SMITH-PURCELL BWO WITH ELECTRON BEAM FOCUSING *

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Abstract

We have recently studied the electron beam requirement for successful operation of Smith-Purcell Backward Wave Oscillator and found that one requires a flat electron beam. Without focusing, the requirement leads to a very stringent criterion on vertical emittance. In this paper, we discuss a possible way to focus the flat beam and show that this leads to an improved performance of Smith-Purcell BWO.

INTRODUCTION

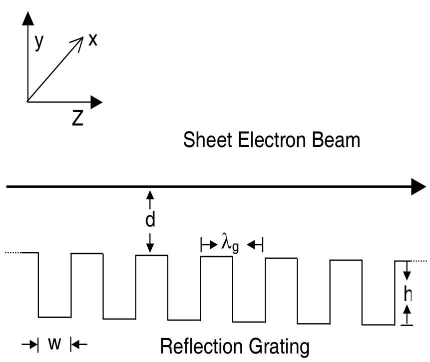

Figure 1: Schematic of a Smith-Purcell BWO.

Smith-Purcell (SP) free-electron laser in the terahertz region using a low energy electron beam is a backward wave oscillator (BWO) [1, 2]. The schematic of a SP-BWO is shown in Fig. 1. Here, a sheet electron beam is shown to be propagating at a height d from the top surface of a reflection grating, with a speed βc along the z-axis, where c is the speed of light. The grating has grooves of width w and depth h and extends uniformly to the positive and negative x-direction. The length of the grating and its period are L and λ_g respectively. In a BWO, the electron beam current is required to be higher than a threshold, known as start current for the electromagnetic oscillation to grow. The start current depends on grating parameters and electron beam parameters. Recently, we have studied the electron beam requirements for a SP-BWO and shown that one requires a flat electron beam in a SP-BWO [3]. The electron beam size and emittance requirement in the vertical direction are very stringent since the electron beam needs to maintain very close distance from the top surface of the grating. In this paper, we discuss a possible way of focusing the flat beam in a SP-BWO using undulator magnetic field and show that this can help relaxing the stringent requirement on vertical emittance and also help reducing the start current and thus improve the performance of SP-BWO.

In the next section, we discuss the focusing of flat beam in SP-BWO and show how we can use undulator magnetic field to focus the electron beam. The following section describes calculations to show how the emittance requirement can be relaxed and focusing can be utilized to improve the performance of SP-BWO. Finally, some conclusions are presented.

FOCUSSING OF FLAT ELECTRON BEAM

The start current I_s in a SP-BWO, as derived in Ref. [3] is given by

$$I_s = \mathcal{J}_s \frac{I_A \Delta x}{4\chi} \frac{\beta^4\gamma^4}{kL^3} e^{2\Gamma_0 d}, \tag{1}$$

where $\mathcal{J}_s$ is the dimensionless start current, I_A = 17 kA is the Alfvén current, χ is the residue of the singularity associated with the surface mode as defined in Ref. [2], γ is electron's energy in units of rest energy, $k = \omega/c = 2\pi/\lambda$, λ is the free-space wavelength of the surface mode, $\Gamma_0 = k/\beta\gamma$ and Δx is the half width of the electron beam in the x direction, which is chosen to be

$$\Delta x = 2\sqrt{\frac{\lambda L}{4\pi\beta_g}}, \tag{2}$$

as described in Ref. [3]. Here, β_g is the group velocity of the surface mode in units of c. The dependence of start current on d is given by the factor $e^{2\Gamma_0 d}$ in Eq. (1) and we therefore choose $d = 1/2\Gamma_0$. Assuming that the beam edge just touches the grating surface, we require that the half width Δy of the electron beam in the y direction is given by

$$\Delta y = 1/2\Gamma_0. \tag{3}$$

The unnormalized rms emittance ϵ_y in the vertical direction of the electron beam should satisfy [3]

$$\epsilon_y \le \frac{1}{(4\Gamma_0)^2 L}. \tag{4}$$

For low electron beam energy and terahertz wavelengths, the electron beam requirement in the vertical direction, as given by above formulae is very stringent. The electron beam size is typically required to be few tens of microns and the emittance requirement is few nm-rad. The electron

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beam requirement in the horizontal direction is much relaxed. The electron beam half width Δx in the x-direction is given by Eq. (2). The unnormalized rms emittance in the x direction is given by [3]

$$\epsilon_x \leq \frac{\lambda}{4\pi\beta_g}. \tag{5}$$

The electron beam size and emittance in the x direction are typically required to be in the range of few mm and few μm-rad respectively. The SP-BWO therefore requires flat beam for operation, where the ratio of emittance in the horizontal to vertical direction could be around 1000:1. In Ref. [3], we did not assume any external focusing of the flat beam. The requirements given by Eqs. (4,5) are in the absence of any external focusing. In order to relax the stringent requirement on vertical emittance, one can decrease the grating length as seen in Eq. (4), but this will increase the start current as can be seen from Eq. (1). Hence, the question arises whether we can relax requirement on vertical emittance without requiring to increase the start current with the help of external focusing. This is the question that that we want to address in this paper.

For focusing of the electron beam in SP-BWO, several authors have mentioned about the possibility of using a solenoidal magnetic field [4, 5, 6]. The envelope equation for a non-axisymmetric electron beam in solenoidal magnetic field, in Larmour frame is given by

$$\sigma_x^{''} + \left(\frac{eB_0}{2\gamma mc}\right)^2 \sigma_x - \frac{\epsilon_x^2}{\sigma_x^3} = 0, \tag{6}$$

$$\sigma_y^{''} + \left(\frac{eB_0}{2\gamma mc}\right)^2 \sigma_y - \frac{\epsilon_y^2}{\sigma_y^3} = 0, \tag{7}$$

where σ_x and σ_y are the rms electron beam radii along x and y axes respectively,'$\,'\,$' denotes differentiation with respect to z, e is the electronic charge, m is mass of the electron, B_0 is the uniform magnetic field along the z axis. The rms beam size in the x as well as y direction gets focused according to the above equation in the Larmour frame, which rotates with Larmour frequency $\omega_L = eB/2\gamma m$. As a result of the Larmour rotation, the electron beam effectively gets focused and simultaneously rotates. Consequently, the vertical electron beam size increases by a large amount. As a result of this, solenoidal magnetic field can not be used to focus flat beam in SP-BWO. However, if one uses round beam in SP-FEL, one can use solenoidal magnetic field for focusing, as has been discussed by several authors [4, 5, 6]. As per our analysis in Ref. [3], flat beam is more suitable and practical for SP-BWO and hence, we need to find out suitable mechanism for focusing of flat beam.

For focusing the flat beam in SP-BWO, we propose that one can use the undulator magnetic field, as shown in Fig. 2. The envelope equation for a non-axisymmetetric beam in an undulator having y as well as x component of

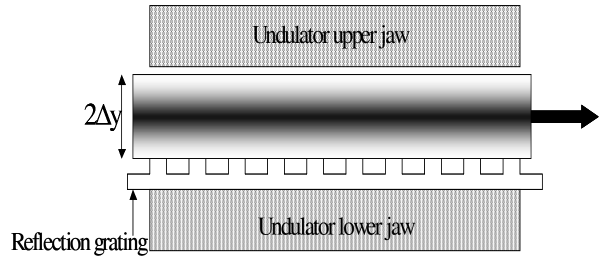

Figure 2: Schematic of focusing in a Smith-Purcell BWO using Undulator.

the magnetic field varying sinusoidally along z is given by

$$\sigma_x{}'' + \left(\frac{a_{ux}k_u}{\sqrt{2}\beta\gamma}\right)^2 \sigma_x - \frac{\epsilon_x^2}{\sigma_x^3} - \frac{I/I_A}{(\sigma_x+\sigma_y)\beta^3\gamma^3} = 0, \tag{8}$$

$$\sigma_y{}'' + \left(\frac{a_{uy}k_u}{\sqrt{2}\beta\gamma}\right)^2 \sigma_y - \frac{\epsilon_y^2}{\sigma_y^3} - \frac{I/I_A}{(\sigma_x+\sigma_y)\beta^3\gamma^3} = 0, \tag{9}$$

where $a_{ux,y} = eB_{ux,y}\lambda_u/2\pi mc$, B_{ux} and B_{uy} are the peak magnetic field in x and y directions respectively, $k_u = 2\pi/\lambda_u$, λ_u is the undulator period and I is the electron beam current. The space-charge effect has also been taken into account in the above equations. In order to make sure that the space-charge does not deteriorate the emittance, one should make sure that the space-charge term (fourth term) in these equations is smaller than the emittance term (third term). Note that such an undulator can be realized using pole shaping as proposed by Scharlemann [7]. The undulator magnetic field provides focusing in x as well as y direction as seen from the above equations where the second term specifies the betatron focusing strength. Note that the use of planar undulator for focusing sheet beam in a narrow waveguide channel has been earlier discussed by Bookse et al. [8]. The matched electron beam size σ_x^0 and σ_y^0 in the x and y directions respectively , as derived from Eqs. (8) and (9) are given by

$$\sigma_{x,y}^0 = \sqrt{\frac{\sqrt{2}\beta\gamma\epsilon_{x,y}}{a_{ux,y}k_u}}. \tag{10}$$

We choose $\sigma_x^0 = 1/2\Delta x$ and $\sigma_y^0 = 1/2\Delta y$, where Δx and Δy are given by Eqs. (2,3) such that the electron beam sizes are matched inside the undulator in both directions. For a given value of ϵ_x and ϵ_y, we can find suitable value of a_{ux} and a_{uy} such that the above equation is satisfied. The electron beam sizes are then matched and in that case, one can increase the interaction length and thus reduce the start current and improve the performance of SP-BWO. Note that in the presence of focusing fields, we do not need to satisfy Eq. (4). On the other hand, by choosing higher value of a_{uy}, one can tolerate higher emittance ϵ_y and thus the stringent requirement given by Eq. (4) can be relaxed. The maximum possible emittance that can be tolerated will depend on the maximum possible focusing strength in the given situation, and also on the maximum energy spread that can be tolerated by SP-BWO. A matched electron beam of rms size σ_y and normalized emittance ϵ_y gives rise to spread Δv_z in the longitudinal velocity v_z given by

$$\Delta v_z = \frac{v_z \epsilon_y^2}{2\sigma_y^2}, \quad (11)$$

under paraxial approximation. Assuming that the maximum energy spread that can be tolerated by SP-BWO corresponds to phase mismatch of $\pi/2$ between the electrons at the exit of the grating, meaning that $\Delta v_z \times L/v_z = \lambda_z/4$, we get the following criterion to be satisfied by ϵ_y

$$\epsilon_y < \sigma_y \sqrt{\frac{\lambda_z}{2L}}, \quad (12)$$

where $\lambda_z = \beta\lambda$ is the period of the surface wave along z-axis.

Hence, we conclude in this section that for a given set of parameters for SP-BWO, we first choose vertical beam size as given in Eq.(3) and then we choose the maximum possible focusing strength in Eq. (10) to maximize the tolerance on vertical emittance keeping in mind the constraint given by Eq. (12). In the next section, we will illustrate these using an example case.

AN EXAMPLE CASE

We now discuss an example case mentioned in Ref. [3]. The electron beam energy is taken as 35 keV, corresponding to β = 0.352 ($\beta\gamma = 0.376$). For grating, we take λ_g = 173 μm, L = 19.03 mm, w = 110 μm, h = 130 μm. For these parameters, we found in Ref. [3] that λ = 761 μm, v_g = 0.184 c and χ = 5.83 per mm. Using Eqs. (2) and (3), we find that Δx = 5 mm and Δy = 22.6 μm. The start current can be determined using Eq. (1). Note that the dimensionless start current $\mathcal{J}_s$ is a function of αL, where α is the attenuation coefficient due to the finite conductivity of the grating material. Choosing the material of the grating as copper with a DC conductivity of 5.76 $\times 10^7 \Omega$-m, and the electron relaxation time of 2.4 $\times 10^{-14}$s, it can be shown that $\alpha L = 0.59(1-i)$ for the surface mode supported by the above grating parameter. The calculation of α is done as per the procedure described in Ref. [9]. For this value of αL, it can be shown that $\mathcal{J}_s$ = 10.67. The calculation of $\mathcal{J}_s$ is discussed in Ref. [10]. Putting these numbers in Eq. (1), we obtain I_s = 37.2 mA.

If we do not use any external focusing, the normalized emittance $(\beta\gamma\epsilon_y, \beta\gamma\epsilon_x)$ in the y and x directions are required to be better than 2.5×10^{-9} m-rad and 120×10^{-6} m-rad as calculated from Eqs. (4) and (5). In order to decrease the start current, as it is obvious from Eq. (1), we can increase the grating length L, but that would require us to decrease the ϵ_y as per Eq. (4). This is because if there is no external focusing, the emittance has to be even smaller if a small electron beam size is to be maintained over a longer distance. However, if we use external focusing and choose the electron beam size to be equal to the matched beam size, the grating length can be increased to reduce the requirement on start current without requiring to reduce the vertical emittance. The peak undulator magnetic field required in the vertical direction such that the rms matched size σ_y = 11.3 μm as required, is obtained to be 0.47 kG using Eq. (10). This kind of magnetic field is practically possible to generate using pure permanent magnet based undulator in Halbach configuration. The peak magnetic field in the Halbach configuration with four magnets per period in a planar undulator is given by [11]

$$B_{uy} = 1.43 B_r \exp(-\frac{\pi g}{\lambda_u}), \quad (13)$$

where g is the gap between the jaws of the undulator and B_r is the remanent magnetic field of the magnetic material. Choosing B_r = 1.2 T for NdFeB magnets, one needs to choose g = 2.3 mm to obtain B_{uy} = 0.47 kG in the above equation assuming λ_u = 2 mm. By putting a reflection grating of thickness 1.1 mm (along the y direction) at the top surface of the lower jaw of the undulator as shown in Fig. 2, one can pass the electron beam of half width 22.6 μm grazing over the grating top surface such that the electron beam passes through the undulator axis. The electron beam can thus be focused in the vertical direction over a long distance using this kind of undulator. If we increase the grating length by two times by choosing L = 38.06 mm, with the same emittance, we can maintain the matched beam size over this entire length. The attenuation factor αL increases by two times and we get αL = 1.18 $(1-i)$. For this value of αL, $\mathcal{J}_s$ = 14.43 [10]. For this case, Δx increases by $\sqrt{2}$ times. Putting all these in Eq. (1), we find that the start current reduces to 8.9 mA. This is a significant reduction in the start current with the same electron beam emittance, but by increasing the grating length if the required undulator focusing is provided. The build-up of power for an electron beam current of 12 mA is shown in Fig. 3, where calculations have been done using nonlinear numerical simulation [2]. It is seen that the power saturates at around 460 mW.

Note that here we have not taken the effect of energy spread present in the electron beam. As we increase the grating length, the deteriorating effect of energy spread becomes more prominent [12]. Also, the effect of attenuation becomes more prominent as we increase the grating length. The start current decreases due to inverse cubic dependence on grating length in Eq. (1), but at the same time, it increases due to attenuation and energy spread effects becoming more important as the length is increased. The optimum grating length will therefore dependent on all these considerations.

The requirement of the magnetic field in the horizontal direction is much more relaxed. Using Eq. (10), the peak magnetic field in the horizontal direction required such that horizontal beam size is equal to the matched horizontal beam size is obtained to be 9.65 G, which is very modest and can be obtained by pole shaping [7]. Note that this

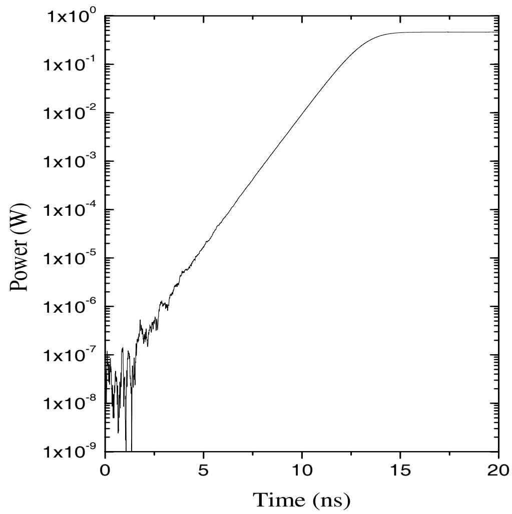

Figure 3: Evolution of power in SP-BWO for the parameters mention in the text. The electron beam current is assumed to be 12 mA.

calculation assumes a normalized rms horizontal emittance of 2.5 $\times 10^{-6}$ m-rad, following Ref. [3].

It is interesting to point out that a peak undulator magnetic field much higher than that discussed in the previous paragraphs can also be produced. Using hybrid undulator, with period as small as 2.7 mm, it is possible to generate peak undulator field as high as 5 kG as discussed in Ref. [13]. In this way, as it can be seen from Eq. (10), the emittance requirement in the vertical direction can be relaxed by an order of magnitude. It can be easily checked that this relaxed emittance still satisfiers the criterion given by Eq. (12). The relaxation in the vertical emittance by an order of magnitude for the example case described here is very significant. As proposed in Ref. [3], without external focusing, one requires a flat electron beam having normalized vertical and horizontal emittance of 2.5 $\times$ 10^{-9} m-rad and 2.5 $\times$ 10^{-6} m-rad respectively for this example case. This implies generation of a flat beam having a ratio of 1000:1 for horizontal and vertical emittances. With the relaxed criterion on vertical emittance using undulator focusing discussed in this paper, this ratio comes down to 100:1. It is interesting to point out that a flat beam of this kind of emittance ratio has already been demonstrated [14]. Also, the emittance of the round beam that needs to be generated for the flat beam generation reduces significantly. It therefore seems feasible to generate such electron beams for the realization of SP-BWO.

CONCLUSIONS

In this paper, we have discussed that the flat electron beam required in a SP-BWO can be kept focused in the interaction region with the help of undulator magnetic field. This improves the performance of SP-BWO and also relaxes the emittance requirement in the vertical direction. Use of undulator focusing is therefore expected to play an important role in the operation of SP-BWOs.

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