SASE FEL PULSE DURATION ANALYSIS FROM SPECTRAL CORRELATION FUNCTION*

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Abstract

A new method to measure the X-rays pulse duration through the analysis of the statistical properties of the SASE FEL spectra has been developed. The information on the pulse duration is contained in the correlation function of the intensity spectra measured after a spectrometer. The spectral correlation function is derived analytically for different profile shapes in the exponential growth regime and issues like spectral central frequency jitter and shot by shot statistical gain are addressed. Numerical simulations will show that the method is applicable also in saturation regime and that both pulse duration and spectrometer resolution can be recovered from the spectral correlation function. The method has been experimentally demonstrated at LCLS, measuring the soft X-rays pulse durations for different electron bunch lengths, and the evolution of the pulse durations for different undulator distances. Shorter pulse durations down to 13 fs FWHM have been measured using the slotted foil.

INTRODUCTION

Radiation from relativisitic electrons constitutes a large spectral range from terahertz to x-rays and provides widespread scientifc and industrial applications. The advent of x-ray free-electron lasers (FELs) [1] has opened up many new research directions: it is now possible to observe atoms and molecules in motion because of the brevity and the intensity of these x-ray pulses. In exploration of this ultrafast world, the accurate knowledge of the x-ray pulse duration is critical.

For an x-ray FEL based on self-amplified spontaneous emission (SASE), the radiation originates from the initial random distribution of the electrons within a bunch. Information about the electron bunch length can be obtained from the statistical fluctuation of the incoherent radiation intensity [2, 3], as demonstrated by earlier experiments using spontaneous radiation sources [4, 5, 6, 7]. However, a SASE FEL differs from a spontaneous source in two aspects. Firstly, although the amplification process is a linear amplifier and does not change the statistical properties of the radiation, it can modify the x-ray pulse duration compared to the electron bunch length due to the exponential gain and slippage effects. Secondly, a SASE FEL typically operates in the saturation regime for purposes of intensity stability. In the saturation regime, the FEL process is very nonlinear, and statistical fluctuations may not be useful to retrive the radiation pulse duration. Recent studies about the x-ray pulse duration and the statistical fluctation in the exponential gain regime can be found in Refs. [8, 9].

In this paper, we discuss a new method to measure the x-ray pulse duration through the analysis of the statistical properties of the SASE FEL spectra. We first develop theoretically how this method can be applied to x-ray pulse duration measurements in the exponential growth regime. Then we show through numerical simulations that this method is still applicable in the FEL saturation regime where the machine typically operates. Finally, we apply this technique to experimental data from the world's first hard x-ray FEL, the Linac Coherent Light Source (LCLS) at various machine settings where the x-ray pulse durations are measured from 200 fs to 13 fs. This technique can be applied to any SASE FELs at arbitrary wavelengths as long as the FEL spectrum can be determined on a single shot basis.

THEORY

In the exponential growth regime, the SASE FEL behaves as a narrow band linear amplifier, which selectively amplifies a wideband random input signal. The fluctuations result from the shot noise of the electron beam current

$$I(t) = (-e)\sum_{k=1}^{N}\delta(t-t_k) \tag{1}$$

at the undulator entrance, where the arrival times t_k are random variables with the probability density $f(t)$. The process of amplification, within a one-dimensional model, can be described by a Green function $h(t,\tau)$, and the electric field calculated as the convolution

$$E(t) = \int_{-\infty}^{+\infty} h(t,\tau)I(\tau)d\tau. \tag{2}$$

For electron beams with constant parameters, the SASE FEL Green function has the form [10, 11]

$$h_{ti}(t-\tau) = A_0(z)e^{ik_0 z}e^{-i\omega_0(t-\tau)-\frac{(t-\tau-z/v_g)^2}{4s_t^2}\left(1+\frac{i}{\sqrt{3}}\right)}, \tag{3}$$

where $A_0(z)$ is the exponential growth factor and is proportional to $e^{\frac{z}{l_g}}$ where l_g is the field gain length. To describe the growth process in case of a non flat current profile, one can model the gain length as function of τ [12]. However, gain mechanism depends on other local beam parameters. Besides the electron beam current, also quantities such as transverse emittance, energy spread and undulator taper contributes to have different growth rates along the

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electron bunch. To model this, we introduce a time dependent gain function h_{td}, and write a time dependent SASE FEL impulse response function as

$$h(t,\tau) = h_{ti}(t-\tau)h_{td}(\tau). \quad (4)$$

h_{td} is a slow-varying function on the scale of FEL coherence length.

The spectrum of the electric field is calculated as

$$\tilde{E}(\omega) = \int_{-\infty}^{+\infty} E(t)e^{i\omega t}dt = (-e)\tilde{H}_{ti}(\omega)\sum_{k=1}^{N} e^{i\omega t_k}h_{td}(t_k) \quad (5)$$

where $\tilde{H}_{ti}(\omega)$ is the Fourier transform of $h_{ti}(t)$. Spectral correlations between two different frequencies can be calculated as

$$\left\langle \tilde{E}(\omega')\tilde{E}^*(\omega'')\right\rangle \approx e^2 N\tilde{H}_{ti}(\omega')\tilde{H}^*_{ti}(\omega'')\tilde{X}(\omega'-\omega'') \quad (6)$$

$$\frac{\left\langle |E(\omega')|^2|E(\omega'')|^2\right\rangle}{e^4N^2|\tilde{H}_{ti}(\omega')|^2|\tilde{H}_{ti}(\omega'')|^2} \approx \left(\tilde{X}(0)^2 + |\tilde{X}(\omega'-\omega'')|^2\right) \quad (7)$$

where angle brackets represent an ensemble average, $X(t) = |h_{td}(t)|^2 f(t)$ is the x-ray average pulse shape and $\tilde{X}(\omega)$ is it's Fourier function. In the following, without loss of generality, we can consider $\tilde{X}(0) = 1$. We denote single-shot spectrum taken after the spectrometer as

$$S(\omega) = \int_{-\infty}^{+\infty} \frac{e^{\frac{(\omega'-\omega)^2}{2s_m^2}}}{\sqrt{2\pi}s_m}|\tilde{E}(\omega')|^2 d\omega', \quad (8)$$

where s_m is the rms of the spectrometer resolution function modeled as Gaussian. We define the second order spectral correlation function as

$$G_2(\Delta\omega) = \frac{\langle S(\omega_0+\Delta\omega/2)S(\omega_0-\Delta\omega/2)\rangle}{\langle S(\omega_0+\Delta\omega/2)\rangle\langle S(\omega_0-\Delta\omega/2)\rangle} - 1, \quad (9)$$

where ω_0 is the central frequency of amplification. Substituting Eq. (6) and (7) into Eq. (9) and using $|\tilde{H}_{ti}(\omega)|^2 \propto e^{-\frac{(\omega-\omega_0)^2}{2s_a^2}}$, where $s_a = \frac{1}{\sqrt{3}s_t}$ is the FEL bandwidth, one obtaines

$$G_2(\delta\omega) = \int_{-\infty}^{+\infty} \frac{e^{\frac{-(\xi-\delta\omega\xi_0)^2}{2\sigma^2}}}{\sqrt{2\pi}\sigma}|\tilde{X}(\xi)|^2 d\xi, \quad (10)$$

where $\sigma_a = s_a/\omega_0$, $\sigma_m = s_m/\omega_0$, $\sigma = \sqrt{2}\frac{\sigma_a\sigma_m}{\sqrt{\sigma_a^2+\sigma_m^2}}\omega_0$ and $\xi_0 = \frac{\sigma_a^2}{\sigma_a^2+\sigma_m^2}\omega_0$, $\delta\omega = \Delta\omega/\omega_0$. Equation (10) can be particularized for different $X(t)$ shapes. Modelling the x-ray average pulse shape as Gaussian, with rms length σ_t, we have

$$G_2(\delta\omega) = \frac{e^{-\frac{\delta\omega^2\xi_0^2\sigma_t^2}{1+2\sigma^2\sigma_t^2}}}{\sqrt{1+2\sigma^2\sigma_t^2}}, \quad (11)$$

while for a flat top average lineshape with full length duration T,

$$G_2(\delta\omega) = 2\int_0^1 e^{-\zeta^2\sigma^2T^2/2}(1-\zeta)\cos(\delta\omega\xi_0T\zeta)d\zeta, \quad (12)$$

the latter can be strightforwardly integrated to obtain a closed form.

To measure both the average x-ray pulse duration and the spectrometer resolution, we start from a large set of recorded spectra. Then the average spectrum is calculated. By fitting the average spectrum, one obtains $\sigma_a^2+\sigma_m^2$ and ω_0. Thus only the relative spectrometer resolution σ_m and the pulse duration are unknown. Finally, the experimental G_2 function is calculated and by fitting it with the analytical model, we derive both the pulse duration and the spectrometer resolution. For typical measurement conditions $\sigma_a >> \sigma_m$, $\sigma \approx \sqrt{2}\sigma_m\omega_0$, $\xi_0 \approx \omega_0$, thus the measurement is insensitive to errors on σ_a.

The method described above holds true for the exponential growth regime, since within our linear time dependent model, the random Gaussian process properties of the input signal are retained [13]. However, in the saturation regime, first and second order correlations Eq. (6) and (7) do not hold true in general. The statistical properties in this regime have been studied by numerical simulations in [14], and a simplified analytical model is described in [11]. One conclusion, relevant for our treatment, drawn from these studies is that the quasi Gaussian statistics is retained for narrow band instantaneous power fluctuations also in the saturation regime.

Shot-to-shot fluctuations of the electron bunch energy, peak current and beam paramenters can have an impact on the G_2 function. We consider a jitter of the central frequency of amplification by letting it fluctuate as a random Gaussian variable with rms $\sigma_\omega\omega_0$ and average ω_0. Following the procedure to derive Eq. (10), we obtain

$$G_2(\delta\omega) = K(\delta\omega)\int_{-\infty}^{+\infty} \frac{e^{\frac{-(\xi-\delta\omega\xi_0)^2}{2\sigma^2}}}{\sqrt{2\pi}\sigma}|\tilde{X}(\xi)|^2 d\xi \quad (13)$$

where

$$K(\delta\omega) = \frac{\left(\sigma_a^2+\sigma_m^2+\sigma_w^2\right)e^{-\frac{\delta\omega^2\sigma_w^2}{4(\sigma_a^2+\sigma_m^2)(\sigma_a^2+\sigma_m^2+\sigma_w^2)}}}{\sqrt{\left(\sigma_a^2+\sigma_m^2\right)\left(\sigma_a^2+\sigma_m^2+2\sigma_w^2\right)}} \quad (14)$$

Also a shot-to-shot gain jitter has an impact on the G_2 function, we write a measured spectrum as $GS(\omega)$, where $G = \overline{G} + \Delta G$ represents the gain, and calculate the second order correlation between the spectral intensities S_1 and S_2 measured at the frequencies ω_1 and ω_2, as

$$G_{2g} = \frac{\langle(\overline{G}+\Delta G)(\overline{S}_1+\Delta S_1)(\overline{G}+\Delta G)(\overline{S}_2+\Delta S_2)\rangle}{\langle(\overline{G}+\Delta G)(\overline{S}_1+\Delta S_1)\rangle\langle(\overline{G}+\Delta G)(\overline{S}_2+\Delta S_2)\rangle} \quad (15)$$

yielding

$$G_2 = \frac{\langle\Delta S_1\Delta S_2\rangle}{\overline{S}_1\overline{S}_2} = \frac{G_{2g}}{\left(1+\frac{\langle\Delta G^2\rangle}{\overline{G}^2}\right)} - 1 \quad (16)$$

The effect of the statistical gain can be canceled by normalizing each collected spectrum by its integral over the frequency. Eventually, since for large values of $|\omega_1-\omega_2|$,

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$\langle \Delta S_1 \Delta S_2 \rangle = 0$, one can evaluate $\left(1 + \frac{\langle \Delta G^2 \rangle}{\overline{G}^2}\right)$ as offset of G_{2g}. This allows to calculate G_2 by using Eq. (16).

NUMERICAL SIMULATIONS

We run numerical simulations with a 1D FEL code to show that the proposed method is also applicable to a large extent at saturation and in deep saturation. We simulated a flat top electron beam with an energy of 5.9 GeV, a peak current of 3 kA, and a 1 mm mrad normalized transverse emittance. The undulator has a period of 3 cm and a strength parameter of 3.5, yielding a radiation wavelength of 0.8 nm and saturation occurring close to 40 m. 4000 independent shots were simulated for a 30 μm electron bunch and 2000 independet shots were simulated for a 3 μm electron bunch. Two different spectrometer resolutions σ_m, 10^{-4} and 2×10^{-4} have been used to show that prior knowledge of the spectrometer resolution is not needed for the pulse duration measurement. Figure 1 shows

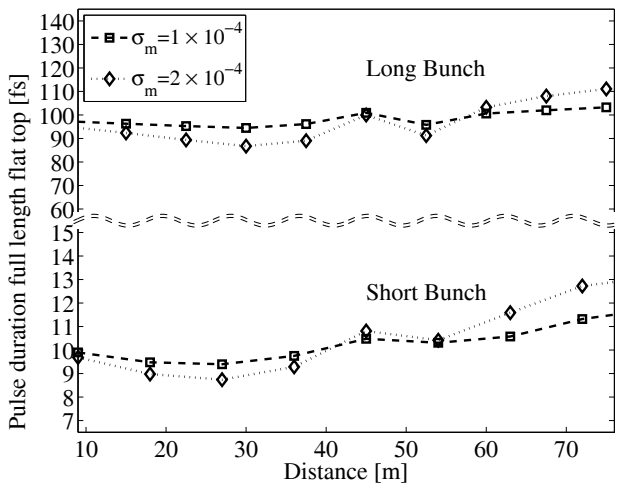

Figure 1: Simulated measurement of x-rays pulse duration vs undulator distance. (Squares) spectrometer resolution $\sigma_m = 10^{-4}$, (Diamonds) spectrometer resolution $\sigma_m = 2 \times 10^{-4}$.

the simulated pulse duration measurement for both electron bunch lengths and spectrometer resolutions. For the short bunch case, the slippage and the edge effects played a more important role as compared to the longer bunch, showing that the method can measure the x-ray pulse duration evolution even for ultra-short bunches. Spectrometer resolutions have been retrieved as $(1.01 \pm 0.01) \times 10^{-4}$ and $(2.05 \pm 0.05) \times 10^{-4}$ for the long bunch case, and as $(0.98 \pm 0.03) \times 10^{-4}$ and $(1.98 \pm 0.09) \times 10^{-4}$ for the short bunch case.

EXPERIMENTAL RESULTS

The experimental demonstration of the method was performed at the LCLS to measure soft x-rays pulse durations. The machine was set to operate at 1.5 keV photon energy and the electron charge was set to 250 pC. The spectra were recorded by the LCLS soft x-rays spectrometer [15]. For each machine setting around 40 000 spectra have been recorded. For each spectrum we have also acquired quantities such as electron beam energy and charge, peak current, trajectory information and x-ray pulse energy. These quantities have been used to select a subset of the collected spectra to analyze. In particular electron beam energy and peak current, were used to select a subset with similar pulse durations and aligned spectra. Aligned spectra are needed to minimize the effect of the central frequency jitter described in Eq. (14). A typical subset contained around 5% of the originally recorded data. To reduce the effect of the FEL intensity jitter, measured spectra were normalized by their integral over frequency. For our analytical treatment we have assumed full transverse coherence. FEL simulations suggest that the transverse coherence decreases after saturation to about 50%-60% [16]. Additional analysis of Genesis simulations [17] have shown that the degree of transverse coherence in the vertical and the horizontal directions are $\approx$ 80% and $\approx$ 70% respectively. To reduce the effect of the transverse modes on the statistical analysis, from the 2D recorded spectral images, only the subset of the data on the horizontal coordinate has been used. This is equivalent to using a vertical slit for improving the transverse coherence in the horizontal direction.

During our experiments we measured the x-ray pulse durations for different machine conditions. In particular, we varied the undulator length and the electron bunch peak current. To obtain shorter x-ray pulses, we also applied the slotted foil technique to change the effective electron bunch length that was able to lase [12].

In the first experiment we measured the pulse duration for different undulator lengths. The peak current was set to 3 kA, which yields an 83 fs electrons bunch length for a flat top shape. Pulse duration measurements are presented in Tab. 1 showing that x-ray pulses were shorter than electron bunches and that, with our post saturation taper configuration, the pulse duration increases when the deep saturation is reached. The measured spectrometer resolution was similar for the different analyzed data sets, and was equal to $\sigma_m = (1.00 \pm 0.04) \times 10^{-4}$. The designed spectrometer resolution at 1.5 keV is 0.85×10^{-4} [15]. Measurements performed at LCLS pointed out that the SXR instrument resolution at that photon energy should be closer to 1.4×10^{-4} [18]. The discrepancy could be attributed to the fact that this resolution was derived from an averaged spectra, and it could be influenced by such instabilities as vibrations and photon beam jitters. The resolution derived by our method is based on the intensity interferometry principle. Therefore, it is much less sensitive to such instabilities. Further in our analytical model we have considered a Gaussian resolution function of the spectrometer. A different shape for the resolution function could also contribute to the differences in the resolution derived from the two methods.

In the second experiment pulse durations were measured

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Table 1: Measured x-ray pulse duration vs undulator distance. Pulse duration expressed as full length flat top.

Undulator Distance [m]	Measured x-ray full length [fs]	Spectrometer measured σ_m
43.5	51	1.02×10^{-4}
53.6	48	0.99×10^{-4}
63.5	49	0.98×10^{-4}
73.7	50	1.03×10^{-4}
83.7	59	0.99×10^{-4}
93.8	73	1.04×10^{-4}

for different peak currents at a fixed electron bunch charge. For these data sets, undulator taper has been applied in order to maximize the output power with 28 undulator segments present. Experimental results have been collected for the peak current of 1.5 kA to 3 kA. Figure 2 shows the measured x-ray pulse duration compared to the electron bunch length in the hypothesis of a flat top electron bunch distribution. Higher peak current electron bunches yield clearly shorter average FEL pulses. Finally, we measured

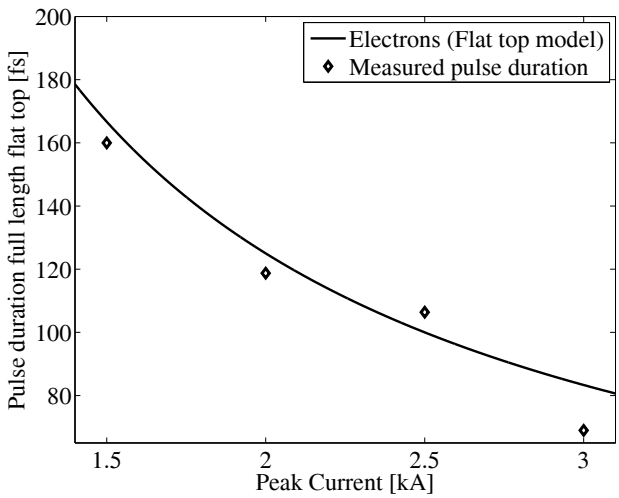

Figure 2: (Diamonds) Measured x-rays pulse duration vs different peak currents, duration expressed as flat top full length. (Solid) Electrons bunch length for a 250 pC flat top profile as function of the peak current

shorter x-rays pulse durations by controlling the electron bunch length using the slotted foil technique. The shortest pulse duration was measured for the slotted foil configuration corresponding to active electron bunch length of 10 fs FWHM [19]. With this setting, the measured average x-ray pulse duration was 13 fs FWHM.

CONCLUSION

We have developed a new approach for measuring the average pulse duration of SASE FEL x-ray pulses by using the statistical characteristics in the spectral domain.

Table 2: Electron bunch length controlled using the slotted foil and measured x-ray pulse duration as FWHM Gaussian. Electron bunch length is calculated with the formula presented in [19]. σ_m is the spectrometer resolution measured for each dataset.

Electron bunch FWHM [fs]	x-ray FWHM [fs]	Spectrometer measured σ_m
10	13	0.86×10^{-4}
18	24	0.90×10^{-4}
28	39	0.88×10^{-4}
56	52	0.85×10^{-4}

The method allows to measure the pulse duration without prior knowledge of the spectrometer resolution and can be also used to measure the spectrometer resolution as a cross check to other direct experimental techniques. By analyzing numerical simluations, we shown that the method is applicable also in the nonlinear region of the SASE amplification process. Experimental soft x-rays pulse durations measurements at LCLS were consistent with the manipulated electron bunch length.

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