USING LASER HARMONICS TO INCREASE BUNCHING FACTOR IN EEHG*

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Abstract

We show that using the second and third harmonics of the laser frequency one can substantially increase the beam bunching factor in the echo-enabled harmonic generation (EEHG): for a cold beam one can obtain the bunching factor ≈ 0.4 in the range of harmonic numbers 100-200. We also discuss an option of using nonlinear dispersive elements to increase the bunching factor.

INTRODUCTION

Echo-enabled harmonic generation (EEHG) for FEL seeding uses two undulator-modulators and two chicanes to introduce a fine structure into the beam longitudinal phase space which, at the end of the system, transforms into high harmonic modulation of the beam current [1, 2]. For harmonics $n \approx 100$, in a perfect EEHG setup, it can provide a bunching factor b in the range $b \approx 0.06-0.08$ sufficient for seeding a soft x-ray FEL.

In this paper we consider two methods that could allow to increase the bunching factor. The ultimate goal in increasing b would be to achieve a level of modulation comparable with the bunching at saturation in an FEL. This would lead to elimination of the need of the lasing process using instead the coherent radiation of a pre-bunched beam in the undulator-radiator as a bright source of x-rays. It would also require a shorter undulator-raduator and relax some of the beam parameters (e.g., its peak current). Simulations of bunching factors for LCLS in soft x-ray regime show that at saturation the bunching factor of the beam is close to 0.5 [3].

We consider two approaches to the problem. First, we analyze how much one can gain in the bunching factor if one uses in both EEHG modulators, in addition to the fundamental, second and third harmonics of the laser frequency to optimize the modulation energy profile of the beam. Second, we consider an option of using nonlinear dispersive elements in the system (with the fundamental laser harmonic only) to increase the bunching factor.

ANALYSIS

In what follows we will use notations defined in [2]: $A_1 = \Delta E_1/\sigma_E$, $B_1 = R_{56}^{(1)} k\sigma_E/E_0$, $A_2 = \Delta E_2/\sigma_E$, $B_2 = R_{56}^{(2)} k\sigma_E/E_0$ where ΔE_1 and ΔE_2 are the amplitudes of the energy modulation in the first and second modulators, respectively, $R_{56}^{(1)}$ and $R_{56}^{(2)}$ are the R_{56} parameters for the first and second chicanes, σ_E the rms energy spread of the incoming beam (it is assumed that the beam initially has a Gaussian energy distribution), E_0 the beam energy, and $k = \omega_L/c$ with ω_L the laser frequency (assumed the same in both modulators). A general expression for the bunching factor b_n at harmonic n of the laser frequency in the case of sinusoidal modulation profile (that is generated with only one laser harmonic) can be found in [2]. From this expression it follows that the maximal modulation in an optimized EEHG system depends only on A_1. For $n \gg 1$ is is given by $b_n = F(A_1)n^{-1/3}$, where the function $F(A_1)$ is defined in [2]. In the limit $A_1 \gg 1$, $F(A_1) \approx 0.39$. In this limit one finds for 50th, 100th and 200th harmonics: $b_{50} = 0.11$, $b_{100} = 0.084$ and $b_{200} = 0.067$. Such relatively low bunching factors are due to the fact that in the final phase space of the beam only a small fraction of the beam particles contribute to the modulation at harmonic n, see Fig. 1. To increase the bunching factor we now assume

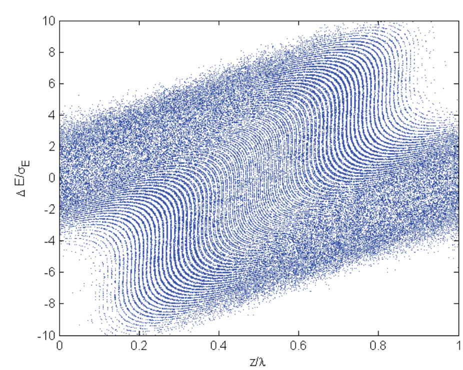

Figure 1: The EEHG phase space at the exit from the system. The horizontal axis is the coordinate z in the beam normalized by the laser wavelength, and the vertical axis is the energy deviation normalized by the initial rms energy spread in the beam. The parameters of the chicanes were optimized for $n = 100$, with $A_1 = 5$, $A_2 = 3$. The bunching factor at 100 harmonic is 0.08. Only particles that form a pattern of vertical stripes contribute to b_{100}.

that one has a capability to vary the energy modulation profiles in both modulators (and is not limited to the function $\sin(kz)$ as was assumed in [1, 2]), that is

$$\Delta E(z) = \Delta E_1 C_1(kz) \text{ in first modulator,}$$
$$\Delta E(z) = -\Delta E_2 C_2(kz) \text{ in second modulator,} \quad (1)$$

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where C_1 and C_2 are dimensionless periodic functions with the period of 2π and ΔE_1, and ΔE_2 are the amplitudes of the modulation (the minus sign in the second equation is chosen for convenience). Of course, an unconstrained freedom in the choice of C_1 and C_2 can dramatically increase the bunching factor. For example, it is easy to show that a "sawtooth" profile for both C_1 and C_2 (that is a linear slope periodically interrupted by discontinuous jumps), in the limit of large A_1 and A_2, gives 100% bunching at all harmonics [4]! Such profile, however, involves an infinite number of laser harmonics, and hence cannot be considered as a practical one. On the other hand, adding a small number of harmonics of the laser frequency might be feasible if the laser beam energy in higher harmonics is reasonably small.

THEORY

To get an insight into the optimal choice of functions C_1 and C_2, we first develop an analytical model of EEHG with arbitrary modulation profiles in which we express the bunching factor b_n in terms of C_1 and C_2 and other beam parameters. For simplicity, we consider the case of equal fundamental laser frequencies in both modulators. Following the derivation of b_n in Appendix A of [2] one can express the bunching factor as an integral over the initial beam distribution function (Eqs. (A5) and (A6) in [2]) . We found it more convenient however to express the bunching factor as an integral of the distribution function after the first chicane. It has the following form (cf. (A5) in [2])

$$b_n = \frac{1}{2\pi}\left|\int_{-\infty}^{\infty} dp' \int_0^{2\pi} d\zeta' \, f(p',\zeta')\langle e^{-in\zeta''(\zeta',p')}\rangle\right|,$$

where $p = (E - E_0)/\sigma_E$ and $\zeta = kz$, p' and ζ' refer to the position after the first chicane, ζ'' is the coordinate ζ after the second chicane,

$$\zeta'' = \zeta' + B_2(p' + A_2 C_2(\zeta')), \quad (2)$$

and $f(p',\zeta')$ is the distribution function after the first chicane given by

$$f(p',\zeta') = f_0(p' - A_1 C_1(\zeta' - B_1 p')), \quad (3)$$

with $f_0(p) = (2\pi)^{-1/2} e^{-p^2/2}$ the initial distribution function of the beam.

To further simplify analysis, we consider the case of a cold beam, $f(p) = \delta(p)$:

$$b_n = \frac{1}{2\pi}\bigg|\int_{-\infty}^{\infty} dp' \int_0^{2\pi} d\zeta' \, \delta(p' - A_1 C_1(\zeta' - B_1 p')) \times e^{-in(\zeta' + B_2(p' + A_2 C_2(\zeta')))}\bigg|. \quad (4)$$

Using the representaion $\delta(x) = (2\pi)^{-1}\int_{-\infty}^{\infty} e^{i\kappa x} d\kappa$ we obtain

$$b_n = \frac{1}{(2\pi)^2}\bigg|\int_{-\infty}^{\infty} dp' d\kappa \int_0^{2\pi} d\zeta' \, e^{i\kappa(p' - A_1 C_1(\zeta' - B_1 p'))} \times e^{-in(\zeta' + B_2(p' + A_2 C_2(\zeta')))}\bigg|. \quad (5)$$

The function $e^{-i\kappa A_1 C_1(\zeta' - B_1 p')}$ is a periodic function of $\zeta' - B_1 p'$ and can be expanded in Fourier series

$$e^{-i\kappa A_1 C_1(\zeta' - B_1 p')} = \sum_{m=-\infty}^{\infty} R_m(\kappa A_1) e^{im(\zeta' - B_1 p')}, \quad (6)$$

with R_m given by

$$R_m(t) = \frac{1}{2\pi}\int_0^{2\pi} dx e^{-i(mx + tC_1(x))}. \quad (7)$$

Substituting this expansion into (5) and integrating over p' and κ we obtain

$$b_n = \left|\sum_{m=-\infty}^{\infty} R_m(A_1(mB_1 + nB_2)) S_{n,m}(A_2 B_2)\right| \quad (8)$$

with

$$S_{n,m}(t) = \frac{1}{2\pi}\int_0^{2\pi} d\zeta \, e^{im\zeta - in(\zeta - tC_2(\zeta))}. \quad (9)$$

Analysis shows that for a given n in an optimized sum (8) the dominant contribution comes from one term, either with $m = 1$ or $m = -1$ [2]. In the $m = 1$ case the chicanes have opposite signs of R_{56}, and for $m = -1$ $R_{56}^{(1)}$ and $R_{56}^{(2)}$ have the same sign. In what follows we assume $m = -1$. With only one term selected in (8) the optimization strategy consists in choosing the function C_1 in such a way as to maximize R_{-1} and then choosing C_2 to maximize $S_{n,-1}$ in (9).

In the next section we will develop an optimization procedure for b_n considering the limit $n \gg 1$.

OPTIMIZATION

Mentioned previously the "sawtooth" profile corresponds to the choice $C_1(x) = ax$ and $C_2(x) = bx$ for $0 < x \leq 2\pi$ with a and b constant. Then R_m is easily integrated and its maximal value is equal to 1 achievable at $A_1(mB_1 + nB_2) = m/a$. Similarly, the integral $S_{n,m}$ can be calculated and again its maximal value is equal to 1 achievable at $nB_2 A_2 = -m/b$. Hence, we conclude that in this limit the bunching factor can be optimized to achieve its maximal value of 1 [4].

We now consider optimization that involves two or three laser harmonics. As a reference case, let us start with the sinusoidal modulation $C_1(\zeta) = C_2(\zeta) = \sin(\zeta)$. In this case R_{-1} reduces to the Bessel function $J_1(A_1(nB_2 - B_1))$ with $\max|R_{-1}| = 0.58$ at $A_1(nB_2 - B_1) = 1.84$. Eq. (9) now takes the form

$$S_{n,-1}(t) = \frac{1}{2\pi}\int_0^{2\pi} d\zeta \, e^{-i((n+1)\zeta - nt\sin(\zeta))}. \quad (10)$$

In the limit of large n, the integrand is a rapidly oscillating function of ζ. It is maximized if t is chosen in such a way that the linear term in the argument of the exponential $(n + 1)\zeta - nt\sin(\zeta)$ almost cancels in the vicinity of point

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$\zeta = 0$, that is $t = (n+1)/n \approx 1$ (a more thorough consideration shows that the optimal linear term should have a coefficient of order of $n^{1/3}$). In this case the main contribution to the integral comes from the region near $\zeta = 0$ where $e^{-i((n+1)\zeta - nt\sin(\zeta))} \approx e^{-i(n+1-nt)\zeta - int\zeta^3/6}$ and the integral reduces to the Airy function with $\max|S_{n,-1}| \approx 0.67n^{-1/3}$. This gives for the maximal value of the bunching factor $\max|R_{-1}| \max|S_{n,-1}| = 0.39/n^{1/3}$, as mentioned in the Introduction.

Let us now consider the case of two harmonics in the first modulator, $C_1(\zeta) = \sin(\zeta) + \alpha\sin(2\zeta)$. Numerical maximization of the integral (7) gives $\max|R_{-1}| = 0.74$ at $\alpha = 0.45$. For three harmonics in the first modulator, $C_1(\zeta) = \sin(\zeta) + \alpha\sin(2\zeta) + \beta\sin(3\zeta)$, we find numerically the optimal values $\alpha = -0.48$, $\beta = 0.29$ with $\max|R_{-1}| = 0.81$. In both cases we find that in the optimum $A_1(nB_2 - B_1) \approx 2$. Plot of the optimized modulation profile for three harmonics is shown in Fig. 2.

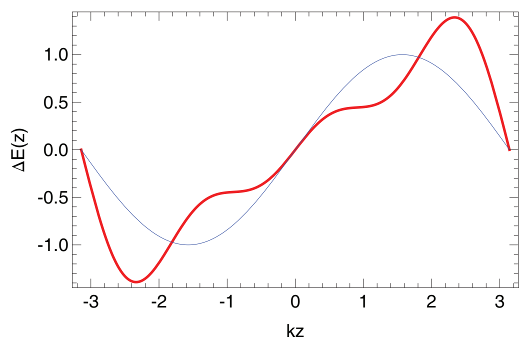

Figure 2: Plot of the optimized modulation profile (red line). For reference, the blue thin line shows the function $\sin(x)$.

Note that in the region $-2.4 < kz < 2.4$ the optimized profile looks like a rough approximation to the "sawtooth" slope. This is not surprising because the optimal values of α and β are close to those obtained by truncation of the Fouries expansion of the "sawtooth": $\sin(x) - \frac{1}{2}\sin(2x) + \frac{1}{3}\sin(3x)$.

We will now turn to optimization of function $S_{n,-1}$. As above, we consider two cases: one with the first and the second harmonics, $C_2(\zeta) = \sin(\zeta) + a\sin(2\zeta)$, and the other with three harmonics in the modulation profile, $C_2(\zeta) = \sin(\zeta) + a\sin(2\zeta) + b\sin(3\zeta)$. From analysis similar to that at the beginning of this section we expect that for two harmonics the optimized asymptotic dependence in the limit of large n of $S_{n,-1}$ versus n is $n^{-1/5}$, and for the three harmonics $S_{n,-1} \propto n^{-1/7}$, that is the bunching factor will have a slower decay with n than in the case of the sinusoidal modulation. The results of a direct numerical optimization in both cases (as well as the reference case of one harmonic $C_2(\zeta) = \sin(\zeta)$) are shown in Fig. 3.

As expected, adding more harmonics increases the maximized value of $S_{n,-1}$ for a given n, and hence the bunching factor. Note also that the numerically found powers in the asymptotic dependence $S_{n,-1} \propto n^{-1/p}$ for 2 and 3 harmonics show a somewhat slower decay than what follows from a simple analysis outlined above: $p = 5.4$ instead $p = 5$ for two harmonics, and $p = 8$ instead of $p = 7$ for 3 harmonics.

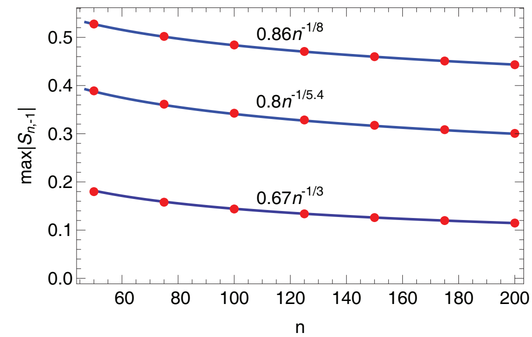

Figure 3: Maximized values of the function $S_{n,-1}$ versus harmonic number n (red dots). The lower set of points corresponds to $C_2(\zeta) = \sin(\zeta)$, the middle one is for the case of two harmonics, and the upper one is for the case of 3 harmonics in the modulation profile. Blue lines show fitted analytical approximations indicated near each line.

For $n = 100$ in the case of three harmonics the optimized modulation profile has $a = -0.273$ and $b = 0.053$. The plot of function $C_2(\zeta)$ in this case (together with the reference $\sin(\zeta)$) is shown in Fig. 4. Note almost a perfect linear slope for $-1.5 < \zeta < 1.5$.

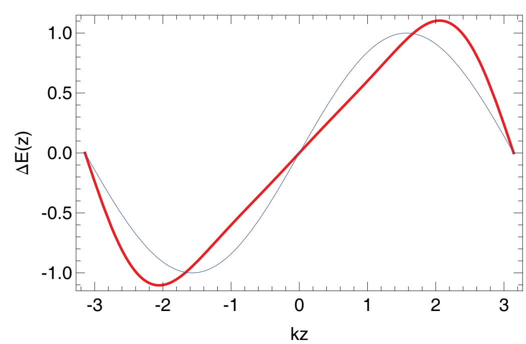

Figure 4: Plot of $\sin(\zeta) - 0.273\sin(2\zeta) + 0.053\sin(3\zeta)$ (red line) and $\sin(\zeta)$ (blue line).

The combined effect of optimization of both factors R_{-1} and $S_{n,-1}$ gives the optimized bunching factor, and is shown in Fig. 5. For $n = 100$ the three-harmonic case gives four times larger bunching factor than with one harmonic.

A considerable gain in the magnitude of the beam modulation with the optimized modulation profiles can be understood if one considers the phase space of the beam. Such a phase space at the end of the system for the 3 harmonics case is shown in Fig. 6. It was obtain by simulations with $A_1 = A_2 = 5$. The final bunching factor for that case at 100th harmonic is 0.37. Comparing this plot with Fig. 1

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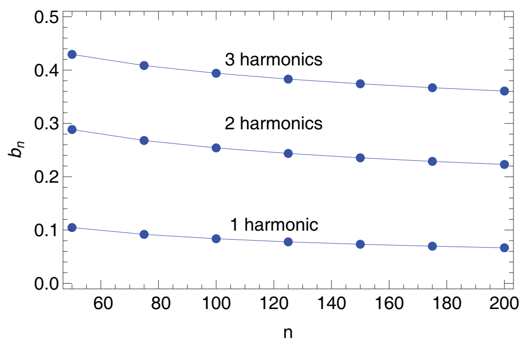

Figure 5: The optimized bunching factor as a function of the harmonic number for 1, 2 and 3 harmonics.

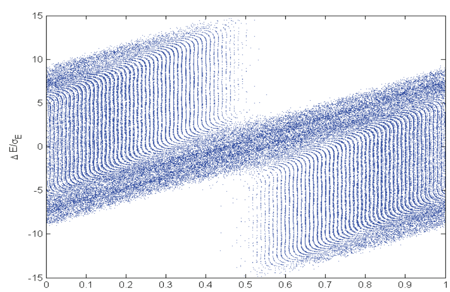

Figure 6: Phase space for an optimized modulation profile.

one can see that much more particles are now involved into the formation of the bunching pattern.

To implement two- or three-harmonic scheme one needs, in addition to the fundamental laser frequency ω_L, to generate harmonics at $2\omega_L$ and $3\omega_L$. These can be obtained by frequency doubling or tripling of the main laser beam using a nonlinear crystal. Note that one has to implement a phase control of all harmonics involved in the modulation in order to achieve profiles of the modulation shown in Figs. 2 and 4. One also needs undulator-modulators resonant with the beam at the second an third harmonics. Instead of using three separate undulators, it might be advantageous to modulate the beam in one undulator which is tuned at the fundamental laser frequency. Such an undulator is capable to couple the beam with the fundamental and third harmonic propagating along the undulator axis, and also the second harmonic propagating at a small angle to the axis. One can also use an undulator with a composite magnetic field which incorporates all three harmonics of the fundamental undulator period.

NONLINEAR DISPERSION

While using higher laser harmonics holds a promise of considerable increase of the bunching factor, it also results in a more complicated seeding device. As another approach to the problem we considered maximization of the EEHG bunching using nonlinear dispersive elements in the system.

Specifically, assuming now a sinusoidal energy modulation, we also assumed that the strength of the dispersive elements R_{56} is a nonlinear function of δ ($\delta = (E - E_0)/E_0$). We then approximate $R_{56}(\delta)$ by its Taylor expansion $R_{56}(\delta) \approx \bar{R}_{56}(1 + a\delta'^2 + b\delta'^4)$ (analysis shows that odd powers of δ do not contribute to the increase of the bunching, and are dropped in the Taylor series). This gives the following transformation for one modulator-chicane stage:

$$\delta' = \delta + \Delta E \sin k_L z$$
$$z' = z + \delta' \bar{R}_{56}(1 + a\delta'^2 + b\delta'^4). \tag{11}$$

Assuming a cold beam and using numerical optimization we found that a nonlinear chicane after the first undulator increases the factor R_{-1} from 0.58 to 0.62 with optimal values $a = -0.6$ and $b = 0$. A nonlinear chicane after the second undulator increases $S_{m,-1}$ from 0.14 to 0.37, with optimal values $a = 0$ and $b = 0.34$. Combined, this leads to an increase of b_{100} from 8.4% to 22%.

CONCLUSIONS

In this paper we considered optimization of the echo-enable harmonic generation by introducing higher (2nd and 3d) harmonics into the energy modulation profile of the beam. With optimal profiles, for a cold beam, this would result in the bunching factor at 100th harmonic $b_{100} \approx 0.4$. In principle, energy modulation in this scheme can be implemented in a single undulator tuned at the fundamental wavelength. We emphasize that high enough bunching factors can eliminate the need for the FEL lasing and can lead to a shorter undulator-radiator and different requirements for the beam parameters.

We also showed that the bunching can be increased by using nonlinear dispersive elements (and a standard $\sin kz$ modulation profile).

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REFERENCES

[1] G. Stupakov, Phys. Rev. Lett. 102, 074801 (2009).

[2] D. Xiang and G. Stupakov, Phys. Rev. ST Accel. Beams 12, 030702 (2009).

[3] Y. Ding (2011), private communication.

[4] A. Chao and D. Ratner, MOPB21, Proc. of FEL 2011, Shanghai, China (2011).

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