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Statistical Analysis of Crossed Undulator for Polarization Control in a SASE FEL

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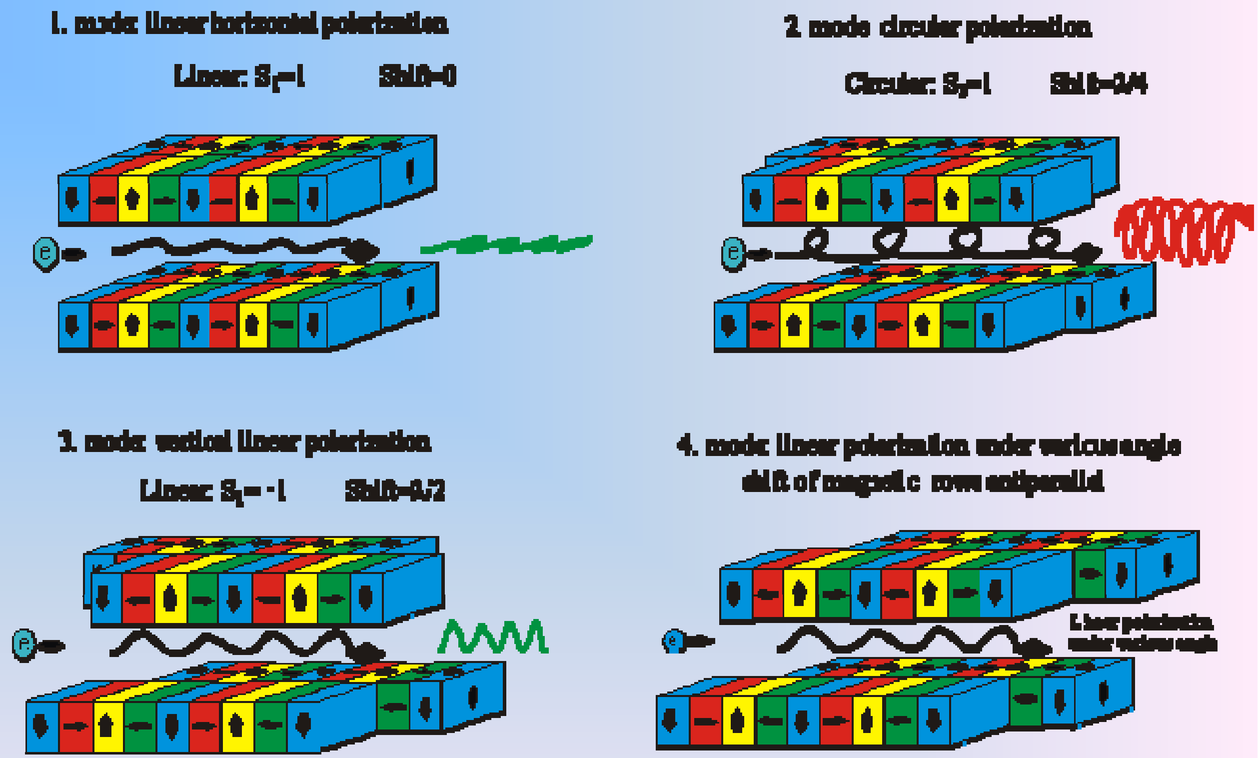
Motivation

- Polarization control is highly desirable for many x-ray FEL applications (magnetism, material studies, etc)

- Elliptical Polarization Undulators (EPUs) are routinely used in spontaneous sources

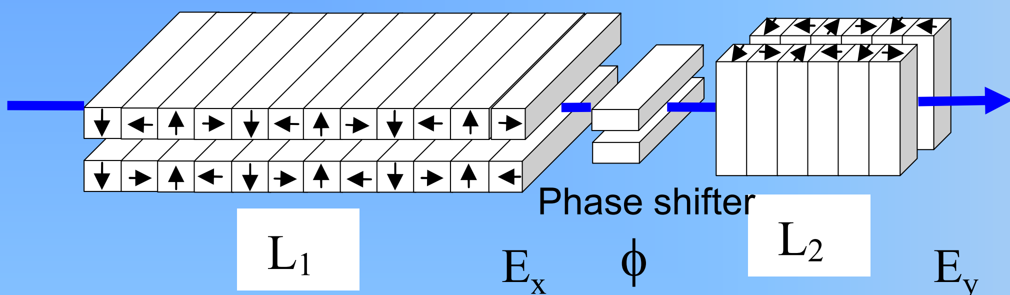


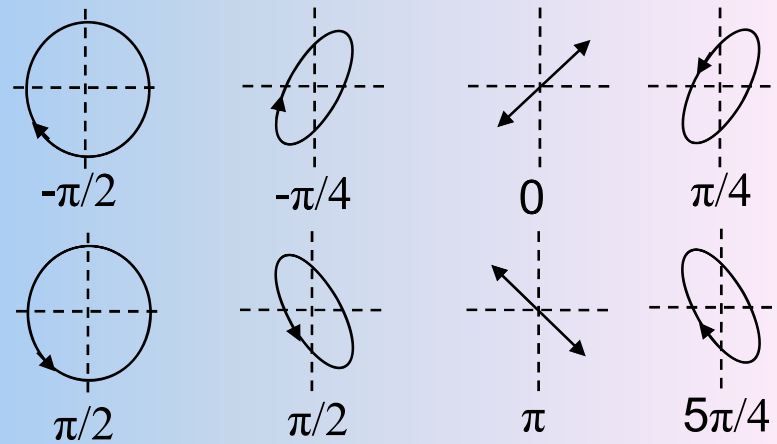
But…

- For x-ray FELs, undulator tolerance is very demanding.
- Slow EPU mechanical movement pretends fast polarization switching

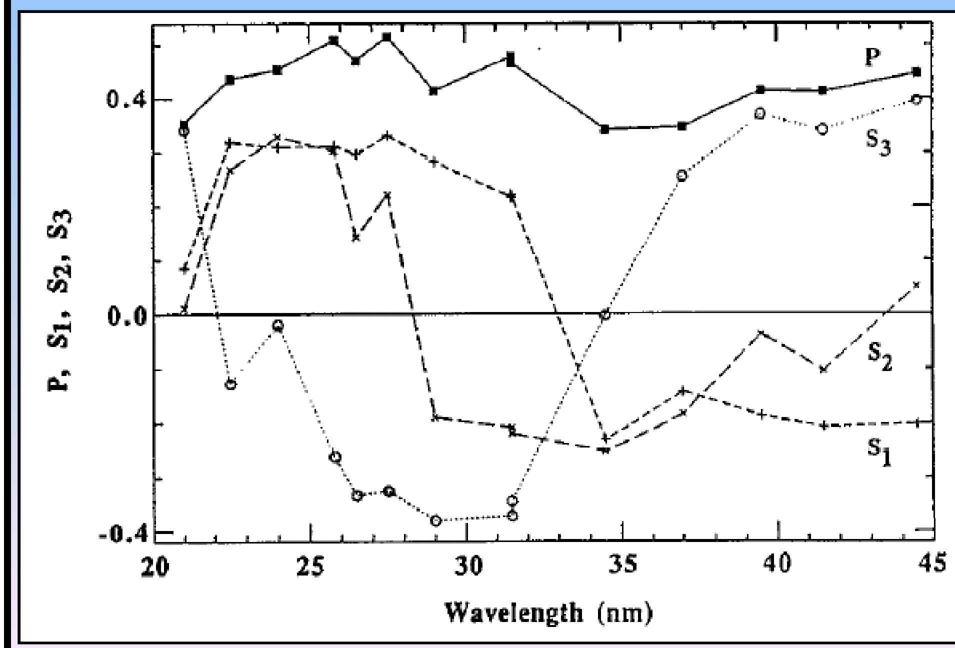
Crossed Planar Undulator

Proposed by K-J. Kim



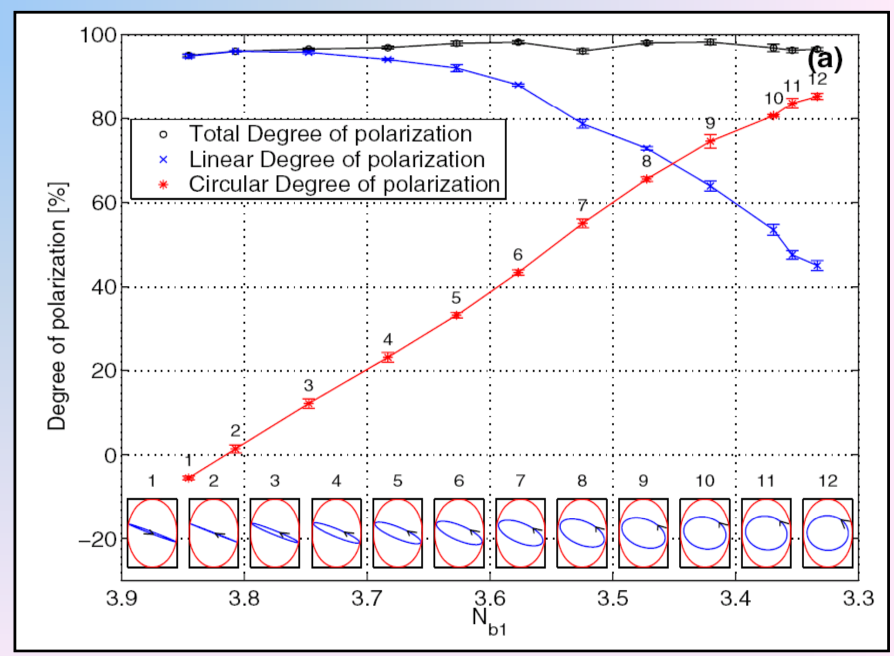


Apply to SR by monochromator (BESSY)



J. Bahrdt et al, Rev. Sci. Instrum. (1992)

Apply to FEL oscillator (DUKE)

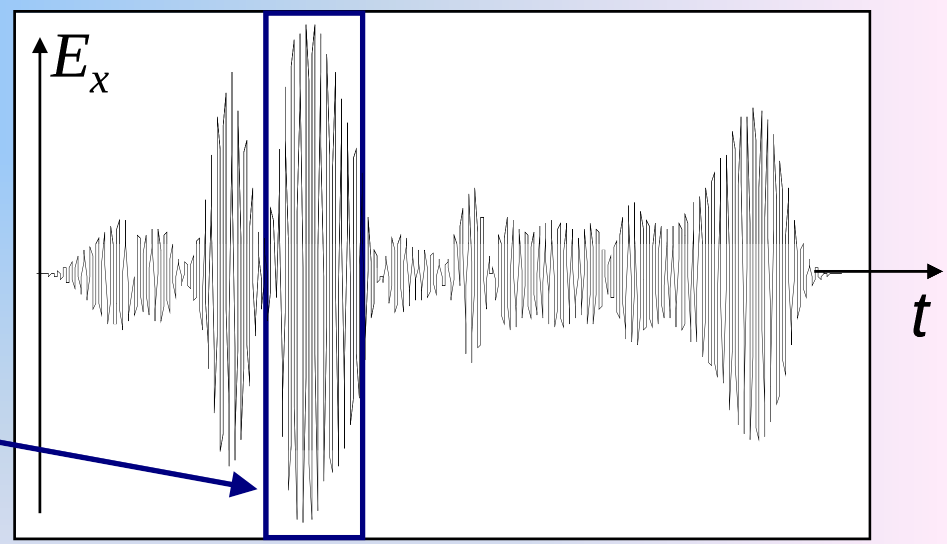


Y. Wu et al, PRL (2006)

Crossed Undulator for SASE FELs

- SASE is temporally chaotic with many random spikes
- Intensity fluctuates shot-to-shot
- Need a statistical analysis to quantify polarized SASE light using crossed undulator

$E_x(t)$ is normal SASE in first undulator, consisting many intensity spikes separately by coherence time

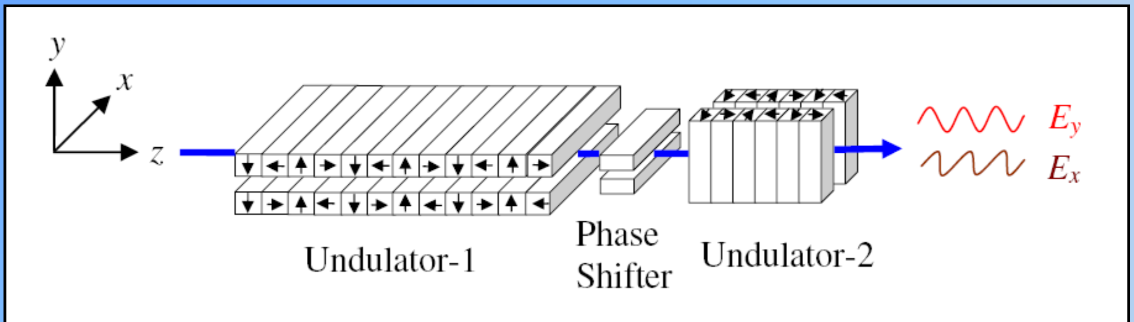


In exponential gain regime before saturation

$$\tau_c = \frac{\sqrt{\pi}}{\sigma_\omega}$$

SASE bandwidth

Vertical Field in Second Undulator



Enhanced radiation due to energy modulation

- E_y is coherent radiation generated by pre-bunched beam Undulator-2. At frequency $\omega = \nu\omega_1$, the vertical field is

$$E^y_\nu(z_2) = E^x_\nu(L_1) e^{i(\phi-\psi/2)} \mathrm{sinc}\left(\frac{\psi}{2}\right) \frac{2i}{\mu_0^2} \left[\rho k_u z_2 - \mu_0 e^{i\alpha}(\rho k_u z_2)^2\right]$$

$e^{i\phi}$ is phase delay by the shifter,

$\psi = \Delta\nu k_u z_2$, and $\Delta\nu = \nu - 1$.

μ_0 is FEL growth rate in Undulator-1

Coherent radiation from a density modulated beam

- Fourier transform $$E_y(t) = \int \frac{\omega_1 d\nu}{\sqrt{2\pi}} E^y_\nu(z_2 = L_2) e^{i\Delta\nu[(k_1+k_u)(L_1+L_2)-\omega_1 t]}$$

Coherency Matrix

- State of polarization can be described by the coherency matrix

$$\mathrm{J} = \begin{bmatrix} \langle E_x(t)E_x^*(t)\rangle & \langle E_x(t)E_y^*(t)\rangle \\ \langle E_y(t)E_x^*(t)\rangle & \langle E_y(t)E_y^*(t)\rangle \end{bmatrix}$$

- Define first order correlation function between E_x and E_y :

$$g_{xy} \equiv \frac{\langle E_x(t)E_y^*(t)\rangle}{[\langle |E_x(t)|^2\rangle\langle |E_y(t)|^2\rangle]^{1/2}}$$

- With equal *x* & *y* intensity

$$\mathbf{J} = \bar{I}\begin{bmatrix} 1 & |g_{xy}|e^{i\theta} \\ |g_{xy}|e^{-i\theta} & 1 \end{bmatrix}$$

Right-hand circular pol.

$$\mathrm{J} = \frac{\overline{I}}{2}\begin{bmatrix} 1 & j \\ -j & 1 \end{bmatrix}$$

Left-hand circular pol.

$$\mathrm{J} = \frac{\overline{I}}{2}\begin{bmatrix} 1 & -j \\ j & 1 \end{bmatrix}$$

Degree of polarization

$$\boxed{P = |g_{xy}|}$$

Degree of Polarization

- Equal x & y intensity in exponential gain regime requires

$$L_2 \approx 1.3 L_G\,, \quad \text{where} \quad L_G = \frac{\lambda_u}{4\pi\sqrt{3}\rho}$$

- Calculate correlation function by

$$\begin{aligned} g_{xy} &= \lim_{T\to\infty} \frac{1}{\bar{I}T} \int_{-T/2}^{T/2} dt\, E_x(t) E_y^*(t) \\ &= \frac{1}{\bar{I}T} \int_{-\infty}^{\infty} \omega_1 d\nu\, E_\nu^x(L_1) E_\nu^{y*}(L_2) e^{-i\Delta\nu k_u L_2}\,, \end{aligned}$$

Assume Gaussian spectrum

$$\langle |E_\nu^x(z)|^2 \rangle = \frac{\bar{I}T}{\sqrt{2\pi}\sigma_\omega} e^{-\frac{(\Delta\nu)^2}{2\sigma_\nu^2}}$$

$$\sigma_\nu = \sigma_\omega/\omega_1 = \sqrt{\frac{9\rho}{\sqrt{3}k_u L_1}}$$

$$|g_{xy}| \approx \frac{1}{\sqrt{2\pi}} \left| \int_{-\infty}^{\infty} d\bar{\nu} \frac{\exp\left(-\frac{\bar{\nu}^2}{2} - i\frac{\bar{\nu}\sigma_\nu k_u L_2}{2}\right) \mathrm{sinc}\left(\frac{\bar{\nu}\sigma_\nu k_u L_2}{2}\right)}{1 + \left(-\frac{1}{2} + i\frac{\sqrt{3}}{2}\right)\frac{\bar{\nu}\sigma_\nu}{3\rho}} \right|$$

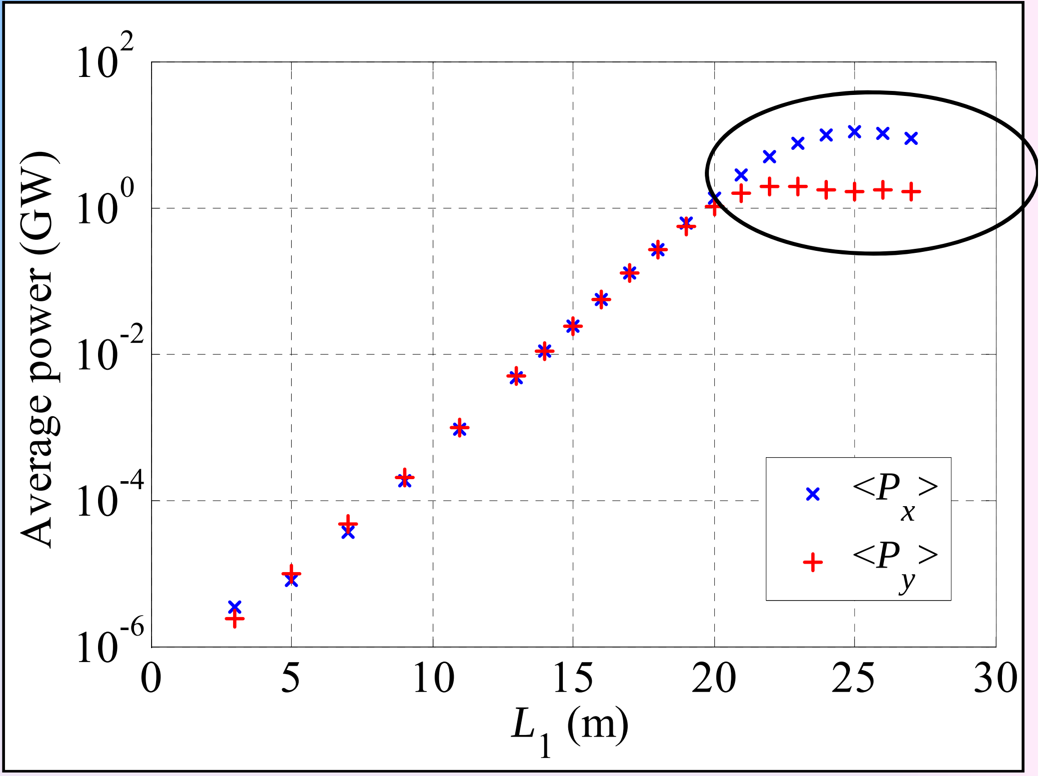
1D simulation to Compare w/ Theory

- Use soft x-ray (1.5 nm) LCLS parameters

Parameter	value	unit
electron beam energy	4.3	GeV
relative energy spread	0(0.023)	%
bunch peak current	2	kA
transverse norm. emittance	1.2	μm
average beta function	8	m
undulator period λ_u	3	cm
undulator parameter K	3.5	
FEL wavelength	1.509	nm
FEL ρ parameter	0.119	%
1D power gain length L_G	1.17	m
3D power gain length L_G^{3D}	1.48	m

$L_2 = 1.3L_G = 1.53$ m is fixed

P_x, P_y vs. L_1 (each point averaged over 200 SASE runs)



Stokes Parameters

- Polarization can be generally described by Stokes parameters
- They are related to the coherency matrix

$$
\begin{aligned}
S_0 &= J_{xx} + J_{yy}, \\
S_1 &= J_{xx} - J_{yy}, \\
S_2 &= J_{xy} + J_{yx} = 2\langle A_x(t)A_y(t)\cos(\theta(t))\rangle, \\
S_3 &= i(J_{yx} - J_{xy}) = 2\langle A_x(t)A_y(t)\sin(\theta(t))\rangle.
\end{aligned}
$$

A(t): field amplitudes; *θ(t):* relative phase between x and y

- Total degree of polarization

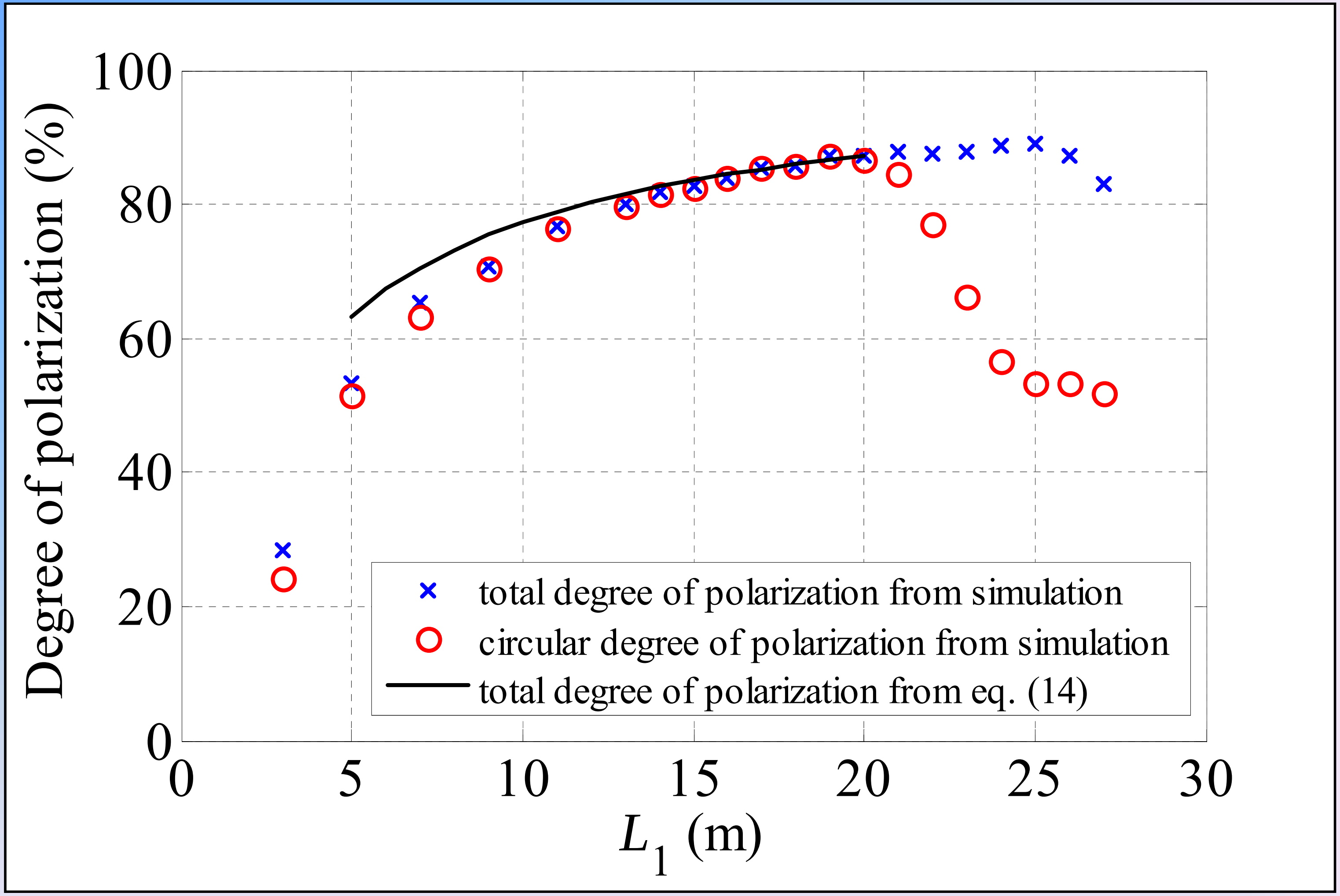
$$P = \frac{\sqrt{S_1^2 + S_2^2 + S_3^3}}{S_0}.$$

- Circular degree of polarization

$$P_c = \frac{|S_3|}{S_0}.$$

Polarization comparison

$L_2 = 1.3L_G = 1.53$ m; each point averaged over 200 SASE runs



Spontaneous and low-gain

exponential gain regime

saturation regime

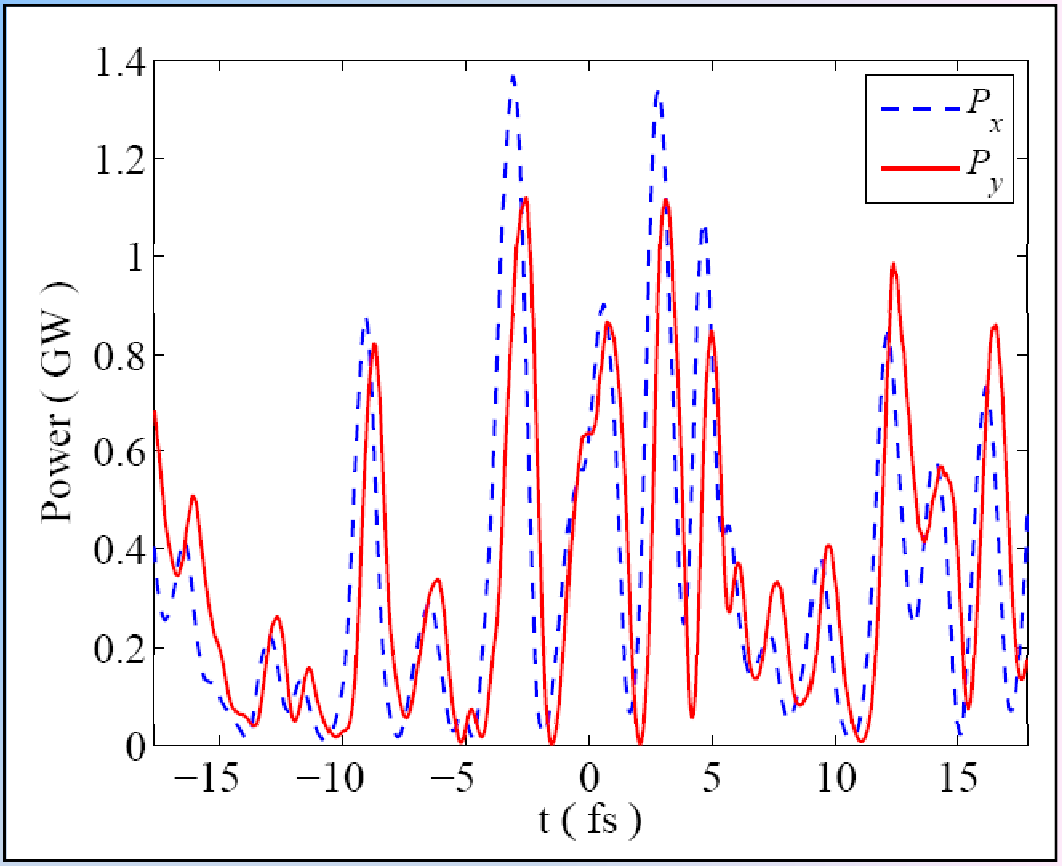
3D Simulation & Discussions

- GENESIS simulation:

L_2 = 2.0m (≈1.3L_G), L_G is the 3D gain length (1.48m);

E_x diffraction in L2 included.

Calculated polarization from simulations P ~ 0.87 at L1 = 23m.



- SASE selects a single transverse mode by exponential gain ➔ 1-D analysis holds for the guided mode;
- Diffraction effect (for free propagating E_x) is small for x-rays in a short L_2 undulator

Summary

- Arbitrarily polarized light from SASE FELs can be generated using crossed undulator

  ➔ polarization over 80% near SASE saturation;

- Analytical results agree with simulations in exponential gain regime;

- Pulsed dipole magnets may be used for fast polarization switch, ~100Hz is possible.