SEEDED RADIATION SOURCES WITH SAWTOOTH WAVEFORMS*

Daniel Ratner † and Alex Chao (SLAC, Menlo Park, California)

Abstract

Seed radiation sources have the ability to increase longitudinal coherence, decrease saturation lengths, and improve performance of tapering, polarization control and other FEL features. Typically, seeding schemes start with a simple sinusoidal modulation, which is manipulated to provide bunching at a high harmonic of the original wavelength. In this paper, we consider seeding from sawtooth modulations. The sawtooth creates a clean phase space structure, providing a maximal bunching factor without the need for an FEL interaction. While a pure sawtooth modulation is a theoretical construct, it is possible to approach the waveform by combining two or more of the composite wavelengths. We give examples of sawtooth seeding for HGHG, EEHG and other schemes, and note that the sawtooth modulation may aid in suppression of the microbunching instability.

INTRODUCTION

A Free Electron Laser (FEL) has the potential to produce fully coherent X-rays, a promising tool for the fields of physics, chemistry and biology. To improve coherence properties, there is strong interest in seeding FELs with optical or UV lasers. In general, seeding schemes produce a low level of microbunching, which the FEL process then maximizes.

In this paper we consider the possibility of seeding with sawtooth waveforms. By increasing the initial level of microbunching it may be possible to simplify, or even avoid, an FEL stage. The flatter phase space of sawtooth waveforms can aid in the suppression of the microbunching instability (MBI). Finally, we use the sawtooth formalism as a tool for analyzing conventional sinusoidal seeding schemes.

MOTIVATIONS

Direct Seeding

While seeding schemes generally incorporate an FEL section to amplify the bunching factor, in principle it is possible to drive a radiation source directly with the seeded beam. Avoiding an FEL stage may allow for simpler radiation sources, or may be essential in schemes when an FEL stage would ruin the beam quality. (For example, we have previously considered sawtooth seeding in combination with compressed harmonic generation, steady-state microbunching and reversible seeding [1, 2, 3, 4]). Without the amplifying effect of the FEL, the final radiated power scales as the square of the initial seeded bunching. In this case, maximizing the seeded bunching factor is essential, and a sawtooth scheme may be of interest.

If an FEL amplifies b_f following the seeding stage, the FEL properties determine the final bunching level. However, the bunching amplitude must exceed the noise level to produce a longitudinally coherent beam, so a sawtooth modulation may still have advantages. Moreover, a larger initial bunching level allows for a shorter FEL amplification stage.

Reversible Laser Heater

The microbunching instability (MBI) is a potentially dangerous instability that can seriously degrade FEL properties (see e.g. [5, 6, 7, 8]). By increasing the incoherent energy spread of the beam with a laser heater it is possible to suppress the MBI [9, 10]. However, a simple sinusoidal energy modulation results in a two horn energy distribution, giving a slow Bessel function suppression instead of the desired exponential damping (Fig. 1). As a result, laser heater schemes include additional means of smearing the energy modulation; at the Linac Coherent Light Source the laser heater is placed in the middle of a chicane.

The sawtooth modulation presents an alternative approach to damping the MBI. Because the sawtooth produces a uniform energy distribution, the subsequent smearing in a dispersive region produces a true incoherent energy spread. The sawtooth modulation could be a stand alone laser heater stage, or could be part of a larger seeding scheme (e.g. compressed harmonic generation [1]).

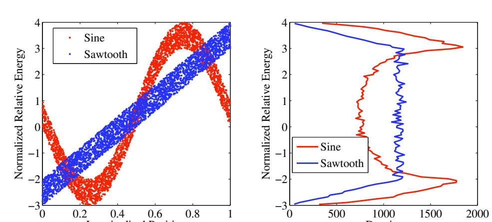

Figure 1: Sinusoidal and sawtooth modulations of the same amplitude (left) produce different energy distributions (right). The double horns of the sinusoidal distribution is less effective at suppressing MBI.

* Work supported by Department of Energy contract DE-AC02-76SF00515

† dratner@slac.stanford.edu

ISBN 978-3-95450-117-5

Pedagogical Model

The mathematical simplicity of the sawtooth is useful as a pedagogical model. The inherently linearized waveforms allow closed form solutions of optimal bunching conditions, and often solutions can be found from simple geometric arguments. We will see that the results of sawtooth optimization can also be applied to find approximate solutions for sinusoidal seeding.

ANALYTICAL DESCRIPTION

Sawtooth HGHG

The simplest use of sawtooth seeding is with High Gain Harmonic Generation (HGHG). To quantify the bunching amplitude we define the bunching factor

$$b_f(k) = \int dz_f \int dp_f e^{ikz_f} \Psi(z_f, p_f) \tag{1}$$

with final longitudinal particle position, z_f, normalized energy, $p_f = (E_f - E_b)/E_b$, wavevector, $k = 2\pi/\lambda$, and distribution $\Psi(z_f, p_f)$. We can evaluate Eq. 1 analytically if we assume a simple initial distribution, $\Psi(z_i, p_i) = I \exp[-p_i^2/2\sigma_\delta^2]$, with initial coordinates, z_i and $p_i = (E_i - E_a)/E_a$, and energy spread, $\sigma_\delta = \sigma_E/E_a$.

We can then write down the sawtooth scheme as

$$\begin{aligned} z_1 &= z_i, & p_1 &= p_i + \frac{2A_L}{\lambda} \bmod_\lambda (z_i) \\ z_f &= z_1 + R_{56} p_1, & p_f &= p_1 \end{aligned} \tag{2}$$

To evaluate Eq. 1, we change to the initial coordinates, $dz_f dp_f \rightarrow dz_i dp_i$, where we can integrate over the simple initial $\Psi(z_i, p_i)$.

To maximize bunching, we stand up each sawtooth filament in phase space, maximizing bunching (Fig. 2). As a result, we find the optimal condition, $R_{56} A_L = -\lambda/2$. We note that the sawtooth requires a larger modulation amplitude, A_L, than the normal HGHG, which requires $R_{56} A_L \approx \lambda/2\pi$. (The difference is due to the extra factor of π in the slope of a sine wave compared to a sawtooth for equal modulation amplitudes.)

We can integrate Eq. 1 to find bunching at the harmonics

$$b_f(hk_L) = \exp[-\frac{(hk_L R_{56}\sigma_\delta)^2}{2}] \tag{3}$$

which is generally larger than from sinusoidal HGHG, with $b_{\text{sine}}(hk_L) = \exp[-\frac{(hk_L R_{56}\sigma_\delta)^2}{2}] J_h(hk_L R_{56} A_L)$ [11]. To avoid suppression by the energy spread, both sine and sawtooth modulations require $|hk_L R_{56}\sigma_\delta| < 1$, giving significant bunching at the harmonic h when $A_L > m\sigma_\delta$. However, in the sawtooth modulation case the bunching is larger by a factor of $\sim 1/J_h(h)$, with the final radiated power scaling as the square of b_f.

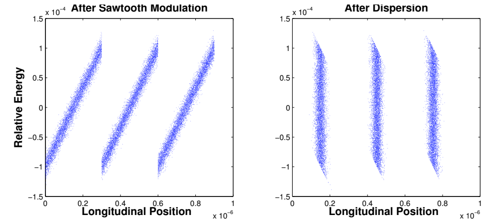

Figure 2: Illustration of the sawtooth HGHG scheme. At left a sawtooth modulation with uniform density can drive sharp density spikes following a dispersive section, right.

Sawtooth EEHG

The sawtooth modulation could also drive a variation of Echo Enabled Harmonic Generation (EEHG) [12]. We describe the EEHG process as follows: an initial modulation (amplitude V_a) and dispersive section ($R_{56}^{(a)}$) filament the beam. A second modulation and dispersive section ($V_b, R_{56}^{(b)}$) then bunch each filament individually. The harmonic is determined by the separation of filaments (Fig. 3).

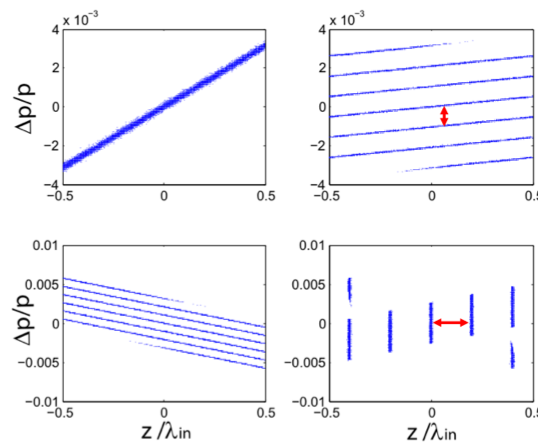

Figure 3: Illustration of the Echo mechanism for a sawtooth modulation. The first dispersive section filaments the beam (top right). The second modulation and dispersive sections then individually bunch the filaments. The vertical separation of filaments (arrow, top right), becomes horizontal separation after the final dispersive section (arrow, bottom right), determining harmonic number.

We can solve for the optimal EEHG manipulation condition through a simple geometric argument; after introducing the sawtooth modulation, our goal is leave each segment with a vertical slope (upright in phase space). We trace the slope through each section of Fig. 3 to find

$$\begin{aligned} S_1 &= V_a/\lambda_{\text{in}} \\ S_2 &= \frac{V_a}{\lambda_{\text{in}} + R_{56}^{(a)} V_a} \\ S_3 &= S_2 + V_b/\lambda_{\text{in}} = \frac{V_a}{\lambda_{\text{in}} + R_{56}^{(a)} V_a} + \frac{V_b}{\lambda_{\text{in}}}, \end{aligned} \tag{4}$$

ISBN 978-3-95450-117-5

and we maximize bunching (final vertical segment) with

$$R_{56}^{(b)} = -\frac{1}{S_3} = -\frac{\lambda_{\text{in}} + R_{56}^{(a)}\ V_a}{V_a + V_b + R_{56}^{(a)}\ V_a V_b/\ \lambda_{\text{in}}}\,. \qquad (5)$$

The horizontal spacing between the vertical filaments, $\Delta z = R_{56}^{(b)}\ \Delta p$, then determines the harmonic, $H = 1/\Delta z$, where Δp is the vertical separation of filaments. We then find optimized harmonic

$$H = \frac{1}{\Delta z} = -(1 + \frac{V_b}{V_a} + \frac{R_{56}^{(a)}\ V_b}{\lambda_{\text{in}}})\,. \qquad (6)$$

For $V_b = -V_a$, we find $H = R_{56}^{(a)}\ V_b/\ \lambda_{\text{in}}$.

The sawtooth model also provides a simple pedagogical model for sinusoidal EEHG. While the general EEHG relations have no closed-form solution, we can find an approximate solution for the sine wave by treating the linear portions as sawtooth modulations. Applying the sawtooth results, we substitute $V_a \rightarrow h_a \equiv V_a k$ and $V_b \rightarrow h_b \equiv V_b k$, giving

$$R_{56}^{(b)} = \frac{\lambda_{\text{in}} + R_{56}^{(a)}\ h_a}{h_a + h_b + R_{56}^{(a)}\ h_a h_b/\ \lambda_{\text{in}}}\,. \qquad (7)$$

We must be careful to allow for $h_a \rightarrow \pm h_a$, due to the two linear regions of a sine wave (phase 0 and π). For given values of $R_{56}^{(a)}$ and h_a, we now find two slopes in the second stage,

$$S_{2\pm} = \frac{h_a}{R_{56}^{(a)}\ V_a \pm \lambda_{\text{in}}}\,. \qquad (8)$$

The result is a more complicated harmonic structure. If we choose a value of $R_{56}^{(b)}$ to satisfy Eq. 5 for one of the initial slopes, we now find two harmonics corresponding to the two initial slopes $\pm h_a$, resulting in

$$H_{\pm} = \frac{1}{\Delta z} = \frac{1}{R_{56}^{(b)}\ S_{\pm 2}} = H\frac{R_{56}^{(a)}\ h_a \pm \lambda_{\text{in}}}{R_{56}^{(a)}\ h_a + \lambda_{\text{in}}}\,, \qquad (9)$$

where H is the harmonic solution given for the sawtooth (Eq. 6). We then expect to find two harmonics (Fig.4). We note that the exact optimal condition (which slightly overbunches the modulation, see [12]), enhances only one of the two harmonics.

COMPARISON OF SAWTOOTH AND SINE WAVEFORMS

While a true sawtooth is a theoretical construct, it may be possible to approximate a sawtooth of amplitude A by including the first few terms of the sawtooth expansion,

$$F(z) = \sum_n \frac{2A}{\pi n}\sin(nkz)\,. \qquad (10)$$

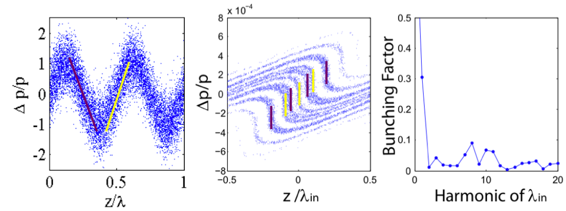

Figure 4: Illustration of the Echo mechanism for a sine modulation. Because a sine has two linear regions ($h_a = \pm Vk$, purple and yellow lines), we will have two different solutions for $R_{56}^{(b)}$, giving two different harmonics $H_{\pm}$. For the parameters above, we find $H_+ = 10$ and $H_- \approx 8$. The optimal bunching condition of [12] will suppress the secondary harmonic.

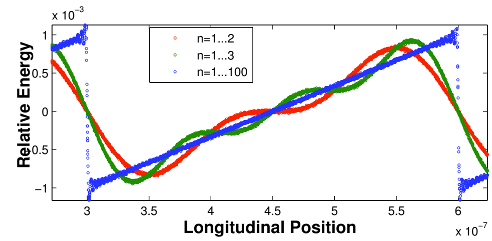

Figure 5: To generate an approximate sawtooth modulation, we can seed the beam with several seeds of different frequencies. Even two or three frequencies can produce a quasi-sawtooth modulation.

For example, using just the first two terms of the expansion produces a comparatively uniform energy spread that would suppress the MBI more efficiently.

Though sawtooth seeding allows for higher bunching factors, the larger dispersion requirement suggests that sine modulations may be more efficient at higher harmonics. To evaluate the usefulness of the sawtooth, we compare equal amplitude sawtooth and sine modulations. We find that the sawtooth increases the bunching factor at low harmonics, but the need for larger R_{56} (and thus greater smearing of the incoherent energy spread), results in worse performance at high harmonics.

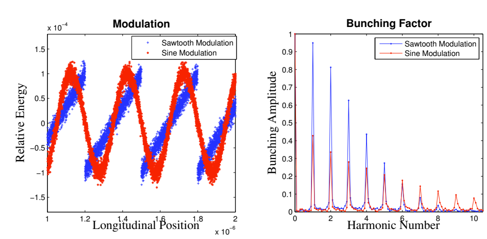

Figure 6: Comparison of equal amplitude sawtooth and sine modulations. The sawtooth increases bunching at low frequencies, but has worse performance at high harmonics.

The difficulty of producing short wavelength seed lasers limits the final radiation wavelength in HGHG. For this reason, we would like to compare the sawtooth and sine mod-

ISBN 978-3-95450-117-5

ulations for a case of equal modulation power at the shortest seed wavelength. In the case of a sawtooth constructed from two frequencies ($n = 1, 2$ in Eq. 10), we then use the same power as the sine HGHG case for the $n = 2$ term, and double the modulation amplitude for $n = 1$. The result is given in Fig. 7. Again we find that the sawtooth aids low harmonic microbunching, but inhibits higher harmonics.

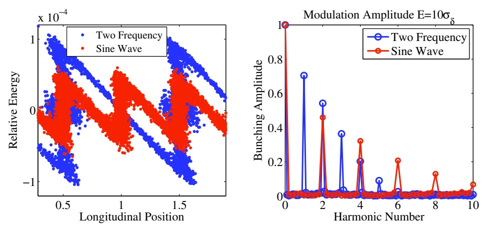

Figure 7: Comparison of one and two wavelength HGHG. The power at the higher frequency of the two wavelength case matches that of the single wavelength case. Adding the second (longer) wavelength modulation to approximate the sawtooth appears to increase the bunching factor at low harmonics, but does not help at high frequencies.

It may be possible to optimize the bunching factor by deviating from the sawtooth expansion (Eq. 10). If available short wavelength laser power limits the final radiated wavelength, then our goal is to improve the bunching factor by adding power at longer wavelengths. As an example, in Fig. 8 we add an additional modulation at double the wavelength with four times the modulation amplitude. By optimizing the dispersion, we find it is possible to enhance the microbunching amplitude compared to a single wavelength modulation. We note this scheme has some similarity to single-chicane compressed harmonic generation [13] and pre-density modulation [14]. Optimization work is ongoing.

We have considered practical examples only for the case of HGHG, but the same approach can be applied to any seeding scheme. For example, Stupakov and Zolotorev have completed a detailed analysis of optimized multi-frequency seeding for EEHG [15].

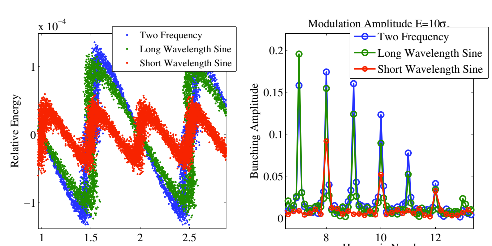

Figure 8: On the left we show the phase space from a high frequency seed (red), low frequency seed (green), and combined seed (blue). At right, we see that the combined seed produces stronger microbunching around the tenth harmonic than the individual seeds. Harmonic number is given relative to the longest seed wavelength.

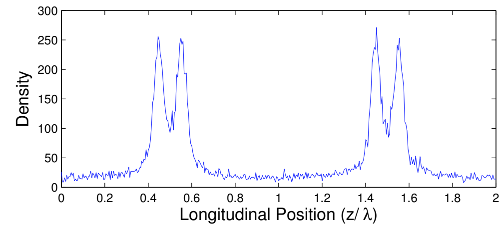

Figure 9: Electron beam density for the simulation of Fig. 8. Rather than following the sawtooth expansion, we tweak the relative amplitudes and dispersive strength to produce two spikes separated by approximately $\lambda/10$.

CONCLUSION

We have considered seeding radiation sources with sawtooth modulations. We find that multi-frequency modulations can optimize the bunching factor at targeted harmonics and help suppress MBI. The sawtooth model is also useful as a pedagogical model for studying seeding schemes.

ACKNOWLEDGEMENTS

We thank G. Stupakov for helpful discussions.

REFERENCES

[1] D. Ratner, A. Chao, and Z. Huang. In *Proceedings of the 2009 FEL Conference*, Liverpool, UK, 2009.

[2] D. Ratner and A. Chao. *Phys. Rev. Lett.*, 105:154801, 2010.

[3] Y. Jiao, D. Ratner and A. Chao. In *Proceedings of the 2011 FEL Conference*, Shanghai, China, 2011.

[4] D. Ratner and A. Chao. In *Proceedings of the 2011 FEL Conference*, Shanghai, China, 2011.

[5] M. Borland. elegant: A flexible SDDS-compliant code for accelerator simulation. Technical Report LS-287, Argonne National Laboratory, 2000.

[6] E. L. Saldin, E. A. Schneidmiller, and M. V. Yurkov. *Nucl. Instrum. Meth. A*, 490:1, 2002.

[7] G. Stupakov S. Heifets and S. Krinsky. *Phys. Rev. ST-AB*, 5:064401, 2002.

[8] Z. Huang and K.J. Kim. *Phys. Rev. ST Accel. Beams*, 5:074401, 2002.

[9] Z. Huang, M. Borland, P. Emma, J. Wu, C. Limborg, G. Stupakov, and J. Welch. *Phys. Rev. ST-AB*, 7:074401, 2004.

[10] Z. Huang et al. *Phys. Rev. ST Accel. Beams*, 13:110703, 2010.

[11] L.H. Yu. *Phys. Rev. A*, 44:5178, 1991.

[12] G. Stupakov. radiation. *Phys. Rev. Lett.*, 102:074801, 2009.

[13] T. Shintake. Internal Report AccelLab-99-1, KEK, 1999.

[14] C. Feng and D. Wang and Z. Zhao. In *Proceedings of the 2010 FEL Conference*, Malmo, Sweden, 2010.

[15] G. Stupakov and M. Zolotorev. In *Proceedings of the 2011 FEL Conference*, Shanghai, China, 2011.

ISBN 978-3-95450-117-5