in amplitude and $\Delta\phi$ in phase, we generate a final error in energy for a particle of

$$\Delta E_f \approx A\Delta\phi\cos(k_{\rm in}\, z_0) + \Delta A\sin(k_{\rm in}\, z_0)\,. \quad (4)$$

(Note that we are not concerned with the final position errors, which are negligible compared to the bunch length.) For the reverse dispersion, an error of $\Delta\ R_{56}$ produces a phase error of $k\Delta\ R_{56}\ [A\sin(k_{\rm in}\, z_0) + E_0]$, where E_0 is the normalized particle energy prior to the seeding process, and is on the order of the equilibrium energy spread, σ_E. This phase error then produces an energy error following the modulation reversal of approximately

$$\Delta E_f \approx A\frac{\Delta\ R_{56}}{R_{56}}\frac{\sin(2\ k_{\rm in}\, z_0)}{2}\,. \quad (5)$$

where we have used the typical HGHG requirement of $R_{56}= 1/\ k_{\rm in}\ A$ and assumed $\sigma_E\ \ll\ A$ in the final line. To prevent the seeding process from increasing the energy spread of the beam, we need to keep the energy errors smaller than the total quantum excitation per manipulation

$$\langle\Delta E^2\rangle < \epsilon^2/T\,, \quad (6)$$

with single turn excitation ϵ and average duty factor of the manipulation T. Averaging Eqs. 4 and 5 over a uniform distribution, we find the constraints on the manipulation parameters of

$$\frac{\Delta A}{A},\ \Delta\phi,\ \frac{\Delta R_{56}}{2R_{56}} < \frac{\sqrt{2}\epsilon}{\sqrt{T}A}\,. \quad (7)$$

The individual tolerances add in quadrature, so in practice they must be considered in total. We can check the tolerances numerically by tracking particles around a ring, and measuring the energy spread as a function of manipulation errors. We find close agreement with Eq. 7 (Fig. 2) For a proof-of-principle set of parameters we find these tolerances are relatively simple to meet (Table 1).

More challenging is the need to avoid longitudinal particle movement between the HGHG and reverse HGHG sections; we will call these the beam tolerances. We consider a positional error Δz and an energy error ΔE that occur prior to the reversal stage. The reverse dispersion turns the energy error into a position error, $\Delta z_f = \Delta z - R_{56}\ \Delta E$, and the modulation reversal turns the total positional error into a new energy error,

$$\Delta E_f = \left[1 - A\ k_{\rm in}\cos(k_{\rm in}\, z_0)\ R_{56}\ \right]\Delta E + A\ k_{\rm in}\cos(k_{\rm in}\, z_0)\Delta z\,. \quad (8)$$

We again find the expectation value of the final energy spread, $\langle\Delta E_f^2\rangle$, by integrating over longitudinal position in the bunch. Taking $R_{56}\approx 1/(A\ k_{\rm in})$, we find constraints on the errors of

$$\sqrt{3}\Delta E,\ \frac{\Delta z}{R_{56}} < \frac{\sqrt{2}\epsilon}{\sqrt{T}}\,. \quad (9)$$

We can loosen all tolerances by reducing the duty cycle, T, of the seeding process. To reduce T, we can operate the seed laser at a lower repetition rate than the storage ring, or alternatively we can seed a rotating fraction of the beam at a higher repetition rate; the key is that each particle sees the modulation only once per $1/T$ turns on average. Once T reaches the damping factor, δ, the reverse seeding is no longer needed because simple damping will recover the initial beam conditions. It may also be possible to improve tolerances by artificially enhancing both the damping factor and quantum excitation by inserting additional damping wiggleres around the ring.

Finally, we note that we have not included higher order dispersion in the analysis above. In general it is difficult to build dispersive regions that reverse the sign of both R_{56} and second order dispersion, T_{566}. Though the proof-of-principle example described below is not affected by T_{566}, many of the more interesting cases (such as the beating and EEHG sources) will likely have to optimize parameters to avoid such terms. It is possible to suppress T_{566} by adding sextupoles, etc., but these magnets complicate storage ring operation.

HGHG PROOF OF PRINCIPLE

As a proof of principle, we consider an optical reversible seeding experiment with HGHG. To simulate the process, we track longitudinal coordinates of particles traveling around a ring with parameters taken from the SPEAR3 ring (Table 1). We evaluate the seeding efficacy by calculating the bunching factor, B_f, at the location of the radiator following the HGHG stage. To account for longitudinal errors arising from second order and transverse effects, we introduce random position and energy errors to each particle after calculating the bunching factor. We treat the manipulation errors in dispersion and modulation amplitude and

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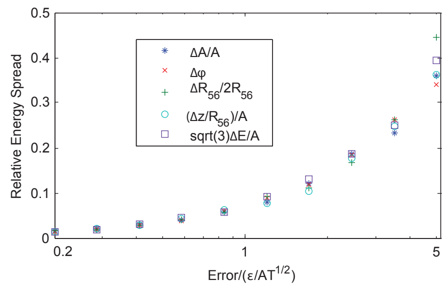

Figure 2: Equilibrium energy spread as a function of manipulation and beam errors. Simulations track each point for several damping periods to reach steady state. Parameters are given in Table 1, with the exception of harmonic number, $H = 10$, so that we can ignore the E_0 term and use Eq. 5. We find that when relative errors are larger than the dimensionless parameter ϵ/A, the energy spread increases significantly, confirming Eqs.7 and 9.

phase as fixed quantities (not changing turn by turn). By setting $T = 1/100$ (1% duty cycle), the error tolerances are relatively loose.

To achieve the modulation of amplitude A, we require laser power of around $P_L = 800$ MW focused to a radius of roughly $\sigma_L = 100$ μ m (see e.g. [14]). The modulation does not remove energy from the beam, so the laser power can be stored in a cavity. To make the power level manageable for a cavity, we set the duty cycle to T =1% (each particle only sees the modulation on average once per 100 revolutions). In this case, the average stored power in the cavity is 8 MW. We conclude that a proof-of-principle experiment at optical wavelengths would be feasible in a SPEAR3-like ring (Fig. 3).

Table 1: Parameters and Tolerances for the Example Sources Simulated in Fig. 3

Simulation Parameters	HGHG (400 nm)	Tolerance
R_{56}	70 μm	18%
A	1.8×10^{-3}	18%
λ	800 nm	...
Harmonic (H)	2	...
Turns per shot ($1/T$)	100	...
Damping (δ)	8×10^{-4}	...
E-spread (σ_δ)	9×10^{-4}	...
$\Delta\phi$	...	10°
Δz	...	20 nm
ΔE	...	3×10^{-4}

CONCLUSION

We describe a mechanism for producing steady-state microbunching in a storage ring. By reversing external seeding, it is possible to generate strong microbunching at a high repetition rate in a storage ring. We describe several

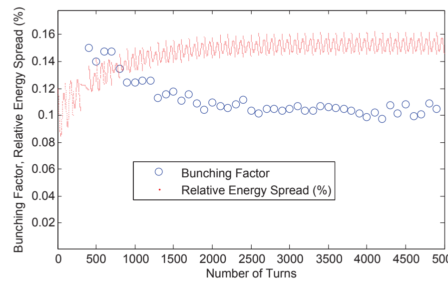

Figure 3: Bunching factor and energy spread are given as a function of turn number for simulation of HGHG in a SPEAR3-like ring. The simulation parameters and tolerances are given in Table 1. In blue, we show the energy spread (dots) and bunching factor at the second harmonic (circles). As the energy spread increases due to reversal errors, the bunching factor drops in response.

examples, including long wavelengths (beating) and high harmonics (EEHG) and provide simulations to illustrate a proof-of-principle experiment with reversible HGHG.

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