FEL COHERENCE BELOW THE SHOT-NOISE LIMIT AND ITS FUNDAMENTAL LIMITS

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SASE (Noise) Minimization

New Concept:

•Is it possible to enhance coherence and relax seeding power requirements for XUV FELs by reducing the input noise?

•What are the fundamental limits of FEL coherence?

Notes:

•The velocity and current shot noise terms **are** correlated at the entrance to the wiggler!

•In the art of microwave tubes, schemes for low-noise e-beam tube amplifiers were developed.

•Electron beam noise dynamics in the optical regime was observed in SLAC and Bessy.

Small Signal FEL Model (Linear Response)

$$\mathrm{n}(\underline{\mathrm{r}},\mathrm{t}) = \mathrm{n}_0(\underline{\mathrm{r}}) + \mathrm{Re}\left[\tilde{\mathrm{n}}^{\omega}(\underline{\mathrm{r}}_{\perp},\mathrm{z})\mathrm{e}^{-\mathrm{i}\omega\mathrm{t}}\right]$$

$$\underline{v}(\underline{r},t) = \underline{v}_0(\underline{r}) + \mathrm{Re}\left[\tilde{\underline{v}}^{\omega}(\underline{r}_{\perp},z)e^{-i\omega t}\right]$$

$$\underline{\mathrm{J}}(\underline{\mathrm{r}},\mathrm{t}) = \underline{\mathrm{J}}_0(\underline{\mathrm{r}}) + \mathrm{Re}\left[\tilde{\underline{\mathrm{J}}}^{\omega}(\underline{\mathrm{r}}_{\perp},\mathrm{z})\mathrm{e}^{-\mathrm{i}\omega\mathrm{t}}\right]$$

$$\underline{\mathrm{E}}(\underline{\mathrm{r}},\mathrm{t}) = \mathrm{Re}\left[\sum_{\mathrm{q}} \mathrm{C}_{\mathrm{q}}^{\omega}(\mathrm{z})\tilde{\underline{\mathrm{E}}}_{\mathrm{q}}(\underline{\mathrm{r}}_{\perp})\mathrm{e}^{\mathrm{i}k_{z_q}\mathrm{z}-\mathrm{i}\omega\mathrm{t}}\right]$$

$$\underline{\underline{\tilde{\mathrm{H}}}}_{\mathrm{q}}(\mathrm{z}) = \mathrm{Laplace}^{-1}(\underline{\underline{\overline{\mathrm{H}}}}_{\mathrm{q}}(\mathrm{s}))$$

$$\begin{bmatrix} C_q^{\omega}(z) \\ \tilde{i}^{\,\omega}(z) \\ \tilde{v}_z^{\omega}(z) \end{bmatrix} = \sum_{j=1}^{3} e^{i\delta k_j z} \cdot \begin{bmatrix} h_j^{EE} & h_j^{Ei} & h_j^{Ev} \\ h_j^{iE} & h_j^{ii} & h_j^{iv} \\ h_j^{vE} & h_j^{vi} & h_j^{iv} \end{bmatrix} \cdot \begin{bmatrix} C_q^{\omega}(0) \cdot e^{ik_z z} \\ \tilde{i}^{\,\omega}(0) \cdot e^{i(k_z + k_w)z} \\ \tilde{v}_z^{\omega}(0) \cdot e^{i(k_z + k_w)z} \end{bmatrix}$$

$$\delta\mathrm{k}_{\mathrm{j}}\left(\delta\mathrm{k}_{\mathrm{j}} - \theta - \theta_{\mathrm{pr}}\right)\left(\delta\mathrm{k}_{\mathrm{j}} - \theta + \theta_{\mathrm{pr}}\right) + \Gamma^3 = 0$$

$$\underline{\underline{\mathrm{h}}}_{\mathrm{j}} = \mathrm{Res}\left[\underline{\underline{\mathrm{H}}}(\mathrm{s})\right]\Big|_{\mathrm{s}=\mathrm{i}\delta\mathrm{k}_{\mathrm{j}}}$$

E. Dyunin, A. Gover, NIM A 593, 49 (2008)

Coherent Start-Up

$$P(L) = \left| C_q^\omega(L) \right|^2 = \left| \tilde{H}^{EE}(\omega) \cdot C_q^\omega(0) + \tilde{H}^{Ei}(\omega) \cdot \tilde{i}^{\,\omega}(0) + \tilde{H}^{Ev}(\omega) \cdot \tilde{v}_z^{\omega}(0) \right|^2$$

FEL Amplifier (Seed Radiation Injection):

$$P(L) = \left| \tilde{H}^{EE}(\omega) \right|^2 \left| C_q^\omega(0) \right|^2 = G(\theta(\omega)) P(0)$$

Pre-bunched (superradiant) FEL*:

$$P(L) = \left| \tilde{H}^{Ei}(\omega) \cdot \tilde{i}^{\,\omega}(0) + \tilde{H}^{Ev}(\omega) \cdot \tilde{v}_z^{\omega}(0) \right|^2$$

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*I.Schnitzer, A.Gover, NIM, A237, 124 (1984),
M. Arbel, A. Gover, PRL, 86, 2561 (2001), A. Gover, PR-ST-AB, **8**, 030701 (2005)

Incoherent Start-Up

Spectral Power: $$\left\langle \frac{dP(L)}{d\omega} \right\rangle_{incoh} = \frac{2}{\pi} \frac{\left\langle \left| C_q^{\omega}(L) \right|^2 \right\rangle_T}{T}$$

EM noise amplification:

$$\left\langle \frac{dP(L)}{d\omega} \right\rangle_{rad-noise} = \left| \tilde{H}^{EE}(\omega) \right|^2 \frac{\left\langle \frac{dP_{bb}(0)}{d\omega} \right\rangle_T}{T}$$

Spontaneous emission & SASE:

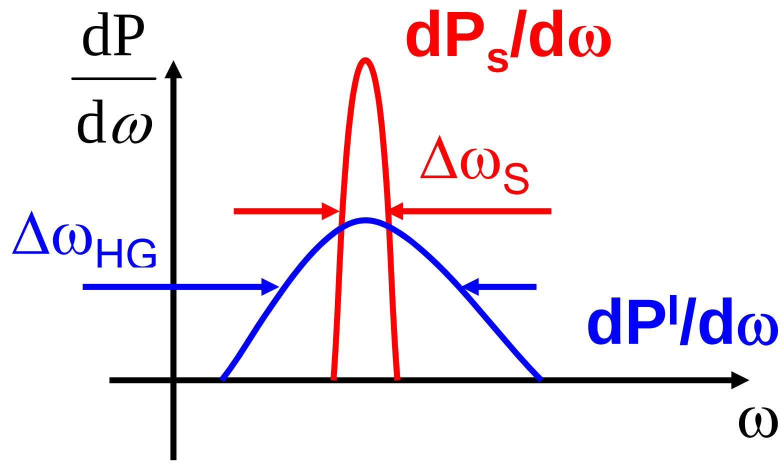
$$\left\langle \frac{dP(L)}{d\omega} \right\rangle_{beam-noise} = \frac{2}{\pi} \frac{\left\langle \left| \tilde{H}^{Ei}(\omega) \cdot \tilde{i}^{\omega}(0) + \tilde{H}^{Ev}(\omega) \cdot \tilde{v}_z^{\omega}(0) \right|^2 \right\rangle_T}{T}$$

SEED-Radiation *vs.* Current Shot-Noise (SASE)

Coherent power dominates TOTAL SASE power:

$$P_s(L) > \frac{dP^i_{noise}}{d\omega}\Delta\omega_{HG}$$

$$P_s(0) > \frac{2}{\pi}\frac{s_{pb}}{\Gamma^2} e I_b \Delta\omega_{HG}$$



$$\left(s_{pb} = \frac{1}{32 A_{em}}\sqrt{\frac{\mu}{\varepsilon}}\left(\frac{a_w}{\gamma\beta_z}\right)^2\right)$$

Coherent power dominates FILTERED SASE power:

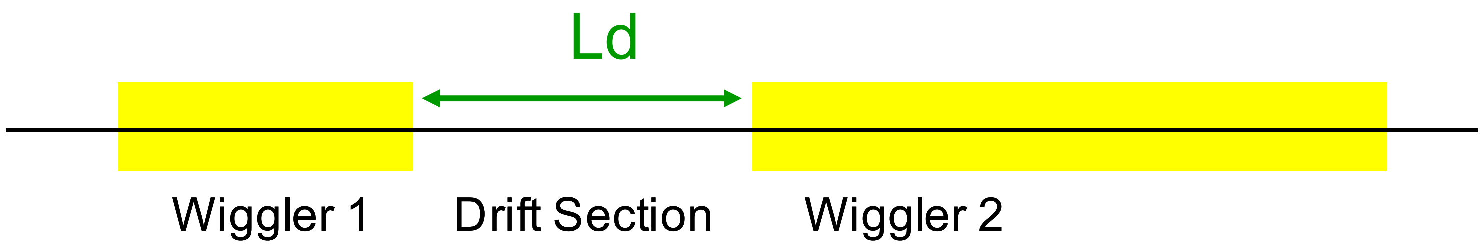
$$P_s(L) > \frac{dP^i_{noise}}{d\omega}\Delta\omega_s$$

$$P_s(0) > \frac{2}{\pi}\frac{s_{pb}}{\Gamma^2} e I_b \Delta\omega_s$$

VISA

	$\Delta\omega_{HG} \cong 2.5$ THz	$\Delta\omega_S \cong 1$ THz
$P_s(0) >$	0.5 W	0.2 W

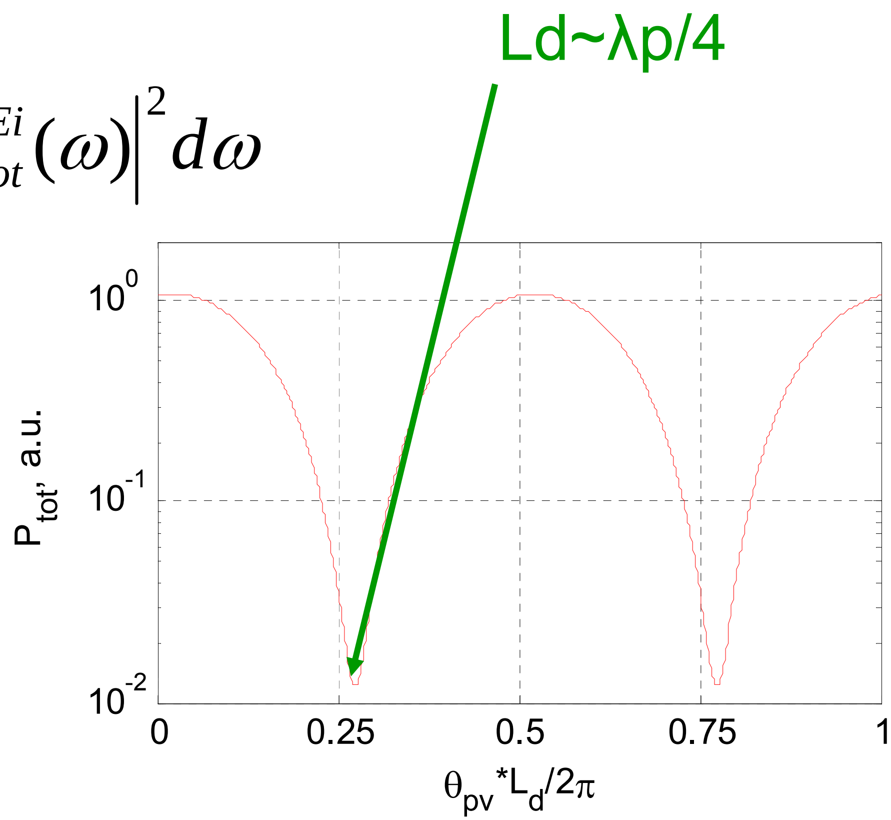
SASE Power Control



$$P_{tot} = \int \frac{dP}{d\omega} d\omega = \frac{2}{\pi} e I_b \int \left| H_{tot}^{Ei}(\omega) \right|^2 d\omega$$

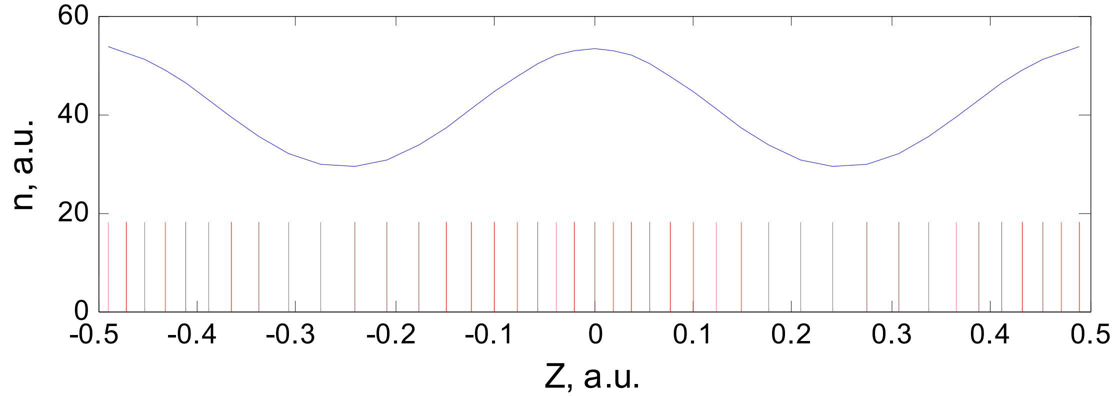
(starting from current shot-noise at z=0, $\Delta E=0$)

E. Dyunin, A. Gover, NIM **A 593**, 49 (2008)

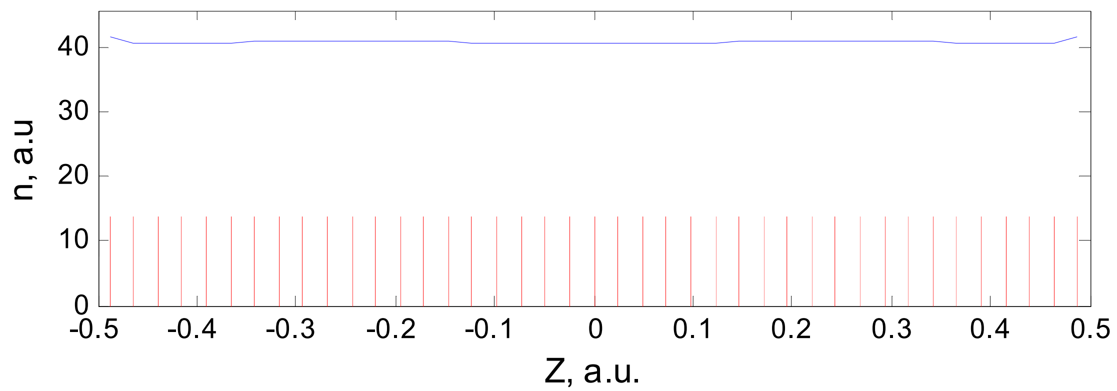


Dynamics of beam plasma longitudinal oscillation in a drift region (moving frame)

$$z = 0$$



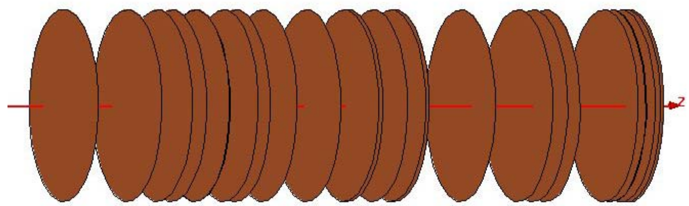
$$z = \pi / 2\theta_{pr}$$



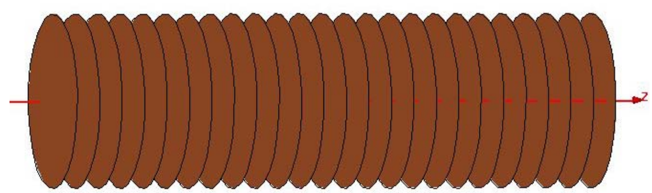
$$(\theta_{pr} = \omega'_{pr} / v_{z0})$$

Dynamics of random beam plasma longitudinal oscillation in a drift section (moving frame)

$$z = 0$$



$$z = \pi / 2\theta_{pr}$$



E-Beam Plasma oscillation dynamics in the non-radiating Sections: acceleration and drift sections.

Beam modulation transfer matrix $\underline{\underline{M}}(z)$

$$\begin{pmatrix} \tilde{i}(L) \\ \tilde{V}(L) \end{pmatrix} = \underline{\underline{M}}(L) \begin{pmatrix} I(0) \\ \tilde{V}(0) \end{pmatrix}$$

$$(\tilde{V}(z) \equiv \left[\gamma_{z0}^2(z) \gamma_0(z) v_{z0}(z) m / e \right] \tilde{v}_z(z) -$$

a relativistic "Chu's kinetic voltage")

General Non-dissipative section:

(K > 0   W > 0)

$$\begin{pmatrix} K\cos\phi & -i\sin\phi / w \\ -iw\sin\phi & \cos\phi / K \end{pmatrix} e^{i\phi_b}$$

Fast Acceleration:

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} e^{i\phi_b}$$

Drift Section:

$$\begin{pmatrix} \cos\phi_{prd} & -i\sin\phi_{prd} / W_d \\ -iW_d \sin\phi_{prd} & \cos\phi_{prd} \end{pmatrix} e^{i\phi_b}$$

$$\phi_{prd} = \theta_{prd} L \qquad \phi_b = \frac{\omega}{v_z} L \qquad W_d = \sqrt{\mu_0 / \varepsilon_0} \Big/ \left(k\theta_{pr} A_e \right)$$

Quarter Period Plasma Oscillation in a Drift Section

$\underline{\text{For} \quad \mathrm{L} = \pi/2\theta_{\mathrm{prd}}:}$

$$\underline{\underline{M}} = \begin{pmatrix} 0 & -i/W_d \\ -iW_d & 0 \end{pmatrix} e^{i\phi_b}$$

$\underline{If \quad \tilde{V}(0) = 0,}$

$$then: \qquad \tilde{I}(L) = 0$$

$$\tilde{V}(L) = -ie^{i\phi_b} W_d \tilde{I}(0)$$

Accelerator + FEL

$$\begin{pmatrix} \tilde{C}_q(L_w) \\ \tilde{i}(L_w) \\ \tilde{V}_z(L_w) \end{pmatrix} = \underline{\underline{H}}_{FEL} \begin{pmatrix} \tilde{C}_q(0) \\ \tilde{i}(0) \\ \tilde{V}_z(0) \end{pmatrix} \qquad \begin{pmatrix} \tilde{i}(0) \\ \tilde{V}(0) \end{pmatrix} = \underline{\underline{\tilde{M}}}_{acc} \begin{pmatrix} \tilde{I}(z_c) \\ \tilde{V}(z_c) \end{pmatrix}$$

$$\left(\frac{dP}{d\omega}\right)_{incoh} = \frac{2}{\pi}\frac{\left\langle \left|C_q^{\omega}\right|^2 \right\rangle_T}{T} = \frac{2}{\pi}\overline{\left|\breve{C}_q(L_w)\right|^2} =$$

$$= \frac{2}{\pi}\overline{\left|\tilde{H}_{TOT}^{Ei}\breve{i}(z_c) + \tilde{H}_{TOT}^{EV}\breve{V}(z_c)\right|^2} + \frac{2}{\pi}\left|\tilde{H}^{EE}\right|^2\overline{\left|\breve{C}_q(0)\right|^2}$$

$$\tilde{H}_{TOT}^{Ei} = \tilde{H}_{FEL}^{Ei}\tilde{M}_{acc}^{ii} + \tilde{H}_{FEL}^{EV}\tilde{M}_{acc}^{Vi}$$

$$\tilde{H}_{TOT}^{EV} = \tilde{H}_{FEL}^{Ei}\tilde{M}_{acc}^{iV} + \tilde{H}_{FEL}^{EV}\tilde{M}_{acc}^{VV}$$

With drift section preceding the FEL $\left(\phi_p = \theta_{prd} L_d\right)$

FEL: $$A \equiv \left| \frac{\tilde{H}^{EV}_{FEL}}{\tilde{H}^{Ei}_{FEL}} \right| = \frac{\omega_0 e I_b}{\Gamma \gamma_{z0}^2 \gamma_0 m v_{z0}^2}$$

Effective beam noise input powers at FEL entrance:

Current transfer — Velocity transfer <<1

$$\left(\frac{dP}{d\omega}\right)^i_{in} = \frac{2}{\pi}\frac{s_{pb}}{\Gamma^2} e I_b \left| \cos\phi_p - i e^{i\pi/3} A W_d \sin\phi_p \right|^2$$

$$\left(\frac{dP}{d\omega}\right)^V_{in} = \frac{2}{\pi}\frac{s_{pb}}{\Gamma^2} \left(\frac{\delta E_{ef}}{W_d}\right)^2 \left| \sin\phi_p + i e^{i\pi/3} A W_d \cos\phi_p \right|^2$$

$$\left(\frac{dP}{d\omega}\right)^{iV}_{in} = \frac{2}{\pi}\frac{s_{pb}}{\Gamma^2} \delta E_{ef} \frac{\delta v_c}{v_{c0}} A$$

With ^out drift section preceding the FEL $\left(\phi_p = \theta_{prd} L_d\right)$ → 0

FEL:

$$A \equiv \left| \frac{\tilde{H}^{EV}_{FEL}}{\tilde{H}^{Ei}_{FEL}} \right| = \frac{\omega_0 e I_b}{\Gamma \gamma_{z0}^{2} \gamma_0 m v_{z0}^2}$$

Effective beam noise input powers at FEL entrance:

$$\left(\frac{dP}{d\omega}\right)^{i}_{in} = \frac{2}{\pi}\frac{s_{pb}}{\Gamma^2} e I_b$$

$$\left(\frac{dP}{d\omega}\right)^{V}_{in} = \frac{2}{\pi}\frac{s_{pb}}{\Gamma^2} e I_b \cdot \left(\frac{\omega_0 \delta v_{z0}}{\Gamma v_{z0}^2}\right)^2$$

$$\left(\frac{dP}{d\omega}\right)^{iV}_{in} = \frac{2}{\pi}\frac{s_{pb}}{\Gamma^2} e I_b \cdot \left(\frac{\omega_0 \delta v_{z0}}{\Gamma v_{z0}^2}\right)\left(\frac{\delta v_{zc}}{v_{z0c}}\right)$$

Minimum beam - noise:

For $AW_d \ll 1$ Minimum noise at: $\phi_p = \pi/2 + \sqrt{3}AW_d/2$

$$\left(\frac{dP}{d\omega}\right)_{in}^{i}\Bigg|_{\phi_p \cong \pi/2} = \left(\frac{AW_d}{2}\right)^2 \left(\frac{dP}{d\omega}\right)_{in}^{i}\Bigg|_{\phi_p = 0}$$

$$\left(\frac{dP}{d\omega}\right)_{in}^{V}\Bigg|_{\phi_p \cong \pi/2} = K_v^{\,2}\left(\frac{dP}{d\omega}\right)_{in}^{i}\Bigg|_{\phi_p \cong \pi/2}$$

$$\Bigg\} \ll \left(\frac{dP}{d\omega}\right)_{in}^{i}\Bigg|_{\phi_p = 0}$$

$$K_v \equiv 2r_p \frac{\gamma_{z0}^2 \gamma_0 \beta_{z0}^3}{\gamma_{z0d}^2 \gamma_{0d} \beta_{z0d}^3} (A_e k \Gamma) \frac{\delta E / e}{\sqrt{\mu_0/\varepsilon_0}\, I_b}$$

Fundamental Coherence Limits

A Conservative of motion in a non-dissipative e-beam transport section:

$$\overline{|\breve{i}_c|^2}\;\overline{|\breve{V}_c|^2} = (\delta E_c)^2$$

Minimum input noise:

$$\left(\frac{dP_{in}}{d\omega}\right)_{min} = \frac{\delta E_c}{\pi} + \frac{1}{1+e^{-\hbar\omega/kT}}$$

Microwave/THz regime:

$$\left(\frac{dP_{in}}{d\omega}\right)_{min} = \frac{\delta E_c}{\pi} + k_B T \qquad \approx \frac{\delta E_c}{\pi}$$

Optical regime:

$$\left(\frac{dP_{in}}{d\omega}\right)_{min} = \frac{\delta E_c}{\pi} + \hbar\omega \qquad \approx \hbar\omega$$

CONCLUSION

- It is possible to adjust the e-beam current shot-noise level by controlling the longitudinal plasma oscillation dynamics.
- This can be used to enhance FEL coherence and relax seeding power requirement.
- After elimination of shot noise, IR/XUV FEL coherence is ultimately limited by the quantum input noise $dP/d\omega = \hbar\omega$.
- Warning: Coherence enhancement is limited by transverse Coulomb scattering (Boersch effect)

Conventional Theory: $\underline{\underline{M}} = I \quad (z_c = 0)$

$$\begin{pmatrix} \tilde{C}_q(L_w) \\ \tilde{i}(L_w) \\ \tilde{V}_z(L_w) \end{pmatrix} = \underline{\underline{H}}_{FEL} \begin{pmatrix} \tilde{C}_q(0) \\ \tilde{i}(0) \\ \tilde{V}_z(0) \end{pmatrix}$$

$$\begin{pmatrix} \tilde{i}(0) \\ \tilde{V}_z(0) \end{pmatrix} = \underline{\underline{\tilde{M}}}_{acc} \begin{pmatrix} \tilde{I}(z_c) \\ \tilde{V}(z_c) \end{pmatrix}$$

$$\left(\frac{dP}{d\omega}\right)_{incoh} = \frac{2}{\pi}\frac{\left\langle \left|C_q^{\omega}\right|^2 \right\rangle_T}{T} = \frac{2}{\pi}\overline{\left|\breve{C}_q(L_w)\right|^2} =$$

$$= \frac{2}{\pi}\overline{\left|\tilde{H}^{Ei}_{TOT}\breve{i}(z_c) + \tilde{H}^{EV}_{TOT}\breve{V}(z_c)\right|^2} + \frac{2}{\pi}\left|\tilde{H}^{EE}\right|^2\left|\breve{C}_q(0)\right|^2$$

$$\tilde{H}^{Ei}_{TOT} = \tilde{H}^{Ei}_{FEL}$$

$$\tilde{H}^{EV}_{TOT} = \tilde{H}^{EV}_{FEL}$$

Conventional Theory: $\underline{\underline{M}} = I \quad (z_c = 0)$

$$\left(\frac{dP}{d\omega}\right)_{in}^{i} = \frac{2}{\pi}\frac{s_{pb}}{\Gamma^2} eI_b$$

Current shot-noise

$$\left(\frac{dP}{d\omega}\right)_{in}^{V} = \left(\frac{\omega_0 \delta v_{z0}}{\Gamma v_{z0}^2}\right)^2 \left(\frac{dP}{d\omega}\right)_{in}^{i}$$

Velocity shot-noise

$$\left(\frac{dP}{d\omega}\right)_{in}^{iV} = \left(\frac{\omega_0 \delta v_{z0}}{\Gamma v_{z0}^2}\right)\left(\frac{\delta v_{zc}}{v_{z0c}}\right)\left(\frac{dP}{d\omega}\right)_{in}^{i}$$

Kinetic power noise

$$\left(\frac{dP}{d\omega}\right)_{in}^{E} = \frac{\hbar\omega}{1 - e^{-\hbar\omega/k_B T}} \simeq \hbar\omega$$

Spontaneous/black-body

At the cathode or Virtual cathode

Current shot noise: $$\overline{\left|\mathrm{i}\right|^2} = \mathrm{eI_b}$$

Velocity Noise: $$\overline{\left|\breve{\mathrm{V}}\right|^2} = \left(\mathrm{mc^2}\delta\gamma_{\mathrm{ef}}\right)^2$$

Kinetic power: $$\overline{\breve{\mathrm{i}}\breve{\mathrm{V}}^*} = \mathrm{mc^2}\left(\delta \mathrm{v_{zc}} / \mathrm{v_{zc}}\right)^2$$

$$\delta \mathrm{E_{eff}} \equiv \gamma_{\mathrm{z0c}}^2 \gamma_{\mathrm{0c}} \mathrm{v_{z0c}} \delta \mathrm{v_{zc}} \mathrm{mc^2} \qquad \left(= \mathrm{k_b T_c}\right)$$

$$\mathrm{v_{z0c}} \equiv \left\langle \mathrm{v_{zj}} \right\rangle_{\mathrm{j}}$$

$$\left(\delta \mathrm{v_{zc}}\right)^2 \equiv \left\langle \left(\mathrm{v_{zj}} - \mathrm{v_{z0c}}\right)^2 \right\rangle_{\mathrm{j}}$$

Effective Input noise

$$\left(\frac{dP}{d\omega}\right)_{in}^{eff} = \left(\frac{dP(L_w)}{d\omega}\right)_{incoh} \Bigg/ \left|\tilde{H}_{FEL}^{EE}\right|^2$$

Coherence condition:

$$[P_s(0)]_{coh} >> \left(\frac{dp}{d\omega}\right)_{in}^{eff} \Delta\omega$$

Laplace Transform: $\int_0^{\infty} f(z)e^{-sz}dz = L\{f(z)\} = \bar{f}(s)$

$$\begin{bmatrix} \bar{C}_q(s) \\ \bar{i}(s+ik_z+ik_w) \\ \bar{v}_z(s+ik_z+ik_w) \end{bmatrix} = \underline{\underline{\bar{H}}}_q(s) \begin{bmatrix} C_q(0) \\ \tilde{i}(0) \\ \tilde{v}_z(0) \end{bmatrix}$$

$$\underline{\underline{\bar{H}}}_q(s) = \frac{1}{\Delta} \begin{bmatrix} (s-i\theta)^2 + \theta_{pr}^2 & \sqrt{s_{pb}}(s-i\theta) & -I_b\sqrt{s_{pb}}\dfrac{ik_z}{v_{0z}} \\ \sqrt{1/s_{pb}}\, iQ/I_b & \dfrac{s(s-i\theta)}{I_b} & \dfrac{-isk_z}{v_{0z}} \\ -\sqrt{1/s_{pb}}\, Q(s-i\theta)/I_b & \dfrac{-Q}{I_b} & s(s-i\theta)\dfrac{k_z}{v_{0z}} \end{bmatrix}$$

$$\theta = \frac{\omega}{v_{0_z}} - k_z - k_w \qquad \Delta = s\left((s-i\theta)^2 + \theta_{pr}^2\right) - iQ$$

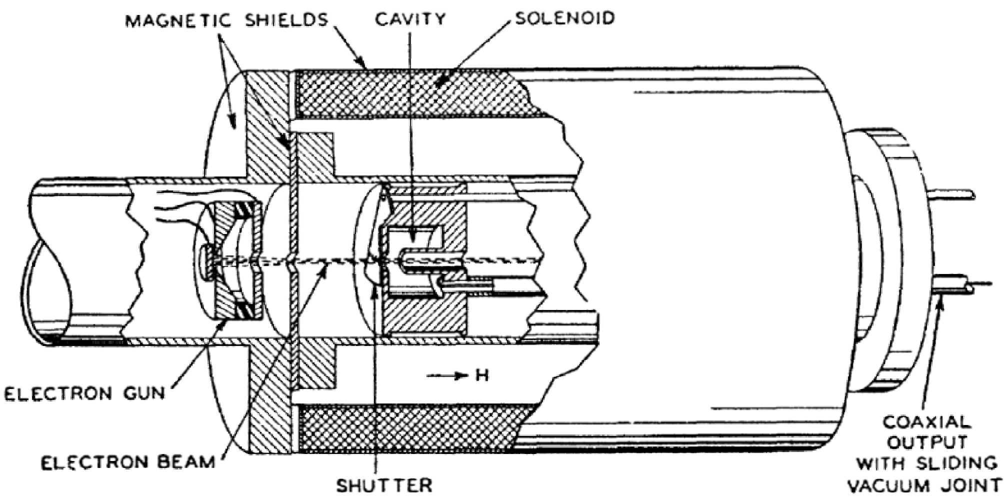
$$s_{pb} = \frac{\sqrt{\mu_0/\varepsilon_0}}{32} \cdot \left(\frac{a_w}{\gamma\beta_z}\right)^2 \cdot \frac{1}{A_{em}} \qquad Q = \frac{\pi}{2} \cdot \frac{a_w^2}{\gamma^3\gamma_z^2\beta_z^5} \cdot \frac{I_b}{I_A}\frac{k_z}{A_{em}} A_{JJ}^2 = \Gamma^3 = (2k_w\rho)^3$$

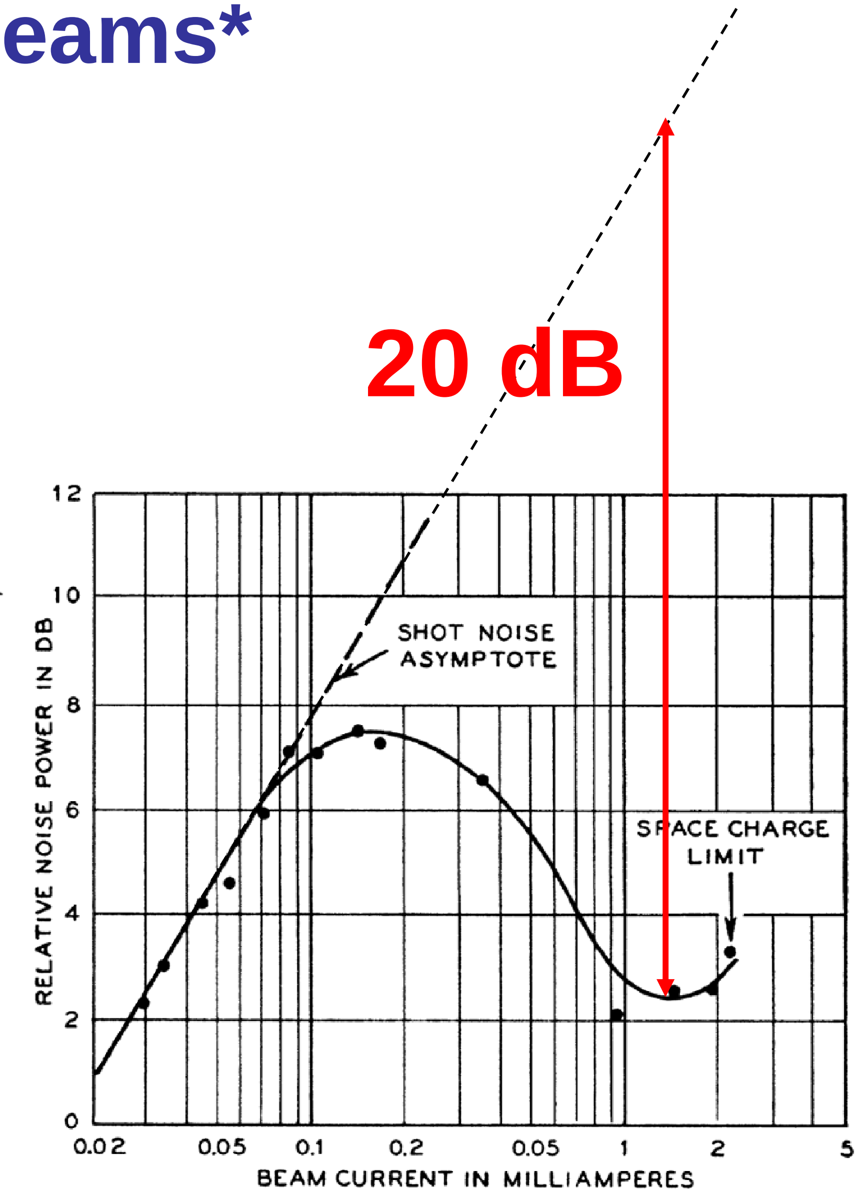
High Gain Tenuous Beam Regime

$$\underline{\underline{\tilde{H}}}_q(z) = \underline{\underline{\tilde{h}}}_1 e^{-\left(1+i/\sqrt{3}\right)(\omega-\omega_0)^2/\left(2\Delta\omega_{HG}^2\right)} e^{\frac{1}{2}\Gamma L\left(\sqrt{3}+i\right)} e^{-i\pi/12}$$

$$\underline{\underline{\tilde{h}}}_1 = \frac{1}{3}\begin{bmatrix} 1 & \sqrt{s_{pb}}\, e^{-i\pi/6} & -\frac{I_b}{4}\sqrt{s_{pb}}\,\frac{k_z}{v_{0z}} e^{i\pi/6} \\ \sqrt{1/s_{pb}}\,\frac{Q}{4I_b} e^{i\pi/6} & \frac{1}{I_b} & \frac{-k_z}{v_{0z}} e^{i\pi/3} \\ -\sqrt{1/s_{pb}}\,\frac{Q}{I_b} e^{-i\pi/6} & \frac{-Q}{4I_b} e^{-i\pi/3} & \frac{k_z}{v_{0z}} \end{bmatrix}$$

Experimental Verification of Space Charge and Transit Time Reduction of Noise in Electron Beams*





*C. C. Cutler, C. F. Quate,
Phys. Rev. **80,** 875 Dec. 1950

Current Pre-bunching *vs.* Shot-Noise (SASE)

μ-Pre-bunching:

$$P_{pb}(L,\omega) = \left|\tilde{H}^{Ei}(\omega)\right|^2 \cdot \left|\tilde{i}_{in}(\omega)\right|^2$$

Shot noise:

$$\frac{dP^{i}_{noise}(L,\omega)}{d\omega} = \frac{2}{\pi}\frac{1}{9}e^{\sqrt{3}\Gamma L} \cdot S_{pb}\frac{e}{I_b\Gamma^2}$$

$$P_{pb} > \frac{dP^{i}_{noise}}{d\omega}\Delta\omega_{HG} \quad \Rightarrow \quad \left|\tilde{i}_{in}(\omega_0)\right| > \sqrt{\frac{3^{3/4}e\lambda_w}{2\pi}\sqrt{\frac{\Gamma}{L}}\omega_0 I_b}$$

(**VISA**: i^{ω} > 0.01% from beam current I_b)

Relativistic Extention of Chu's Kinetic Voltage

$$V(z,t) = -\frac{mc^2}{e}[\gamma(z,t) - 1] =$$

$$= V_0(z) + \mathrm{Re}\left(\tilde{V}(z)e^{-i\omega t}\right)$$

$$V_0(z) = -\frac{mc^2}{e}[\gamma_0(z) - 1]$$

$$\tilde{V}(z) = \frac{dV_0(z)}{dV_{z0}}\tilde{v}_z(z) =$$

$$= -\left[\gamma_{z0}{}^2(z)\gamma_0(z)v_{z0}(z)m/e\right]\tilde{v}_z(z)$$

Non – Dissipative section

$$\begin{pmatrix} \tilde{i}(z) \\ \tilde{V}(z) \end{pmatrix} = \begin{pmatrix} \tilde{I}(z) \\ \tilde{U}(z) \end{pmatrix} e^{i\phi_b(z)}$$

$$\phi_b(z) = \omega \int_0^z dz' / v_{z0}(z')$$

$$\frac{d}{dz}\tilde{U}(z) = \frac{1}{i\omega\varepsilon_0 A_e}\tilde{I}(z)$$

$$\frac{d}{dz}\tilde{I}(z) = i\omega\varepsilon_0 A_e \theta_{pr}^2(z)\tilde{U}(z)$$

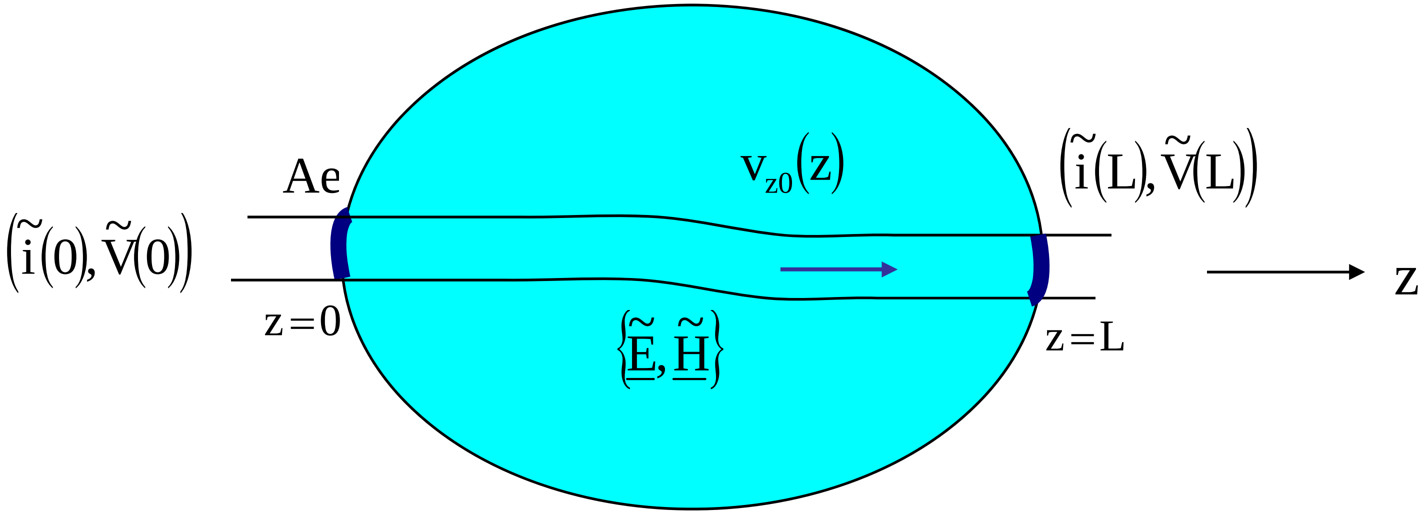
$$\theta_{pr}(z) = \omega'_{pr}(z) / v_{z0}(z) \qquad \omega'_{pr} = r_p\left[e^2 n_0 / \varepsilon_0 m \gamma_0 \gamma_{z0}{}^2\right]^{1/2}$$

$$\frac{d^2}{dz^2}\tilde{U}(z) - \theta_{pr}^2(z)\tilde{U}(z) = 0$$

Poynting Thm. For flowing charge

$$\underline{\nabla}\cdot\left(\underline{\tilde{E}}\times\underline{\tilde{H}}+\tilde{V}\tilde{J}_z{}^{*}\hat{e}_z\right)=$$

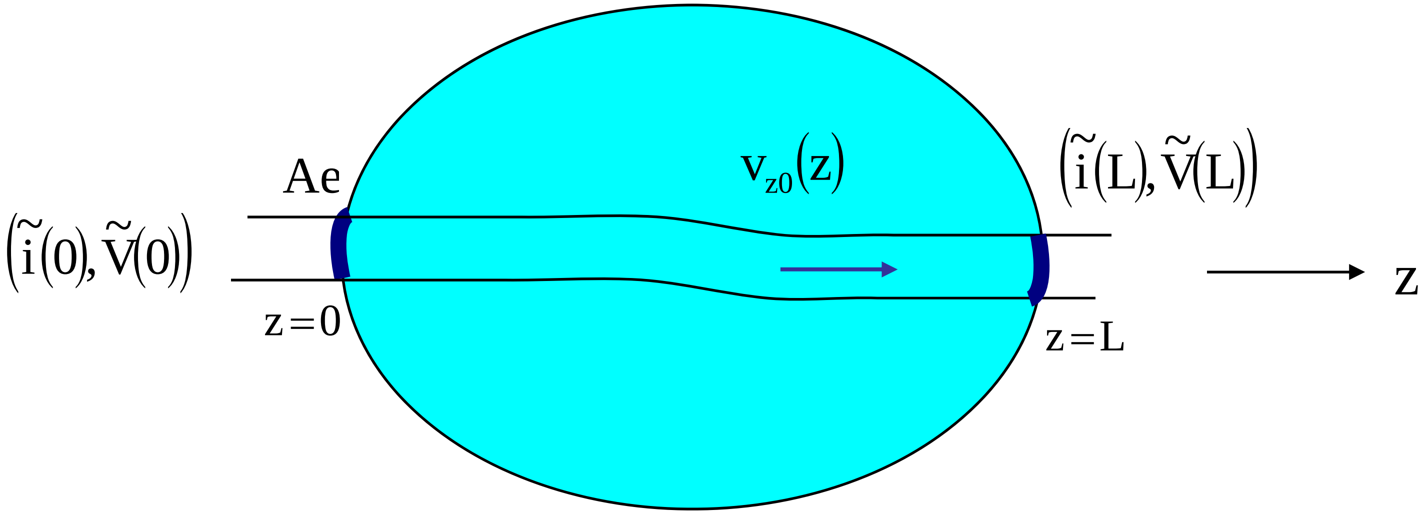
$$=i\omega\left(\mu_0\left|\underline{\tilde{H}}\right|^2-\varepsilon_0\left|\underline{\tilde{E}}\right|^2-\frac{\rho_0}{\frac{m}{e}\gamma_{z0}^2\gamma_0 v_{z0}^2}\left|\tilde{V}\right|^2\right)$$



Non – Dissipative section

Extended Chu's Kinetic power thm:

$$\mathrm{Re}\left[\tilde{i}(L)\tilde{V}(L)^{*}\right] = \mathrm{Re}\left[\tilde{i}(0)\tilde{V}(0)^{*}\right]$$



E-Beam Plasma Dynamics in the non-radiating Sections: acceleration and drift sections

Relativistic Extension of Chu's Kinetic Voltage:

$$V(z,t) = -\frac{mc^2}{e}\left[\gamma(z,t) - 1\right] =$$

$$= V_0(z) + \mathrm{Re}\left(\tilde{V}(z)e^{-i\omega t}\right)$$

$$\tilde{V}(z) = \left[\gamma_{z0}{}^2(z)\gamma_0(z)v_{z0}(z)m / e\right]\tilde{v}_z(z)$$